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Urban Smart Growth Mathematical Model and Application

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Abstract: In view of urban sprawl brought about by urbanization development, this paper establishes a weighted comprehensive evaluation model to measure the city's smart growth status. Bordeaux is selected as the research object, and relevant data are collected and processed. The data is then substituted into the established model to solve the problem. The results show that some indicators in the city are still at a poor level. Combining the indicators with higher weights and lower scores in the evaluation results, a better urban smart growth plan was proposed. Finally, the ARIMA forecasting model is used to predict the indicators in the future more than ten years. The results verify the effectiveness of the urban smart growth plan and the potential of the plans.

Keywords: Urban Smart Growth, Weighted Comprehensive Evaluation, ARIMA Forecast

1. Introduction

Smart growth is the urban planning and management theory proposed by the American planning community in the late 1990s to cope with the disorderly spread of urban suburbanization and the resulting land resource and environmental problems [1]. The smart growth theory aims to build a city with economic prosperity, social equity, and environmentally sustainable development [2]. It integrates the use of other economic and social policies on the basis of urban spatial expansion theory [3], and plays an effective role in the urban growth management practices.

As a new idea of urban development, smart growth concept has gradually matured and made great progress in many countries. The smart growth concept emphasizes the comprehensive and coordinated development of the social economy and the resources and environment, and promotes the compact, centralized and efficient development of the city through the replacement of urban land use function, the delimit of the urban control boundary, the protection of the ecological environment and the old city transformation [4]. With the rapid development of urbanization and urban economy, cities are experiencing a historical process of rapid expansion of space scale and huge changes in spatial structure. The urban land scale is expanding rapidly, the land resources are destroyed by extensive urbanization, and the urban ecological environment and the quality of life of the residents are also adversely affected. Therefore, it is particularly important to choose a reasonable urban smart growth strategy to guide the healthy and orderly development of urbanization.

In view of the smart growth of cities, scholars have studied this issue. In the literature [5], the economy, society, environment and population of the city are used as the main indexes to measure the smart growth, and the differential equation model is set up, the dynamic changes of each index are predicted and analyzed, and the influence of each index on the smart growth of the city and the regulation effect on the sustainable development of the city in the future are studied. In the literature [6], based on the principal component analysis (PCA), the multiple impact factors that affect the realization of the smart growth of a city consist of three comprehensive indicators including economy, environment and energy. And 3E coordination degree theory is used to measure whether a city can achieve smart growth, so as to provide a reasonable basis for the smart growth of a city. The literature [7] constructs the urban smart growth evaluation index system according to the "three E principle", and uses the fuzzy comprehensive analysis and entropy method to set up a smart growth evaluation model for the selected index, and evaluates the designated city. The shrewd growth evaluation, index prediction and index potential evaluation were established by grey theory and Logistic regression, and the smart growth target of the city was predicted and evaluated. Based on the weighted comprehensive evaluation method and the ARIMA time series model, this paper analyzes the urban smart growth, and studies the impact of the indicators on the smart growth of the city and the regulation of the sustainable development of the city in the future.

2. Research Foundation

2.1. Weighted Comprehensive Evaluation

Weighting method is one of the common methods for solving multi-objective evaluation problems [8]. The basic idea of weighted evaluation method is to assign different weight coefficients to each of the selected indicators to calculate each goal. The significance of each target is the corresponding real number weights [9]. It is assumed that there are n objects to be evaluated in an evaluation system, and there are m evaluation factors to form an evaluation index set. The weighted evaluation mathematical model is

$$\max F_j = \max\{F_1,\ F_2,\ \cdots,\ F_n\} \tag{1}$$

$$F_j = \sum_{i=1}^{m} r_{ij} W_i \triangleq r_j W^T \le 1 \tag{2}$$

$$\sum_{i=1}^{m} W_i = 1 \tag{3}$$

$$W_i > 0 \tag{4}$$

Where: represents the utility value of the *i*-th indicator of the *j*-th scheme, and represents the weight of the *i*-th indicator.

