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3D Goursat problem in the non-classical treatment for Manjeron generalized equation with non-smooth coefficients

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Abstract: In this paper substantiated for a Manjeron generalized equation with non-smooth coefficients a three dimensional Goursat problem -3D Goursat problem with non-classical boundary conditions is considered, which requires no matching conditions. Equivalence of these conditions three dimensional boundary condition is substantiated classical, in the case if the solution of the problem in the isotropic S. L. Sobolev's space is found. The considered equation as a hyperbolic equation generalizes not only classic equations of mathematical physics (heat-conductivity equations, string vibration equation) and also many models differential equations (telegraph equation, Aller's equation, moisture transfer generalized equation, Manjeron equation, Boussinesq - Love equation and etc.). It is grounded that the 3D Goursat boundary conditions in the classic and non-classic treatment are equivalent to each other. Thus, namely in this paper, the non-classic problem with 3D Goursat conditions is grounded for a hyperbolic equation of sixth order. For simplicity, this was demonstrated for one model case in one of S.L. Sobolev isotropic space.$W_p^{(2,2,2)}(G)$

Keywords: 3D Goursat Problem, Manjeron Generalized Equation, Hyperbolic Equation, Equation with Non-Smooth Coefficients

1. Introduction

Hyperbolic equations are attracted for sufficiently adequate description of a great deal of real processes occurring in the nature, engineering and etc. In particular, many processes arising in the theory of fluid filtration in cracked media are described by non-smooth coefficient hyperbolic equations.

Urgency of investigations conducted in this field is explained by appearance of local and non-local problems for non-smooth coefficients equations connected with different applied problems. Such type problems arise for example, while studying the problems of moisture, transfer in soils, heat transfer in heterogeneous media, diffusion of thermal neutrons in inhibitors, simulation of different biological processes, phenomena and etc.

In the present paper, here consider three dimensional Goursat problem- 3D Goursat problem for sixth order equation with non-smooth coefficients. The coefficients in this hyperbolic equation are not necessarily differentiable; therefore, there does not exist a formally adjoint differential equation making a certain sense. For this reason, this question cannot be investigated by the well-known methods using classical integration by parts and Riemann functions or classical-type fundamental solutions. The theme of the present paper, devoted to the investigation 3D Goursat problem for sixth order differential equations of hyperbolic type, according to the above-stated is very actual for the solution of theoretical and practical problems. From this point of view, the paper is devoted to the actual problems of applied mathematics and physics.

2. Problem Statement

Consider Manjeron generalized equation

$$(V_{2,2,2}u)(x) \equiv D_1^2 D_2^2 D_3^2 u(x) + \sum_{\substack{i_1=0 \\ i_1+i_2+i_3<6}}^{2} \sum_{i_2=0}^{2} \sum_{i_3=0}^{2} a_{i_1,i_2,i_3}(x) D_1^{i_1} D_2^{i_2} D_3^{i_3} u(x) = Z_{2,2,2}(x) \in L_p(G) \tag{1}$$

Here $u(x) \equiv u(x_1, x_2, x_3)$ is a desired function determined on $G; a_{i_1,i_2,i_3}(x)$ are the given measurable functions on $G = G_1 \times G_2 \times G_3$, where $G_k = (0, h_k),\ k = \overline{1,3};\ Z_{2,2,2}(x)$ is a given measurable function on G; $D_k^{\xi} = \partial^{\xi} / \partial x_k^{\xi}$ is a generalized differentiation operator in S.L. Sobolev sense, D_k^0 is an identity transformation operator.

Equation (1) is a hyperbolic equation with three double real characteristics $x_k = const, k = \overline{1,3}$. Therefore, in some sense we can consider equation (1) as a pseudoparabolic equation [1]. This equation is a Boussinesq - Love generalized equation from the vibrations theory [2] and Aller's equation under mathematical modeling [3, p.261] of the moisture absorption process in biology.

