

Electricity market and its risk management in Nigeria

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Abstract: This paper is on the development of adequate mathematical model of electricity price via Fourier series. Fourier series is the representation of a function $f(x)$ as an infinite series in sine and cosine terms. Our choice of Fourier series model for electricity price is as result of its volatility, fluctuation trends of hydro flow and poor market designs and we use actively-traded natural gas to hedge against electricity price in Nigeria. The natural gas prices are volatile but do not have a clear seasonal pattern, thus eliminating natural gas price volatility through hedging substantially reduce the electricity price, this development of logical mathematical frame work in the form of hedging tools assures an investor of his or her safety in the power sector.

Keywords: Fourier Series, Electricity Market, Seasonality, Hedging Risk

1. Introduction

In this paper we consider investors who have the intention to expand the electricity generation capacity in Nigeria. The newly liberalized electricity markets in Nigeria where, electricity is traded like every other commodity as attracted the attention of both the foreign and local investors into the power sector. The motivation for this work are those of [2] who observe the detection of market-power abuse and price manipulation while an improvement on this fact made by [4] with the hypothesis that a forward-contract seller is relatively less risk averse than buyer, subsequently, In [24], [25], [26] discovered the presence of relatively large premium corroborates of the electricity price, just to mention a few. In this paper we consider logical mathematical frame work via Fourier series model for the pricing of electricity, given as $f(x) = \frac{1}{2}a_0 + \sum_{n=1}^{\infty} a_n \cos nx + b_n \sin nx$ where the coefficients a_0, a_n, b_n , denotes the drift term and volatility respectively and $f(x)$ denotes the i.i.d. random variable and subsequently show that given a contingent claim 'S', such that $\mathbb{E} \gamma^*(S^2) < \infty$ is called attainable if there exists at least one tame self-financing trading strategy \emptyset such that

$V_{(T)}^{\emptyset} = S$ and we show that electricity market is complete (it can be hedge)

2. Mathematical Formulations

(Dynamics of electricity pricing)

Theorem 1

Given $f(x) = \frac{1}{2}a_0 + \sum_{n=0}^2 a_n \cos nx + b_n \sin nx$ where the coefficients a_0, a_n, b_n

Denotes the drift and volatility respectively and $f(x)$ denotes an i.i.d random variable where,

$W = \{W_t, t \in [0, T]\}$ is a Brownian motion defined in a complete probability space $(\Omega, \mathcal{F}, \mathbb{P})$. We will denote by $\{f_t: t \in [0, T]\}$ the filtration generated by the Brownian motion and completed by the $\mathbb{P} - null$ sets

$$f(x) = \frac{1}{2}a_0 + \sum_{n=0}^2 a_n \cos nx + b_n \sin nx \quad x \in [0, \pi] \quad (1)$$

Where $a_0 = \pi$ the drift rate and a_n, b_n the volatility including all the economic factors; where $a_n = \frac{2}{\pi n^2} (\cos n\pi - 1)$, $b_n = \frac{-2 \cos n\pi}{n}$;

And

$$f_{p^L}(x) = \left(\frac{\pi}{2}\right) + \sum_{n=0}^2 \frac{2}{\pi n^2} (\cos n\pi - 1) 1 + e^{-\frac{n^2}{2}t} + \left(\frac{-2 \cos n\pi}{n}\right) e^{\frac{n^2 t}{2}} \quad (1)$$

Proof:

$a_0 = \frac{1}{\pi} \int_0^{\pi} 2x dx$ where $f(x) = 2\pi$, we consider only $0 \leq x \leq \pi$, $0 \leq t \leq \pi$, and $0 \leq n \leq 2$

$$a_0 = \left[\frac{1}{\pi} \cdot \frac{2x^2}{2} \right]_0^{\pi} = \frac{2\pi^2}{2\pi} = \frac{\pi^2}{\pi} = \pi \quad (2)$$

$$a_n = \frac{1}{\pi} \int_0^{\pi} f(x) \cos nx dx$$

$$= \frac{1}{\pi} \int_0^\pi 2x \cos nx dx = \frac{2}{\pi} \int_0^\pi x \cos nx dx$$

$$U dv = U v \int V du$$

Let $u = x$ and $dv = \cos nx$

$$du = dx \text{ and } V = \frac{\sin nx}{n}$$

$$\frac{2}{\pi} \left[\frac{x \sin nx}{n} \Big|_0^\pi - \int_0^\pi \frac{\sin nx}{n} dx \right] = \frac{2}{\pi} \left[\frac{x \sin nx}{n} \Big|_0^\pi + \frac{\cos nx}{n^2} \Big|_0^\pi \right] = \frac{2 \cos nx}{\pi n^2} \Big|_0^\pi$$