2.2. Time Series Analysis

ARIMA time series model is a high accuracy prediction model, the basic idea is that some time series are a set of random variates that depend on t [10]. Although the single sequence value of the time series has is probabilistic, the change of the entire sequence has a certain regularity and can be approximated by the related mathematical model [11]. Proceed as follows:

Step 1: Take n measures on a time series to get the sample $Z_1, Z_2, \cdots, Z_n$;

Step 2: Data preprocessing: set $w_t = Z_t - \overline{Z}$ ($\overline{Z} = \frac{1}{n}\sum_{i=1}^{n} Z_i$), and get n dates: $w_1, w_2, \cdots, w_n$;

Step 3: Calculate the data: the sample self-covariance function r_k , the sample autocorrelation function ρ_k, the partial correlation function φ_k, among them: $k = 0, 1, 2, \cdots, n$;

$$r_k = \frac{w_1 w_{1+k} + w_2 w_{2+k} + \cdots + w_{n-k} w_n}{n} \tag{5}$$

$$\rho_k = \frac{r_k}{r_0} \tag{6}$$

Step 4: Stationary random process $\{Y_t\}$, satisfying the p order stochastic difference equation and the following conditions:

$$Y_t = \varphi_1 Y_{t-1} + \cdots + \varphi_p Y_{t-p} + \alpha_t + \theta_1 \alpha_{t-1} + \cdots + \theta_q \alpha_{t-q} \tag{7}$$

With lag operator that:

$$\phi(L) Y_t = \Theta(L) \alpha_{t-q} \tag{8}$$

$$\phi(L) = 1 - \varphi_1 L - \cdots - \varphi_p L^p \tag{9}$$

$$\Theta(L) = 1 - \theta_1 L - \cdots - \theta_q L^q \tag{10}$$

Among them: L is lag operator; $\phi(L)$ is the L of p order autoregressive coefficient polynomial; $\Theta(L)$ is the L of q order moving average coefficient polynomial; $\varphi_i (i = 1, 2, \cdots, p)$ is autoregressive operator; $\theta_j (j = 1, 2, \cdots, q)$ is moving average operator; $\{\alpha_t\}$ is zero-mean white noise sequence.

3. Model Establishment and Result Analysis

3.1. Weighted Comprehensive Evaluation Model

Based on the principle of smart growth, four primary indicators of growth models and trends, protection of natural resources, environmental quality, and traffic quality were selected, as well as their corresponding secondary and tertiary indicators. In order to reflect the importance of different levels of indicators, we use the analytic hierarchy process to determine the weight of the indicators.

1. Construct a judgment matrix and assign value

Make a pairwise comparison for each element on the same level about the importance of a principle on the previous level, with one to nine scales, structure comparison matrix. In accordance with the guidelines of the importance of indicators for targets, indicators for each of the two criteria for a ratio, build judgment matrix A_1, A_2 and A_3.

For the four dimensions of urban growth models and trends, natural resource protection, environmental quality, and traffic quality, a judgment matrix such as formula (11) was established.

$$A_1 = \begin{pmatrix} 1 & 2 & 2 & 3 \\ 1/2 & 1 & 1 & 3 \\ 1/2 & 1 & 1 & 3 \\ 1/3 & 1/3 & 1/3 & 1 \end{pmatrix} \quad (11)$$

For the three indicators of scale and length, land use, and centralization, we have established a judgment matrix such as formula (12).

$$A_2 = \begin{pmatrix} 1 & 1/3 & 3 \\ 3 & 1 & 7 \\ 1/3 & 1/7 & 1 \end{pmatrix} \quad (12)$$

For the three indicators of park green space, air environment, and acoustic environment, we established a judgment matrix such as formula (13).

$$A_3 = \begin{pmatrix} 1 & 3 & 5 \\ 1/3 & 1 & 3 \\ 1/5 & 1/3 & 1 \end{pmatrix} \quad (13)$$

For other selected indicators, we believe that the impact on smart growth is significant and the weights are evenly divided.

