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Interest Rate Risk - The Impact of the Yield Curve on Treasury Bill Returns

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Abstract: Interest rate risk involves the risk to earnings or capital arising from movement of interest rates. It arises from differences between the timing of rate changes and the timing of cash flows (re-pricing risk); changing rate relationships among yield curves that affect bank activities (basic risk); from changing rate relationships across the spectrum of maturities (yield curve risk); and from interest-rate-related options entrenched in bank products (option risk). This paper assessed the impact of the level, slope and curvature components of the yield curve on treasury bill returns using secondary data to draw quarterly yield curves for the various maturity periods. This approach was extended to capture the sensitivity to changes in the level, slope, and curvature of the term structure using the parameters of the dynamic [14] model to fit the term structure. The results revealed that, the shorter the yield to maturity the stable and better the returns or yield. Applying dynamic factor models, it was seen that, the slope factor representing the short term component had better returns compared to the medium term and the long term components. Also, the results revealed that, the 91 day T-bill which represents the short term component produced better and much stable returns compared with the 182 day T- bill and 1 year note representing the medium and long term components respectively.

Keywords: Yield Curve, Interest Rate Risk, Term Structure, Dynamic Factor

1. Introduction

The recognition and management of financial risk is inherent to the business of banking and insurance roles as financial intermediaries. To meet the demands of customers and communities and to execute business strategies, financial institutions make loans, purchase securities, and take deposits with different maturities and interest rates. These activities may leave banks' and insurance companies earnings and capital exposed to movements in interest rates. This exposure is interest rate risk. The yield curve is the graph of the required interest rates for various maturity debts. The shape of the curve reflects a whole host of economic phenomena and expectations. However it provides a unique opportunity and dataset to analyze what market participants are willing to pay for cash flows that occur far into the future. It also provides an opportunity to see differences between short run and long run expectations as reflected in actual market prices.

The parsimonious model of the yield curve was developed by [14]. The instantaneous forward rate function of [14] was derived from the short rate, which was assumed to be a non-homogeneous second-order differential equation.

Conventionally, banks and insurance companies have served as mediators in transforming financial resources from savings to investments. In this process, financial institutions often act as "qualitative asset transformers" by changing the characteristics of financial claims with respect to risk, size, maturity, and so forth. As a consequence, banks and insurance companies hold predominantly nominal and often fixed-interest rate assets and liabilities, especially in the case of banks swerving maturities. Following the reasoning of [15] and, in more detail, [10], this has been widely claimed to be the specific reason for the interest rate sensitivity of financial institutions.

However, by comparing international financial systems, [18] and [2] found that, for the traditionally bank-based financial system of Germany, the role of financial institutions as qualitative asset transformers in the process of allocating financial resources between savers and borrowers has remained unchanged. Hence, the interest rate risk of financial institutions, that is the variation in the market values of their equity positions persuaded by changing term structures, has been and still is of feasible interest to both investors (e.g. for purposes of hedging and performance provenance) and regulators (e.g. for an assessment of systemic interest rate risk).

On the other hand, the critique relating to the use of the time measure for the management of fixed income securities, which ignores changes other than corresponding ones in the term structure of interest rates, can be dependably applied to the predominant method to measure the interest rate risk of financial institutions. In contrast, the [14] framework models the entire term structure of interest rates for a given point in time by mapping the term structure into three factors. Following [5], these factors can be interpreted as level, slope, and curvature of the term structure of interest rates. Estimating the sensitivity of stock returns to changes in these factors is a more suitable approach to measuring the "true" interest rate risk, since the Nelson-Siegel factors closely reproduce changes in the shape of the entire term structure of interest rates and not just a supposed corresponding change.

Applications of models with more than one single interest rate factor to measure the interest rate risk of financial institutions have been rare. Notable exceptions are studied by [13] and [9] for the US market, where two interest rate factors of different maturities were simultaneously employed. Typically, the yield curve depicts a line that rises from lower interest rates on shorter-term bonds to higher interest rates on longer-term bonds. Researchers in finance have studied the yield curve statistically and have found that, shifts or changes in the shape of the yield curve are attributable to a few unobservable factors [3]. Specifically, empirical studies reveal that more than 99% of the movement of various Treasury bond yields is captured by three factors, which are often called "level," "slope," and "curvature" [12]. The names describe how the yield curve shifts or changes shape in response to a shock.