(since $\frac{x \sin nx}{n} \Big|_0^\pi = 0$) (3)

$$v = \frac{-\cos nx}{n}, \Rightarrow \frac{2}{\pi} \left[\frac{-x \cos nx}{n} \Big|_0^\pi + \frac{1}{n} \int_0^\pi \cos nx dx \right] = \frac{2}{\pi} \left[\frac{-x \cos nx}{n} \Big|_0^\pi + \frac{\sin nx}{n^2} \Big|_0^\pi \right] = \frac{-2}{\pi n} [x \cos nx]_0^\pi \quad \left(\text{since } \frac{1}{n^2} [\sin nx]_0^\pi \right) = 0 \Rightarrow$$

$$\frac{-2}{\pi n} [\pi \cos n\pi - 0 \cdot \cos n0] = \frac{-2}{\pi n} [\pi \cos n - 0] = \frac{-2}{\pi n} (\pi \cos n\pi) = \frac{-2 \cos n\pi}{n} \therefore \Rightarrow b_n = \frac{-2 \cos n\pi}{n} = \frac{2}{n} \text{ for } n \text{ odd } \frac{-2}{n} \text{ for } n \text{ even} \quad (5)$$

Substituting for $a_0, a_n,$ and b_n into $f(x) = \frac{1}{2} a_0 + \sum_{n=1}^2 a_n \cos nx + b_n \sin nx$

$$f(x) = \frac{\pi}{2} + \sum_{n=1}^2 \frac{2}{\pi n^2} (\cos n\pi - 1) \cos nx + \left(\frac{-2 \cos n\pi}{n} \right) \sin nx$$

Thus, we solve for $\cos nw(t)$ and $\sin nw(t)$ stochastically finding the mathematical expectation of

$$\mathbb{E}(\cos nw(t)) \text{ and } \mathbb{E}(\sin nw(t)) \quad (6)$$

$$\mathbb{E}(\cos nw(t))$$

Let $U = \cos nx$ and $\frac{\partial u}{\partial x} = -n \sin nx, \quad \frac{\partial^2 u}{\partial x^2} = -n^2 \cos nx,$
 $\frac{\partial u}{\partial t} = 0$

Using Itô lemma, we have,

$$du(t, x(t)) = \left(\frac{\partial u}{\partial t} + a \frac{\partial u}{\partial t} + \frac{b^2}{2} \frac{\partial^2 u}{\partial x^2} \right) dt + \frac{\partial u}{\partial x} dw(t)$$

$$U(t, x(t)) = U(0, x(0)) + \int_0^t \left(\frac{\partial u}{\partial s} + \frac{1}{2} \frac{\partial^2 u}{\partial x^2} \right) ds + \int_0^t \frac{\partial u}{\partial x} dw(s)$$

$$\cos nx(t) = \cos nx(0) + \int_0^t \frac{-n^2}{2} \cos nx(s) ds + \int_0^t -n \sin nx(s) dw(s)$$

$$\cos nx(t) = 1 + \int_0^t \frac{-n^2}{2} \cos nw(s) ds + \int_0^t -n \sin nx(s) dw(s)$$

$$\mathbb{E}(\cos nx(t)) = \mathbb{E}(1) - \frac{1}{2} \int_0^t \mathbb{E}(n^2 \cos nx(s)) ds - \int_0^t \mathbb{E}(n \sin nx(s)) dw(s)$$

$$\mathbb{E}(\cos nx(t)) = 1 - \frac{1}{2} \int_0^t \mathbb{E}(n^2 \cos nx(s)) ds$$

$$\mathbb{E}(\cos nx(t)) = 1 - \frac{n^2}{2} \int_0^t \mathbb{E}(\cos nx(s)) ds$$

$$L(t) = 1 - \frac{n^2}{2} \int_0^t L(s) ds$$

$$\frac{2}{\pi n^2} (\cos n\pi - \cos n0) = \frac{2}{\pi n^2} (\cos n\pi - 1) \text{ (since } \cos n0 = 1)$$

$$\therefore a_n = \frac{2}{\pi n^2} (\cos n\pi - 1) = \frac{-4}{\pi n^2} \text{ for } n \text{ odd, } 0 \text{ for } n \text{ even.} \quad (4)$$