2. Calculate weight vector and consistency check

In order to determine the allowable range of the inconsistency of the judgment matrix, a consistency check is used to calculate the weight vector. The relative weights of the elements being compared for the criterion are calculated by the judgment matrix, and the consistency check is performed to define the consistency index as formula (14).

$$CI = \frac{\lambda - n}{n - 1} \quad (14)$$

The greater the *CI*, the more inconsistent. To measure the size of the *CI*, introduce a random consistency index *RI*, calculate the *CI* to get *RI*, and define the consistency ratio *CR* as in formula (15).

$$CR = \frac{CI}{RI} \quad (15)$$

Among them: when $CR<0.1$, pass the consistency test. If passed, the eigenvector corresponding to the largest characteristic root serves as a weight vector.

Based on the above analysis, the *CR* of A_1, A_2, and A_3 are shown in Table 1.

Table 1. *CR value of judgment matrix.*

Judgment matrix	*CR* value
A_1	0.0227
A_2	0.0370
A_3	0.0046

Because CR <0.1, so through the consistency test, and find the eigenvector of the largest eigenvalue of the judgment matrix at this time is the basic weight vector, the result is as the formula (16) (17) (18).

$$W_1 = \{0.4146\ \ 0.2436\ \ 0.236\ \ 0.0982\} \quad (16)$$

$$W_2 = \{0.6370\ \ 0.2583\ \ 0.1047\} \quad (17)$$

$$W_3 = \{0.2426\ \ 0.6694\ \ 0.879\} \quad (18)$$

Therefore, we can calculate the weights among the indicators. The results are shown in Table 2- Table 4.

Table 2. *Dimension's indicators weight.*

Dimension's indicators	
Index	**Weights**
Growth Models and Trends	0.4789
Natural resources protection	0.2383
Environmental Quality	0.0838
Traffic quality	0.1990

Table 3. *Secondary indicators weight.*

Secondary indicators	
Index	**Weights**
Scale and length	0.6370
Land use	0.2583
Centralized	0.1047
Policy development	0.2000
Policy effectiveness	0.8000
Parkland	0.2426
Air environment	0.6694
Acoustic environment	0.0879
Land traffic	0.5000
Land traffic	0.5000

Table 4. *Third-level weight.*

Index	Weights
Urban population development trend	0.333
Urban Employment Development Trend	0.333
Economic trends	0.333
Unutilized land ratio	0.500
Change in land use ratio	0.500
Centralization	1.000
Reasonable policy	1.000
Changes in the number of agricultural land	0.333
Agricultural Structure Change Trend	0.333
Utilization of natural resources	0.333
Per capita park green area	0.500
Built-up area greening rate	0.500
Air quality is not good	1.000
Road traffic noise average equivalent sound level	0.500
Urban area noise average equivalent sound level	0.500
Per capita road length	0.333
Number of cars	0.333
Bus line length	0.333
Number of ships	0.500
Number of aircraft	0.500

3. Multi-index weighted evaluation

After determining the weights of the indicators, a multi-index weighted evaluation method is used to evaluate the objects to be evaluated and give corresponding scores. Construct a set of evaluation grades and specify that the score be between (0,1), 0 to 0.2 is very poor, 0.2 to 0.4 is bad, 0.4 to 0.6 is fair, 0.6 to 0.8 is good, 0.8 -Excellent between -1.0

points. After the data is subjected to the same orientation and non-dimensionalization, weighted synthesis method is used to weight the individual evaluation values to obtain the total score of the evaluation.