In the present paper Manjeron generalized equation (1) is considered in the general case when the coefficients $a_{i_1,i_2,i_3}(x)$ are non-smooth functions satisfying only the following conditions:

$$a_{i_1,i_2,i_3}(x) \in L_p(G),\quad a_{2,i_2,i_3}(x) \in L_{\infty,p,p}^{x_1,x_2,x_3}(G),$$

$$a_{i_1,2,i_3}(x) \in L_{p,\infty,p}^{x_1,x_2,x_3}(G),\quad a_{i_1,i_2,2}(x) \in L_{p,p,\infty}^{x_1,x_2,x_3}(G),$$

$$a_{2,2,i_3}(x) \in L_{\infty,\infty,p}^{x_1,x_2,x_3}(G),\quad a_{i_1,2,2}(x) \in L_{p,\infty,\infty}^{x_1,x_2,x_3}(G),$$

$$a_{2,i_2,2}(x) \in L_{\infty,p,\infty}^{x_1,x_2,x_3}(G),\quad i_k = \overline{0,1}\ \ k = \overline{1,3};$$

Under these conditions, we'll look for the solution $u(x)$ of equation (1) in S.L. Sobolev isotropic space

$$W_p^{(2,2,2)}(G) \equiv \{u(x) : D_1^{i_1} D_2^{i_2} D_3^{i_3} u(x) \in L_p(G),\ i_k = \overline{0,2},\ k = \overline{1,3}\},$$

where $1 \le p \le \infty$. We'll define the norm in the space $W_p^{(2,2,2)}(G)$ by the equality

$$\|u\|_{W_p^{(2,2,2)}(G)} \equiv \sum_{i_1=0}^{2} \sum_{i_2=0}^{2} \sum_{i_3=0}^{2} \left\| D_1^{i_1} D_2^{i_2} D_3^{i_3} u \right\|_{L_p(G)}$$

For Manjeron generalized equation (1) we can give the classic form 3D Goursat condition in the form :

$$\begin{cases} D_1^{i_1} u(x)\big|_{x_1=x_1^0} = \varphi_1^{i_1}(x_2, x_3), (x_2, x_3) \in G_2 \times G_3, i_1 = \overline{0,1}; \\ D_2^{i_2} u(x)\big|_{x_2=x_2^0} = \varphi_2^{i_2}(x_1, x_3), (x_1, x_3) \in G_1 \times G_3, i_2 = \overline{0,1}; \\ D_3^{i_3} u(x)\big|_{x_3=x_3^0} = \varphi_3^{i_3}(x_1, x_2), (x_1, x_2) \in G_1 \times G_2, i_3 = \overline{0,1}; \end{cases} \tag{2}$$

where $\varphi_1^{i_1}(x_2, x_3), \varphi_2^{i_2}(x_1, x_3)$ and $\varphi_3^{i_3}(x_1, x_2)$ are the given measurable functions on G. It is obvious that in the case of conditions (2), in addition to the conditions

$$\varphi_1^{i_1}(x_2, x_3) \in W_p^{(2,2)}(G_2 \times G_3), i_1 = \overline{0,1};$$

$$\varphi_2^{i_2}(x_1, x_3) \in W_p^{(2,2)}(G_1 \times G_3), i_2 = \overline{0,1};$$

and

$$\varphi_3^{i_3}(x_1, x_2) \in W_p^{(2,2)}(G_1 \times G_2), i_3 = \overline{0,1};$$

the given functions should also satisfy the following agreement conditions:

$$\begin{cases} D_2^{i_2} \varphi_1^{i_1}(x_2, x_3)\big|_{x_2=x_2^0} = D_1^{i_1} \varphi_2^{i_2}(x_1, x_3)\big|_{x_1=x_1^0}, \\ D_1^{i_1} \varphi_3^{i_3}(x_1, x_2)\big|_{x_1=x_1^0} = D_3^{i_3} \varphi_1^{i_1}(x_2, x_3)\big|_{x_3=x_3^0}, \\ D_3^{i_3} \varphi_2^{i_2}(x_1, x_3)\big|_{x_3=x_3^0} = D_2^{i_2} \varphi_3^{i_3}(x_1, x_2)\big|_{x_2=x_2^0}, \\ i_k = \overline{0,1},\quad k = \overline{1,3}. \end{cases} \tag{3}$$