For $b_n,$

$$b_n = \frac{1}{\pi} \int_0^\pi f(x) \sin nx dx$$

$$b_n = \frac{1}{\pi} \int_0^\pi 2x \sin nx dx \Rightarrow b_n = \frac{2}{\pi} \int_0^\pi x \sin nx dx, \text{ by } u dv = uv - \int v du$$

Let $x = u \quad dx = du \text{ and } dv = \sin nx$

$$L(t) = 1 - \frac{n^2}{2} L(t), \quad L(0) = 1$$

$$\frac{dL(t)}{dt} = \frac{-n^2}{2} L(t)$$

$$\int \frac{dL(t)}{L(t)} = \int_0^t \frac{-n^2}{2} ds$$

$$\ln L(t) = \frac{-n^2 t}{2} + c$$

$$\ln(1) = 0 + c$$

$$0 = c \ln L(0) = 0 + c$$

Then we have, $\ln L(t) = \frac{-n^2 t}{2}$

$$L(t) = e^{-\frac{n^2 t}{2}}$$

Thus,

$$\cos nw(t) = e^{-\frac{n^2 t}{2}} \quad (7)$$

Synonymously,

We have that

$$\sin nw(t) = e^{-inw(t)} \quad (8)$$

$$U(t, X(t)) = e^{-inw(t)} = e^{-inx}$$

$$\frac{\partial u}{\partial t} = 0, \frac{\partial u}{\partial x} = -ine^{-inx}, \frac{\partial^2 u}{\partial x^2} = n^2 e^{-inx}$$

Substituting into Itô lemma below $du(t, x(t)) = \left(\frac{\partial u}{\partial t} + a \frac{\partial u}{\partial t} + \frac{b^2}{2} \frac{\partial^2 u}{\partial x^2} \right) dt + \frac{\partial u}{\partial x} dw(t)$ we have

$$\begin{aligned} \sin nw(t) &= 0 + \int_0^t 0 + 0 + \frac{n^2}{2} e^{-inx} ds + n \int_0^t -ine^{-inw(s)} dw(s) \\ \sin nw(t) &= 0 + \int_0^t 0 + 0 + \frac{n^2}{2} e^{-inw(s)} ds + n \int_0^t -ine^{-inw(s)} dw(s) \end{aligned}$$

$$\sin nw(t) = \frac{n^2}{2} \int_0^t e^{-(nw(s))} ds - ni \int_0^t \mathbb{E}(e^{-inw(s)}) dw(s) = \frac{n^2}{2} \int_0^t e^{\frac{n^2}{2}s} ds = \frac{n^2}{2} \cdot \frac{2}{n^2} \cdot e^{\frac{n^2}{2}t} = e^{\frac{n^2}{2}t} \tag{9}$$

similarly,

$$\mathbb{E}(\cos nw(t))$$

We also recall that $\cos nw(t) = e^{inw(t)}$

$$\text{Let } U = e^{inx}, \frac{\partial u}{\partial x} = ine^{inx}, \frac{\partial^2 u}{\partial x^2} = -n^2 e^{inx}, \frac{\partial u}{\partial t} = 0$$

$$\begin{aligned} \cos nw(t) &= 1 + \int_0^t 0 + 0 - n^2 e^{inw(s)} ds \\ &\quad + \int_0^t ine^{inw(s)} dw(s) \end{aligned}$$

$$\cos nw(t) = 1 - \int_0^t n^2 e^{inw(s)} ds + \int_0^t ine^{inw(s)} dw(s)$$

$$\mathbb{E}(\cos nw(t)) = 1 - \int_0^t \frac{n^2}{2} \mathbb{E}(e^{inw(s)}) ds + \int_0^t in \mathbb{E}(e^{inw(s)}) dw(s)$$

$$= 1 + \frac{n^2}{2} \int_0^t E(e^{inw(s)}) ds + \int_0^t in E(e^{inw(s)}) dw(s)$$

$$= 1 - \frac{n^2}{2} \int_0^t e^{-\frac{n^2}{2}t} ds$$

$$= 1 + \frac{n^2}{2} \cdot \frac{2}{n^2} e^{-\frac{n^2}{2}t}$$

$$= 1 + e^{-\frac{n^2}{2}t}$$

Thus combining (6), (7), (8) i.e. $\sin nw(t) + \cos nw(t)$, we have

$$1 + e^{\frac{n^2}{2}t} + e^{-\frac{n^2}{2}t} \tag{10}$$

But note that $\cos nw(t) + \sin nw(t)$

$$e^{\frac{n^2}{2}t} + e^{-\frac{n^2}{2}t} = 1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} - \frac{x^7}{7!}$$