4. Case analysis solution

We chose Bordeaux, a developed country city located on the coast, to collect data on the city's indicators. Due to the different nature of the collected data, some are positive data and some negative data. Therefore, we need to carry out the same direction of data processing, so that all data from the same point of view to explain the overall. At the same time, due to the different data sizes of different indicators, the indicators should be dimensionlessly processed to calculate their individual evaluation values. In addition, the Min-Max normalization and Arctan function transformation methods were used to correct the maximum and minimum values in the data. The single evaluation values of Bordeaux's indicators were calculated as shown in Table 5.

Table 5. *The evaluation value of Bordeaux indicators.*

Index	Score
Urban population development trend	0.718
Urban Employment Development Trend	0.707
Economic trends	0.717
Land use ratio	0.329
Change in land use ratio	0.732
Centralization	0.729
Reasonable policy	0.741
Change in the number of agricultural land	0.720
Changes in the structure of agricultural land	0.718
The exploitation of natural resources	0.679
Per capita park green area	0.867
Built-up area greening rate	0.9894
Good air quality	0.588
Road traffic noise average equivalent sound level	0.018
Urban area noise average equivalent sound level	0.976
Per capita road length	0.870
Number of cars	0.766
Bus line length	0.849
Number of ships	0.708
Number of aircraft	0.707

After data processing, we will substitute various indicators into the evaluation system of smart growth for evaluation and analysis. The final score of smart growth evaluation was obtained by weighted comprehensive calculation. The score of smart growth of Bordeaux in France was 0.6987, which was in a good level. Judging from the individual evaluation values of various indicators, most of them are above 0.6 (good), and from the overall point of view of the overall rating, the results of urban indicators have also reached a good level. This shows that the city's current growth plan is in line with the principle of smart growth, and the city's smart growth status is at a good level. However, some individual evaluation indicators are at the general level of 0.4-0.6, and some single evaluation indicators are even at a relatively low level of 0.2-0.4. Therefore, in order to obtain better evaluation scores for smart growth, a more reasonable and smart growth plan should be proposed.

3.2. Smart Growth Plan

In order to propose a more reasonable smart growth plan, consideration should be given to the weight of each indicator and the individual evaluation value of each indicator. Indicators with higher weights and lower individual scores should be given priority. Therefore, we combine the weights of individual indicators and the overall situation of individual evaluation values to rank. The results of comprehensive ranking of indicators are shown in Table 6. According to the comprehensive ranking of the indicators, we can reasonably select indicators that should focus on development.

Table 6. *Overall Ranking of Bordeaux Indicators.*

Index	Rank
Unutilized land ratio	1
Exploitation of natural resources	2
Urban Employment Trends	3
Good air quality rate	4
Economic trends	5
Urban Population Trend	6
Road traffic noise average equivalent sound level	7
Number of aircraft	8
Changes in the structure of agricultural land	9
Changes in the number of agricultural land	10
Number of ships	11
Change in land use ratio	12
Concentration	13
Policy rationality	14
Number of cars	15
Bus line length	16
Per capita park green area	17
Average length of road	18
Built-up area greening rate	19
Urban area noise average equivalent sound level	20

From the table, we can see that there are major problems in land use, natural resources, and urban employment in Bordeaux, France, and there is a need for improvement in terms of urban population, economic growth, transportation quality, and environmental quality. In combination with Bordeaux's own economic, geographical and demographic factors, we propose the following growth plan under the framework of three goals and ten smart growth principles.

(1) Rationally use land resources to increase land utilization and avoid blind expansion.

(2) Save natural resources, protect the environment, and improve people's living standards.

(3) Continue to develop the wine industry, improve the export of high-quality wines, improve trade in trade development, and increase employment opportunities.

3.3. Plan Efficiency Evaluation

In order to quantify the impact of the smart growth plan, we should change the values of the corresponding indicators of the plan, and then use the previous smart growth evaluation system to evaluate the improved indicators. For the change of the indicator value, it cannot violate the objective development law. Therefore, we use the ARIMA forecasting model, based on the forecast value of the indicator after 10 years, put forward the expected target value of the indicator, and evaluate the benefit of the smart growth plan we have established.