Consider the following non-classical boundary conditions :

$$\begin{cases} V_{i_1,i_2,i_3} u \equiv D_1^{i_1} D_2^{i_2} D_3^{i_3} u(x)/_{x=x^0} = Z_{i_1,i_2,i_3} \in R, i_k = \overline{0,1}, k = \overline{1,3}; \\ (V_{2,i_2,i_3} u)(x_1) \equiv D_1^2 D_2^{i_2} D_3^{i_3} u(x)/_{x_2=x_2^0, x_3=x_3^0} = Z_{2,i_2,i_3}(x_1) \in L_p(G_1), i_2 = \overline{0,1}, i_3 = \overline{0,1}; \\ (V_{i_1,2,i_3} u)(x_2) \equiv D_1^{i_1} D_2^2 D_3^{i_3} u(x)/_{x_1=x_1^0, x_3=x_3^0} = Z_{i_1,2,i_3}(x_2) \in L_p(G_2), i_1 = \overline{0,1}, i_3 = \overline{0,1}; \\ (V_{i_1,i_2,2} u)(x_3) \equiv D_1^{i_1} D_2^{i_2} D_3^2 u(x)/_{x_1=x_1^0, x_2=x_2^0} = Z_{i_1,i_2,2}(x_3) \in L_p(G_1), i_1 = \overline{0,1}, i_2 = \overline{0,1}; \\ (V_{2,2,i_3} u)(x_1, x_2) \equiv D_1^2 D_2^2 D_3^{i_3} u(x)/_{x_3=x_3^0} = Z_{2,2,i_3}(x_1, x_2) \in L_p(G_1 \times G_2), i_3 = \overline{0,1}; \\ (V_{i_1,2,2} u)(x_2, x_3) \equiv D_1^{i_1} D_2^2 D_3^2 u(x)/_{x_1=x_1^0} = Z_{i_1,2,2}(x_2, x_3) \in L_p(G_2 \times G_3), i_1 = \overline{0,1}; \\ (V_{2,i_2,2} u)(x_1, x_3) \equiv D_1^2 D_2^{i_2} D_3^2 u(x)/_{x_2=x_2^0} = Z_{2,i_2,2}(x_1, x_3) \in L_p(G_1 \times G_3), i_2 = \overline{0,1}; \end{cases} \tag{4}$$

3. Methodology

Therewith, the important principal moment is that the considered equation possesses nonsmooth coefficients satisfying only some p-integrability and boundedness conditions i.e. the considered hyperbolic operator $V_{2,2,2}$ has no traditional conjugated operator. In other words, the Riemann function for this equation can't be investigated by the classical method of characteristics. In the papers [4-6] the Riemann function is determined as the solution of an integral equation. This is more natural than the classical way for deriving the Riemann function. The matter is that in the classic variant, for determining the Riemann function, the rigid smooth conditions on the coefficients of the equation are required.

The Riemann's method does not work for hyperbolic equations with non-smooth coefficients.

In the present paper, a method that essentially uses modern methods of the theory of functions and functional analysis is worked out for investigations of such problems.

In the main, this method it requested in conformity to hyperbolic equations of sixth order with double characteristics. Notice that, in this paper the considered equation is a generation of many model equations of some processes (for example, heat-conductivity equations, telegraph equation, Aller's equation, moisture transfer generalized equation, Manjeron equation, string vibrations equations and etc).

If the function $u \in W_p^{(2,2,2)}(G)$ is a solution of the classical form 3D Goursat problem (1), (2), then it is also a solution of problem (1), (4) for Z_{i_1,i_2,i_3}, defined by the following equalities:

$$Z_{0,0,0} = \varphi_1^0\left(x_2^0, x_3^0\right) = \varphi_2^0\left(x_1^0, x_3^0\right) = \varphi_3^0\left(x_1^0, x_2^0\right);$$

$$Z_{1,0,0} = \varphi_1^1\left(x_2^0, x_3^0\right) = \frac{\partial \varphi_2^0(x_1, x_3)}{\partial x_1}/_{x_1=x_1^0, x_3=x_3^0} = \frac{\partial \varphi_3^0(x_1, x_2)}{\partial x_1}/_{x_1=x_1^0, x_2=x_2^0};$$

$$Z_{0,1,0} = \varphi_2^1\left(x_1^0, x_3^0\right) = \frac{\partial \varphi_1^0(x_2, x_3)}{\partial x_2}/_{x_2=x_2^0, x_3=x_3^0} = \frac{\partial \varphi_3^0(x_1, x_2)}{\partial x_2}/_{x_1=x_1^0, x_2=x_2^0};$$