if $x = \frac{n^2 t}{2}$, Thus Substituting both (2), (3), (4)-(10) into (11), we have

$$\begin{aligned} f_{pL}(x) &= \left(\frac{\pi}{2}\right) + \sum_{n=0}^2 \frac{2}{\pi n^2} (\cos n\pi - 1) 1 + e^{-\frac{n^2}{2}t} + \\ &\quad \left(\frac{-2\cos n\pi}{n}\right) e^{\frac{n^2}{2}t} \end{aligned} \tag{11}$$

Where $e^{-\frac{n^2}{2}t} = 1 + \frac{x^2}{2!} + \frac{x^2}{4!} + \frac{x^6}{6!}$ And $e^{\frac{-n^2}{2}t} = 1 - \frac{x^2}{2!} - \frac{x^3}{3!} - \frac{x^7}{7!}$

Simulation

From (11),

$$\begin{aligned} f_{pL}(x) &= \left(\frac{\pi}{2}\right) + \sum_{n=0}^2 \frac{2}{\pi n^2} (\cos n\pi - 1) 1 + e^{-\frac{n^2}{2}t} + \\ &\quad \left(\frac{-2\cos n\pi}{n}\right) e^{\frac{n^2}{2}t} \end{aligned}$$

Taken values when $0 \leq n \leq 2$
And $0 \leq x \leq \pi$ and $0 \leq t \leq \pi$

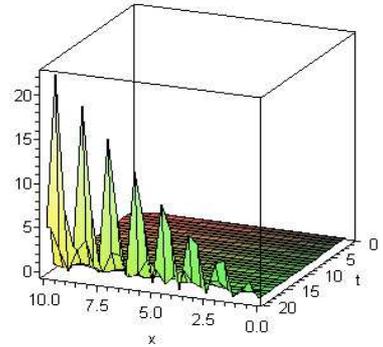


Figure 1.

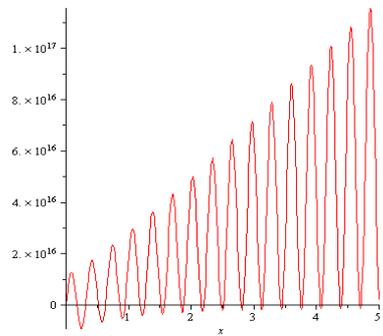


Figure 2.

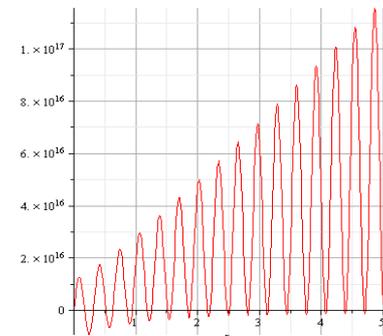


Figure 3.

3. Methodology

(Hedging tools)

We consider a market consisting one stock (risky asset) as the prices of our natural gas and one bond (riskless asset). The prices process of the risky asset is assumed to be of the form $S_t = S_0 e^{N_t}$, $t \in [0, T]$ with

$$N_t = \int_0^t \left(\mu_s - \frac{\sigma_s^2}{2} \right) ds + \int_0^t \sigma_s dw(s) \quad (12)$$

Where, $W = \{W_t, t \in [0, T]\}$ is a Brownian motion defined in a complete probability space $(\Omega, \mathcal{F}, \mathbb{P})$. We will denote by $\{f_t: t \in [0, T]\}$ the filtration generated by the Brownian motion and completed by the \mathbb{P} - null sets. The mean rate of return μ_t and the volatility process σ_t are measurable and adapted satisfying the following integrability conditions

$$\int_0^t |\mu_t| dt < \infty, \quad \int_0^t \sigma_t^2 dt < \infty \text{ a.s.}$$

By Itô's formula, we obtain that S_t satisfies the linear stochastic differential equation:

$$ds_t = \mu_t s_t dt + \sigma_t s_t dw_t \quad (13)$$

The price of the bond $\beta_t, t \in [0, T]$, evolves according to the differential equation $d\beta_t = r_t \beta_t dt$, $\beta_0 = 1$, where the interest rate process is a non-negative measurable and adapted process satisfying the integrability condition $\int_0^t r_t dt < \infty$, a.s. i.e.