According to our smart growth plan, we selected eight relevant indicators, namely land unavailability, natural resources, employment, air quality, economic trends, population trends, changes in the structure of agricultural land and road traffic noise equivalent levels.

In combination with the established ARIMA forecasting model, through the previous data, the future changes in the index values for more than ten years are predicted and analyzed, and on this basis, the expected target values of important indicators in the plan are reasonably specified. The forecast results are shown in Figure 1.

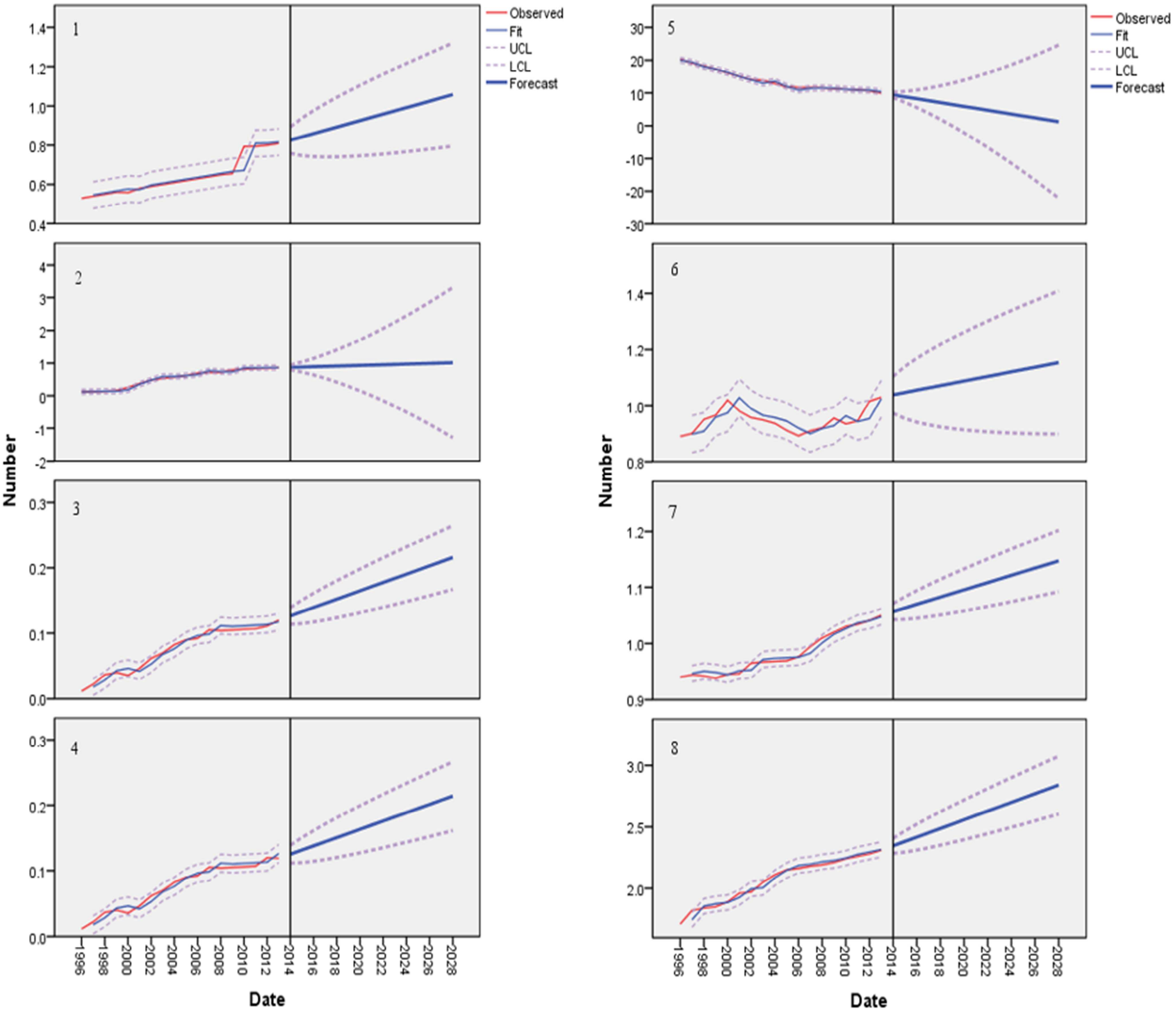

Figure 1. *Forecast result of each index.*

Where: the figures 1-8 are shown as: 1: The rate of unused land, 2: Natural resource exploitation rate, 3: Employment rate, 4: Air quality, 5: Trend of economic change, 6: Population development rate, 7: Structure change of agricultural land, 8: Road noise level

According to the results of the forecast and the proposed plan, the expected target value for the next five years will be set as the forecast value of the indicator after 10 years, and a savvy growth plan will be quantified. Through the rational quantification of the plan, we can quantify the scores of city smart growth after the implementation of the plan. Based on the predicted value of the indicator after 10 years, the expected target value of the indicator can be proposed and a new single evaluation value can be obtained as shown in Table 7.

Table 7. *Bordeaux improved evaluation value.*

Index	Score
Urban population development trend	0.717
Urban Employment Development Trend	0.709
Economic trends	0.720
Land use ratio	0.450
Change in land use ratio	0.739
Centralization	0.729

Index	Score
Reasonable policy	0.741
Change in the number of agricultural land	0.722
Changes in the structure of agricultural land	0.722
The exploitation of natural resources	0.705
Per capita park green area	0.867
Built-up area greening rate	0.974
Good air quality	0.657
Road traffic noise average equivalent sound level	0.989
Urban area noise average equivalent sound level	0.976
Per capita road length	0.870
Number of cars	0.768
Bus line length	0.849
Number of ships	0.708
Number of aircraft	0.707

It can be concluded from Table 7 that most of the individual evaluation values have reached good levels after the implementation of the formulated plan, and the lowest indicators have also reached the general level. After the plan, Bordeaux's comprehensive savvy growth evaluation score is expected to reach 0.7165. The level of growth has significantly improved.