$$Z_{0,0,1} = \varphi_3^1\left(x_1^0, x_2^0\right) = \frac{\partial \varphi_1^0(x_2, x_3)}{\partial x_3}/_{x_2=x_2^0, x_3=x_3^0} = \frac{\partial \varphi_2^0(x_1, x_3)}{\partial x_3}/_{x_1=x_1^0, x_3=x_3^0};$$

$$Z_{1,1,0} = \frac{\partial \varphi_1^1(x_2, x_3)}{\partial x_2}/_{x_2=x_2^0, x_3=x_3^0} = \frac{\partial \varphi_2^1(x_1, x_3)}{\partial x_1}/_{x_1=x_1^0, x_3=x_3^0} = \frac{\partial^2 \varphi_3^0(x_1, x_2)}{\partial x_1 \partial x_2}/_{x_1=x_1^0, x_2=x_2^0};$$

$$Z_{0,1,1} = \frac{\partial \varphi_2^1(x_1, x_3)}{\partial x_3}/_{x_1=x_1^0, x_3=x_3^0} = \frac{\partial \varphi_3^1(x_1, x_2)}{\partial x_2}/_{x_1=x_1^0, x_2=x_2^0} = \frac{\partial^2 \varphi_1^0(x_2, x_3)}{\partial x_2 \partial x_3}/_{x_2=x_2^0, x_3=x_3^0};$$

$$Z_{1,0,1} = \frac{\partial \varphi_1^1(x_2, x_3)}{\partial x_3}/_{x_2=x_2^0, x_3=x_3^0} = \frac{\partial \varphi_3^1(x_1, x_2)}{\partial x_1}/_{x_1=x_1^0, x_2=x_2^0} = \frac{\partial^2 \varphi_2^0(x_1, x_3)}{\partial x_1 \partial x_3}/_{x_1=x_1^0, x_3=x_3^0};$$

$$Z_{1,1,1} = \frac{\partial^2 \varphi_1^1(x_2, x_3)}{\partial x_2 \partial x_3}/_{x_2=x_2^0, x_3=x_3^0} = \frac{\partial^2 \varphi_2^1(x_1, x_3)}{\partial x_1 \partial x_3}/_{x_1=x_1^0, x_3=x_3^0} = \frac{\partial^2 \varphi_3^1(x_1, x_2)}{\partial x_1 \partial x_2}/_{x_1=x_1^0, x_2=x_2^0};$$

$$Z_{2,0,0}(x_1) = \frac{\partial^2 \varphi_2^0\left(x_1, x_3^0\right)}{\partial x_1^2} = \frac{\partial^2 \varphi_3^0\left(x_1, x_2^0\right)}{\partial x_1^2};$$

$$Z_{2,1,0}(x_1) = \frac{\partial^2 \varphi_2^1\left(x_1, x_3^0\right)}{\partial x_1^2} = \frac{\partial^3 \varphi_3^0(x_1, x_2)}{\partial x_1^2 \partial x_2}/_{x_2=x_2^0};$$

$$Z_{2,0,1}(x_1) = \frac{\partial^3 \varphi_2^0(x_1, x_3)}{\partial x_1^2 \partial x_3}/_{x_3=x_3^0} = \frac{\partial^2 \varphi_3^1\left(x_1, x_2^0\right)}{\partial x_1^2};$$

$$Z_{2,1,1}(x_1) = \frac{\partial^3 \varphi_2^1(x_1, x_3)}{\partial x_1^2 \partial x_3}/_{x_3=x_3^0} = \frac{\partial^3 \varphi_3^1(x_1, x_2)}{\partial x_1^2 \partial x_2}/_{x_2=x_2^0};$$

$$Z_{0,2,0}(x_2) = \frac{\partial^2 \varphi_1^0\left(x_2, x_3^0\right)}{\partial x_2^2} = \frac{\partial^2 \varphi_3^0\left(x_1^0, x_2\right)}{\partial x_2^2};$$

$$Z_{1,2,0}(x_2) = \frac{\partial^2 \varphi_1^1\left(x_2, x_3^0\right)}{\partial x_2^2} = \frac{\partial^3 \varphi_3^0(x_1, x_2)}{\partial x_1 \partial x_2^2}/_{x_1=x_1^0};$$

$$Z_{0,2,1}(x_2) = \frac{\partial^2 \varphi_3^1\left(x_1^0, x_2\right)}{\partial x_2^2} = \frac{\partial^3 \varphi_1^0(x_2, x_3)}{\partial x_2^2 \partial x_3}/_{x_3=x_3^0};$$