$$\beta_t = \exp \left(\int_0^t r_s ds \right) \quad (14)$$

Imagine an investor who starts with some initial endowment $x \geq 0$ and invests in the assets describe above. Let α_t be the number of non-risky assets and β_t the number of stocks owned by the investor at time t . The couple $\phi_t = (\alpha_t, \beta_t)$, $t \in [0, T]$, is called a portfolio or trading strategy, and we assume that α_t and β_t are measurable and adapted processes such that

$$\int_0^t |\beta_t \mu_t| dt < \infty, \int_0^t \beta_t^2 \sigma_t^2 dt < \infty, \int_0^t |\alpha_t| r_t dt < \infty \text{ a.s.} \quad (15)$$

Then $x = \alpha_0 + \beta_0 S_0$, and the investor's wealth at time t (also called the value of the portfolio) is

$$V_t(\phi) = \alpha_t \beta_t + \beta_t S_t. \quad (16)$$

The gain $G_t(\phi)$ made by the investor via the portfolio ϕ up to time t is given by

$$G_t(\phi) = \int_0^t \alpha_s d\beta_s + \int_0^t \beta_s dS_s. \quad (17)$$

We say that the portfolio ϕ is self-financing if there is no fresh investment and there is no consumption. The means that the value equals to the initial investment plus the gain:

$$V_t(\phi) = x + \int_0^t \alpha_s d\beta_s + \int_0^t \beta_s dS_s \quad (18)$$

$$= \int_0^t f(t) dB(t) - \int_0^t \frac{g(t)}{\sigma(t)} dB(t) + \int_0^t \frac{g(t)B(t)}{\sigma(t)s(t)} \left(r_t s(t) dt + \sigma_t s(t) d\tilde{N}(t) \right) \quad (24)$$

Thus, we add some regularity conditions on the portfolio to exclude arbitrage opportunities.

4. Problem Statement

Definition 1

A self-financing strategy ϕ is said to be admissible if there exist a constant A such that $V_t(\phi) \geq -A$, a.s. $\forall t \leq T$

Definition 2

An arbitrage opportunity on the time interval $[0, T]$, is an admissible self-financing strategy ϕ such that:

1. $V_{(0)}^\phi = 0$
2. $\mathbb{P}(V_{(T)}^\phi \geq 0) = 1$
3. $\mathbb{P}(V_{(T)}^\phi > 0) = 0$
4. i.e. with zero cost today, a unit probability to have a non-negative value at time T as well as a positive probability value at time T .

Definition 3

A contingent claim S , such that $\mathbb{E} \gamma^*(S^2) < \infty$ is called attainable if there exists at least one tame self-financing trading strategy ϕ such that

$V_{(T)}^\phi = S$ and we show that it is complete

5. Hedging Electricity with the Natural Gas

Theorem 2

If any contingent claims satisfying $\mathbb{E} \gamma^*(S^2) < \infty$ is attainable, then the associated market model is called complete.

Thus let 'S' be a contingent claim such that $\mathbb{E} \gamma^*(S^2) < \infty$ as such we consider the martingale below

Proof:

$$f(t) = \mathbb{E}_{\gamma^*}(e^{-rT} S / w(t)) \quad t \in [0, T] \quad (19)$$

$$f(t) = \mathbb{E}_{\gamma^*} \left(f(t) + \int_0^t g(s) d\tilde{N}(s) \right), \quad t \in [0, T] \quad (20)$$

Since f is a γ^* - martingale, we have

$\mathbb{E}_{\gamma^*}(f(t)) = f(0)$, the dynamics of the risky asset S (natural gas) is given by

$$ds(t) = r_t s_t dt + \sigma_t s_t d\tilde{N}(t) \quad t \in [0, T], \quad (21)$$