3.4. Plan Potential Evaluation

The savvy growth plan we have developed is based on the comprehensive rankings of various indicators. When evaluating the potential of each plan, we should change the corresponding indicator values of each plan by controlling variables, and the obtained indicators should be improved. Through the evaluation system established before, the evaluation scores for smart growth after the implementation of a single plan are shown in Table 8, and the results are used to define the potential of the plan.

Table 8. *Smart growth rating after implementation of each program.*

Initial samrt growth score	Plan A	Plan B	Plan C
	Separate implementation	Separate implementation	Separate implementation
0.6987	0.7118	0.7050	0.7061

The table shows that Bordeaux's planned potential ranking should be:

(1) Make rational use of land resources to increase land utilization and avoid blind expansion;

(2) Continue to develop the wine industry, improve the export of high-quality wines, improve trade in transport development, and increase employment opportunities;

(3) Save natural resources, protect the environment and improve people's living standards.

4. Conclusion

In order to solve the problem of urban sprawl and promote urban smart growth, this paper establishes a weighted comprehensive evaluation model, combines quantitative analysis with qualitative analysis, and converts multi-level, multi-target, and difficult-to-quantify complex problems into multi-level single-target problems. According to the evaluation results, the problems existing in urban development are pointed out, and then according to the unique economic model and cultural background of the city, corresponding smart growth plans are proposed. In order to verify the effectiveness of the plan, an ARIMA forecasting model was established, and the predicted values of the selected indicators after more than a decade were obtained. The prediction results proved the rationality of the smart growth plan. However, when using the analytic hierarchy process, a judgment matrix is obtained based on expert experience, and the prediction result is subjective, easily causing some errors, and needs improvement.

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