$$Z_{1,2,1}(x_2) = \frac{\partial^3 \varphi_1^1(x_2, x_3)}{\partial x_2^2 \partial x_3}/_{x_3=x_3^0} = \frac{\partial^3 \varphi_3^1(x_1, x_2)}{\partial x_1 \partial x_2^2}/_{x_1=x_1^0};$$

$$Z_{0,0,2}(x_3) = \frac{\partial^2 \varphi_1^0\left(x_2^0, x_3\right)}{\partial x_3^2} = \frac{\partial^2 \varphi_2^0\left(x_1^0, x_3\right)}{\partial x_3^2};$$

$$Z_{1,0,2}(x_3) = \frac{\partial^2 \varphi_1^1\left(x_2^0, x_3\right)}{\partial x_3^2} = \frac{\partial^3 \varphi_2^0(x_1, x_3)}{\partial x_1 \partial x_3^2}/_{x_1=x_1^0};$$

$$Z_{0,1,2}(x_3) = \frac{\partial^3 \varphi_1^0(x_2, x_3)}{\partial x_2 \partial x_3^2}/_{x_2=x_2^0} = \frac{\partial^2 \varphi_2^1\left(x_1^0, x_3\right)}{\partial x_3^2};$$

$$Z_{1,1,2}(x_3) = \frac{\partial^3 \varphi_1^1(x_2, x_3)}{\partial x_2 \partial x_3^2}/_{x_2=x_2^0} = \frac{\partial^3 \varphi_2^1(x_1, x_3)}{\partial x_1 \partial x_3^2}/_{x_1=x_1^0};$$

$$Z_{2,2,i_3}(x_1, x_2) = \frac{\partial^4 \varphi_3^{i_3}(x_1, x_2)}{\partial x_1^2 \partial x_2^2}, i_3 = \overline{0,1};$$

$$Z_{i_1,2,2}(x_2, x_3) = \frac{\partial^4 \varphi_1^{i_1}(x_2, x_3)}{\partial x_2^2 \partial x_3^2}, i_1 = \overline{0,1};$$

$$Z_{2,i_2,2}(x_1, x_3) = \frac{\partial^4 \varphi_2^{i_2}(x_1, x_3)}{\partial x_1^2 \partial x_3^2}, i_2 = \overline{0,1};$$

The inverse one is easily proved. In other words, if the function $u \in W_p^{(2,2,2)}(G)$ is a solution of problem (1), (4), then it is also a solution of problem (1), (2) for the following functions:

$$\begin{aligned}\varphi_1^{i_1}(x_2,x_3)=&\sum_{i_2=0}^{1}\sum_{i_3=0}^{1}(x_2-x_2^0)^{i_2}(x_3-x_3^0)^{i_3}Z_{i_1,i_2,i_3}+\sum_{i_2=0}^{1}(x_2-x_2^0)^{i_2}\int_{x_3^0}^{x_3}(x_3-\xi_3)Z_{i_1,i_2,2}(\xi_3)d\xi_3+\\&+\sum_{i_3=0}^{1}(x_3-x_3^0)^{i_3}\int_{x_2^0}^{x_2}(x_2-\xi_2)Z_{i_1,2,i_3}(\xi_2)d\xi_2+\int_{x_2^0}^{x_2}\int_{x_3^0}^{x_3}(x_2-\xi_2)(x_3-\xi_3)Z_{i_1,2,2}(\xi_2,\xi_3)d\xi_2d\xi_3,\ i_1=\overline{0,1};\end{aligned} \tag{5}$$

$$\begin{aligned}\varphi_2^{i_2}(x_1,x_3)=&\sum_{i_1=0}^{1}\sum_{i_3=0}^{1}(x_1-x_1^0)^{i_1}(x_3-x_3^0)^{i_3}Z_{i_1,i_2,i_3}+\sum_{i_1=0}^{1}(x_1-x_1^0)^{i_1}\int_{x_3^0}^{x_3}(x_3-\eta_3)Z_{i_1,i_2,2}(\eta_3)d\eta_3+\\&+\sum_{i_3=0}^{1}(x_3-x_3^0)^{i_3}\int_{x_1^0}^{x_1}(x_1-\eta_1)Z_{2,i_2,i_3}(\eta_1)d\eta_1+\int_{x_1^0}^{x_1}\int_{x_3^0}^{x_3}(x_1-\eta_1)(x_3-\eta_3)Z_{2,i_2,2}(\eta_1,\eta_3)d\eta_1d\eta_3,\ i_2=\overline{0,1};\end{aligned} \tag{6}$$