However,

$dB(t) = r_t B_t dt$, $B(0) = 1$ then the strategy

$\phi(t) = (\alpha_t, \beta_t) = \left(f(t) - \frac{g(t)}{\sigma(t)}, \frac{g(t)B(t)}{\sigma(t)s(t)} \right)$ is the tame self-financing strategy that replicates the contingent claim, the gain process of the strategy is

$$G_{(a)}^\phi = \int_0^a \alpha(t) dB(t) + \int_0^a \beta(t) ds(t) \quad (22)$$

$$= \int_0^a \left(f(t) - \frac{g(t)}{\sigma(t)} \right) dB(t) + \int_0^a \frac{g(t)B(t)}{\sigma(t)s(t)} ds(t) \quad (23)$$

$$= \int_0^a f(t)dB(t) - \int_0^a \frac{g(t)}{\sigma(t)} dB(t) + \int_0^a \frac{g(t)B(t)}{\sigma(t)s(t)} r_t s(t) dt + \int_0^a \frac{g(t)B(t)}{\sigma(t)s(t)} \sigma(t)s(t) d\tilde{N}(t) \quad (25)$$

$$= \int_0^a f(t)dB(t) - \int_0^a \frac{g(t)}{\sigma(t)} dB(t) + \int_0^a \frac{g(t)}{\sigma(t)} dB(t) + \int_0^a g(t)B(t) d\tilde{N}(t) \quad (26)$$

$$= \int_0^a f(t)dB(t) + \int_0^a g(t)B(t) d\tilde{N}(t) \quad (27)$$

$$\text{Recall that } f(t) = \mathbb{E}_{\gamma^*} \left(f(t) + \int_0^t g(s) d\tilde{N}(s) \right), \text{ but } \mathbb{E}_{\gamma^*} f(t) = f(0), \text{ Thus,} \quad (28)$$

Thus substituting (20) and (28) into (27), we have as follows

$$= \int_0^a \left(f(0) + \int_0^t g(s) d\tilde{N}(s) \right) dB(t) + \int_0^a g(t)B(t) d\tilde{N}(t), \quad (29)$$

$$= f(0) \int_0^a dB(t) + \int_0^a \int_s^a g(s) d\tilde{N}(s) dB(t) + \int_0^a g(t)B(t) d\tilde{N}(t) \quad (30)$$

$$= f(0)(B(a) - B(0)) + \int_0^a g(s) \int_s^a dB(t) d\tilde{N}(s) + \int_0^a g(t)B(t) d\tilde{N}(t) \quad (31)$$

$$= f(0) \int_0^a dB(t) + \int_0^a \int_s^a g(s) d\tilde{N}(s) dB(t) + \int_0^a g(t)B(t) d\tilde{N}(t) \quad (32)$$

$$= f(0)(B(a) - B(0)) + \int_0^a g(s)(B(a) - B(s)) d\tilde{N}(s) + \int_0^a g(t)B(t) d\tilde{N}(t) \quad (33)$$

$$= f(0)(B(a) - B(0)) + B(a) \int_0^a g(s) d\tilde{N}(s) \quad (34)$$

$$= f(0)(B(a) - B(0)) + B(a) \int_0^a g(s) d\tilde{N}(s) \quad (35)$$

$$= f(0)(B(a) - B(0)) + B(a)(f(a) - f(0)) \quad (36)$$

$$= f(0)B(a) - f(0)B(0) + f(a)B(a) - f(0)B(a)$$

$$G_{(a)}^\emptyset = f(a)B(a) - f(0)B(0)$$

of the assurance of adequate hedging tools

We have that the wealth process is equals

$$V_{(a)}^\emptyset = \left(f(a) - \frac{g(a)}{\sigma(t)} \right) B(a) + \left(\frac{g(a)B(a)}{\sigma(t)s(a)} \right) s(a)$$

$= f(a)B(a) \geq 0$, thus, \emptyset is a tame strategy i.e

$$V_{(0)}^\emptyset = f(0)B(0) = f(0).$$

$\Rightarrow V_{(T)}^\emptyset = f(T)B(T) = S(T)$, thus

$$V_{(0)}^\emptyset + G_{(a)}^\emptyset = f(0) + f(a)B(a) - f(0)B(0) = f(a)B(a).$$

Showing that \emptyset is self-financing and replicates S thus shows that S is attainable.

6. Results

Dynamics of electricity price in the face of gas availability; there is a subsequent increase in power supply and increase in electricity spot-price

We have that the increase in Electricity price within the first five weeks to 20 weeks of the fourth quarter of the year, indicates the constant power supply.

7. Discussion

We observed that even with an increase in the price tariff, consumers, private and Government organisations will be willing to pay their light bills at regular intervals as well as the investors' interest to invest into the power sector because

8. Conclusion

We had developed meaningful and justified mathematical model for the pricing of electricity as well as laid down mathematical model for its risk aversion. This provides power purchasers with an effectiveness instruments through which they can hedge their investment in the power sector through the natural gas

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