[8] analyzed the same problem using a different approach. They formulated several models with rich macroeconomic dynamics and looked at how the "level", "slope," and "curvature" factors are affected by the structural shocks identified in those models. Their conclusion confirmed [1] that, a substantial portion of short- and medium-term bond yields is driven by macroeconomic variables. However, they also found that, in the long run, macroeconomic variables do indeed explain much of the movement of the long-term bond yields, and the "level" factor responds strongly to macroeconomic variables. Their results indicated that the changes in households' consumption preferences persuaded large, persistent, and significant shifts in the level of the yield curve.

[5]-, (-DL) and [4]-, (-DRA) have shifted attention back to the Nelson and Siegel (NS) model. They considered a statistical three factor model to describe the yield curve over time. The three factors represent the level, slope and curvature of the yield curve and thus carry some economical interpretation. In DRA, the Nelson-Siegel framework is extended to include non-latent factors such as inflation. Further, they framed the Nelson-Siegel model into a state space model where the three factors were treated as unobserved processes and modelled by vector autoregressive processes. A wide range of statistical methods associated with the state space model could be exploited for maximum likelihood estimation and signal extraction, [6]. Parameter estimation in [5] and [4] relied on two simplifying assumptions; first, to allow the time-varying factors to be estimated in a linear setting and second, the factor loadings were kept constant over time for each maturity.

This paper examined the impact of the level, slope and curvature components of the yield curve on T-bill returns since not much studies have been done on it in Ghana. This study could give investors information on their investments strategies and more important on investments in T-bills. It was revealed that, the short term component (91 day T-bill) showed a much stable and better compared to the medium term (182 day T-bill) and the long term (1 year note) components even though they were all risk free investments. The study concludes that investors stay to closer to the short end of the market in T-bill investments.

2. Materials and Methods of Analysis

2.1. Data Source

This paper employed secondary data on treasury yields from the Bank of Ghana website. It consists of the 91 day T-bill rates, 182 day T- bill rates and a 1 year note (360 days) from the year 1999 to 2010.

2.2. Methods of Data Analysis

2.2.1. The Nelson-Siegel Framework

The Nelson-Siegel framework was used to model the term structure (yield curve). Interest rates are denoted by $r_t(\tau)$ at time t and maturity τ. For a given time t, the yield curve $\theta_t(\tau)$ is some smooth function representing the interest rates (yields) as a function of maturity τ. A sparing functional depiction of the yield curve was proposed by [14]. The Nelson-Siegel formulation of the yield curve was modified by ([5], DL) to lower the lucidity between the components of the yield curve. The DL formulation is given by

$$\theta_t(\tau) = \theta(\tau; \lambda, \beta_t) = \beta_{1t} + \beta_{2t}\left(\frac{1-e^{-\lambda\tau}}{\lambda\tau}\right) + \beta_{3t}\left(\frac{1-e^{-\lambda\tau}}{\lambda\tau} - e^{-\lambda\tau}\right) \quad (1)$$

where, $\beta_t = (\beta_{1t}, \beta_{2t}, \beta_{3t})'$ for a given time t, maturity τ and fixed coefficient λ that determines the exponential decay of the second and third component in equation (1).

The shape and form of the yield curve was determined by the three components and their associated weights in β_t. The

first component takes the value one (1) which is constant and can therefore be interpreted as the overall level that influences equally the short and long term interest rates. The second component converges to one (1) as $\tau \to 0$ and converges to zero as $\tau \to \infty$ for a given t. Hence, this component mostly influences short-term interest rates. The third component converges to zero as $\tau \to 0$ and as $\tau \to \infty$ but is concave in τ, for a given t. This component is therefore associated with medium-term interest rates.

In the case of observing a series of interest rates $r_t(\tau_i)$ for a set of N different maturities available at a given time t, the yield curve can be estimated by using a simple regression model;

$$r_t(\tau_i) = \theta_t(\tau_i) + \varepsilon_{it}$$

$$r_t(\tau_i) = \beta_{1t} + \beta_{2t}\left(\frac{1-e^{-\lambda\tau}}{\lambda\tau}\right) + \beta_{3t}\left(\frac{1-e^{-\lambda\tau}}{\lambda\tau} - e^{-\lambda\tau}\right) + \varepsilon_{it} \quad (2)$$

for $i = 1, ..., N$. The disturbances, $\varepsilon_{1t}, ... \varepsilon_{Nt}$ are assumed to be independent with mean zero and constant variance σ^2_t for a given t.