$$\begin{aligned}\varphi_3^{i_3}(x_1,x_2)=&\sum_{i_1=0}^{1}\sum_{i_2=0}^{1}(x_1-x_1^0)^{i_1}(x_2-x_2^0)^{i_2}Z_{i_1,i_2,i_3}+\sum_{i_1=0}^{1}(x_1-x_1^0)^{i_1}\int_{x_2^0}^{x_2}(x_2-\tau_2)Z_{i_1,2,i_3}(\tau_2)d\tau_2+\\&+\sum_{i_2=0}^{1}(x_2-x_2^0)^{i_2}\int_{x_1^0}^{x_1}(x_1-\tau_1)Z_{2,i_2,i_3}(\tau_1)d\tau_1+\int_{x_1^0}^{x_1}\int_{x_2^0}^{x_2}(x_1-\tau_1)(x_2-\tau_2)Z_{2,2,i_3}(\tau_1,\tau_2)d\tau_1d\tau_2,\ i_3=\overline{0,1};\end{aligned} \tag{7}$$

Note that the functions (5)-(7) possess one important property, more exactly, for all Z_{i_1,i_2,i_3}, the agreement conditions (3) possessing the above-mentioned properties are fulfilled for them automatically. Therefore, equalities (5)-(7) may be considered as a general kind of all the functions $\varphi_1^{i_1}(x_2,x_3)$, $\varphi_2^{i_2}(x_1,x_3)$ and $\varphi_3^{i_3}(x_1,x_2)$ satisfying the agreement conditions (3).

We have thereby proved the following assertion.

Theorem. The 3D Goursat problems of the form (1), (2) and the non-classical form (1), (4) are equivalent.

Note that the 3D Goursat problem in the non-classical treatment (1), (4) can be studied with the use of integral representations of special form for the functions $u\in W_p^{(2,2,2)}(G)$,

$$\begin{aligned}u(x)=&\sum_{i_1=0}^{1}\sum_{i_2=0}^{1}\sum_{i_3=0}^{1}(x_1-x_1^0)^{i_1}(x_2-x_2^0)^{i_2}(x_3-x_3^0)^{i_3}D_1^{i_1}D_2^{i_2}D_3^{i_3}u(x_1^0,x_2^0,x_3^0)+\\&+\sum_{i_2=0}^{1}\sum_{i_3=0}^{1}(x_2-x_2^0)^{i_2}(x_3-x_3^0)^{i_3}\int_{x_1^0}^{x_1}(x_1-\tau_1)D_1^2D_2^{i_2}D_3^{i_3}u(\tau_1,x_2^0,x_3^0)d\tau_1+\\&+\sum_{i_1=0}^{1}\sum_{i_3=0}^{1}(x_1-x_1^0)^{i_1}(x_3-x_3^0)^{i_3}\int_{x_2^0}^{x_2}(x_2-\tau_2)D_1^{i_1}D_2^2D_3^{i_3}u(x_1^0,\tau_2,x_3^0)d\tau_2+\\&+\sum_{i_1=0}^{1}\sum_{i_2=0}^{1}(x_1-x_1^0)^{i_1}(x_2-x_2^0)^{i_2}\int_{x_3^0}^{x_3}(x_3-\tau_3)D_1^{i_1}D_2^{i_2}D_3^2u(x_1^0,x_2^0,\tau_3)d\tau_3+\\&+\sum_{i_3=0}^{1}(x_3-x_3^0)^{i_3}\int_{x_1^0}^{x_1}\int_{x_2^0}^{x_2}(x_1-\tau_1)(x_2-\tau_2)D_1^2D_2^2D_3^{i_3}u(\tau_1,\tau_2,x_3^0)d\tau_1d\tau_2+\\&+\sum_{i_2=0}^{1}(x_2-x_2^0)^{i_2}\int_{x_1^0}^{x_1}\int_{x_3^0}^{x_3}(x_1-\tau_1)(x_3-\tau_3)D_1^2D_2^{i_2}D_3^2u(\tau_1,x_2^0,\tau_3)d\tau_1d\tau_3+\\&+\sum_{i_1=0}^{1}(x_1-x_1^0)^{i_1}\int_{x_2^0}^{x_2}\int_{x_3^0}^{x_3}(x_2-\tau_2)(x_3-\tau_3)D_1^{i_1}D_2^2D_3^2u(x_1^0,\tau_2,\tau_3)d\tau_2d\tau_3+\\&+\int_{x_1^0}^{x_1}\int_{x_2^0}^{x_2}\int_{x_3^0}^{x_3}(x_1-\tau_1)(x_2-\tau_2)(x_3-\tau_3)D_1^2D_2^2D_3^2u(\tau_1,\tau_2,\tau_3)d\tau_1d\tau_2d\tau_3.\end{aligned}$$

4. Result

So, the classical form 3D Goursat problems (1), (2) and in non-classical treatment (1), (4) are equivalent in the general case. However, the 3D Goursat problem in non-classical statement (1), (4) is more natural by statement than problem (1), (2). This is connected with the fact that in statement of problem (1), (4) the right sides of boundary conditions don't require additional conditions of agreement type. Note that some boundary -value problems in non-classical treatments for hyperbolic and also pseudoparabolic equations were investigated in the author's papers [7-11].

5. Discussion and Conclusions

In this paper a non-classical type 3D Goursat problem is substantiated for a Manjeron generalized equation with non-smooth coefficients and with a sixth order dominating derivative. Classic 3D Goursat conditions are reduced to non-classic 3D Goursat conditions by means of integral representations. Such statement of the problem has several advantages:

1) No additional agreement conditions are required in this statement;

2) One can consider this statement as a 3D Goursat problem formulated in terms of traces in the S.L. Sobolev isotropic space $W_p^{(2,2,2)}(G)$;

3) In this statement the considered Manjeron generalized equation is a generalization of many model equations of some processes (e.g. heat-conductivity equations , telegraph equation, Aller's equation, moisture transfer generalized equation, Boussinesq - Love equation, string vibrations equations and etc.).

References

[1] D. Colton, "Pseudoparabolic equations in one space variable", J. Different. equations, 1972, vol.12, No3, pp.559-565.

[2] A.P. Soldatov, M.Kh.Shkhanukov, "Boundary value problems with A.A.Samarsky general nonlocal condition for higher order pseudoparabolic equations", DAN USSR, 1987, vol.297, No 3. pp.547-552 .

[3] A.M. Nakhushev, Equations of mathematical biology. M.: Visshaya Shkola, 1995, 301p.

[4] S.S. Akhiev, "Fundamental solution to some local and non - local boundary value problems and their representations ", DAN USSR, 1983, vol.271, No 2, pp.265-269.

[5] S.S. Akhiev, "Riemann function equation with dominant mixed derivative of arbitrary order", DAN USSR, 1985, vol. 283, No 4, pp.783-787.

[6] V.I. Zhegalov, E.A.Utkina, "On a third order pseudoparabolic equation", Izv. Vuzov, Matem., 1999, No 10, pp.73-76.

[7] I. G. Mamedov, "Final-Boundary Value Problem in the Non-Classical Treatment for a Sixth Order Pseudoparabolic Equation", Applied and Computational Mathematics. Vol. 2, No.3, 2013, pp.96-99. doi: 10.11648/j.acm.20130203.15

[8] I. G. Mamedov, "A fundamental solution to the Cauchy problem for a fourth-order pseudoparabolic equation", Computational Mathematics and Mathematical Physics, 2009, volume 49, Issue 1, pp. 93-104.

[9] I.G. Mamedov, "Nonlocal combined problem of Bitsadze-Samarski and Samarski-Ionkin type for a system of pseudoparabolic equations ", Vladikavkazsky Matematicheskiy Zhurnal ,2014, vol.16, No 1, pp.30-41.

[10] I.G. Mamedov, "On a nonclassical interpretation of the Dirichlet problem for a fourth-order pseudoparabolic equation", Differential Equations, 2014, vol. 50, №3, pp. 415-418.

[11] I.G. Mamedov, "Goursat non - classic three dimensional problem for a hyperbolic equation with discontinuous coefficients", Vestnik Samarskogo Gosudarstvennogo Tekhnicheskogo Universiteta, 2010, No 1 (20), pp. 209-213.