2.2.2. The Dynamic Factor Model

Dynamic-factor models have been developed and applied in macroeconomics by researchers such as [11], [16], [17], and [7]. The dynamic factor model makes use of a few latent dynamic factors, f_t, to drive the movements of a high-dimensional vector of time-series variables, X_t, which is also affected by a vector of mean-zero idiosyncratic disturbances, ε_t. These idiosyncratic disturbances arise from measurement error and from special features that are specific to an individual series. The dynamic factor model is given by;

$$X_t = \beta(L)f_t + \varepsilon_t \quad (3)$$

$$f_t = \Psi(L)f_{t-1} + \eta_t \quad (4)$$

where there are N series, X_t and ε_t are $N \times 1$ and for the dynamic factors, f_t and η_t are $q \times 1$, L is the lag operator, and the lag polynomial matrices $\beta(L)$ and $\psi(L)$ are $N \times q$ and $q \times q$ respectively. The ith lag polynomial $\beta_i(L)$ is the dynamic factor loading for the ith series, X_{i_t} and $\beta_i(L)f_t$ is the common component of the ith series.

3. Results and Discussion

3.1. Descriptive Statistics

Table 1, shows the summary statistics for each interest rate maturity, the results revealed that, all the interest rates are positively skewed, showing that majority of these interest rates are clustered at the lower end of the distribution and also a continuous increased in these rates over time. For the entire period, all the interest rates have negative excess kurtosis, showing that they are platykurtic in natures and less peaked as compared to the normal distribution. By their coefficient of variation, the one year note has the largest coefficient of variation; this shows that, it is more volatile than the other interest rates. Their excess kurtosis values showed that, the 182 day T-bill has a higher excess kurtosis, thus more volatile. The 91 day T-bill has first order autocorrelation of 0.983 making it highly persistent compared to the 182 day T-Bill and 1 year note with the least autocorrelation.

Table 1. *Descriptive Statistics.*

Maturity	Mean	SE Mean	Coeff. Var	Min	Max	Skewness	Kurtosis	$\hat{\rho}(1)$	$\hat{\rho}(12)$	$\hat{\rho}(32)$
91 day	22.875	0.880	30.320	8.635	46.685	0.560	-0.540	0.983	0.557	0.145
182 day	24.998	0.855	43.270	10.178	48.450	0.550	-0.390	0.974	0.469	0.110
1year note	21.078	0.538	45.670	12.300	33.543	0.490	-0.940	0.979	0.535	0.180

3.2. Further Analysis

Table 2 shows the parameter estimates of the Nelson Siegel Model. It was seen that, the long term component (level) decreased across the maturity period consecutively from 0.94 for the 91 day T-bill to 0.86 for the 182 day T-bill and to 0.71 for the 1 year note which shows a decrease in the long term yield. Also, an increased in the slope increased short yields such as the 91 day T-bill more than long yields; hence the 91 day T- bill increases across the maturity debts (-1.14 for the 91 day, -1.02 for the 182 day and -0.60 for the 1 year note). The medium term component (curvature) does not have a steady increment but fluctuates with higher uncertain yield indicating unstable returns. An exposure to (at least) level and curvature changes will directly affect the cost of capital of financial institutions. The shorter the yield to maturity, the much stable and better the returns.

Table 2. *Parameter Estimates of the Nelson Siegel Model.*

Maturity	Level	Slope	Curvature
91 day	0.94	-1.14	0.65
182 day	0.86	-1.02	0.53
1 year	0.71	-0.60	0.55

To capture the sensitivity in level (long term), slope (short term) and curvature (medium term), the estimates of the dynamic factor model are shown in Table 3. The Wald statistic estimated was statistically significant at 5% significant level, thus the null hypothesis that all parameters except for variance parameter are zero is rejected. The dynamic factors and variances for the 182 day T-bill and the 1 year note are statistically significant at the 5% significant level whereas the variance for the short term (91 day T-bill) was insignificant at the 5% significant level, showing that it did not vary frequently compared to the medium term (182 day T-bill) and the long term (1 year note); thus produced a much stable and better

returns compared to the 182 day T-bill and 1 year note.

Table 3. Parameter estimates of the dynamic factor model.

	Coeff.	Std. Error	z	P>z	95% conf. interval	
L1	0.689	0.088	7.830	0.000	0.516	0.861
L2	-0.171	0.860	-1.980	0.047	-0.339	-0.002
D. Short term						
F	1.296	0.091	14.310	0.000***	1.118	1.473
D. Medium term						
F	1.397	0.120	11.660	0.000***	1.162	1.632
D. Long term						
F	0.672	0.064	10.460	0.000***	0.546	0.798
Variance						
De. Short term	0.085	0.098	0.870	0.193	0.000	0.278
De. Medium term	1.457	0.209	6.980	0.000***	1.105	1.867
De. Long term	0.505	0.070	7.170	0.000***	0.367	0.643
Loglikelihood	-623.003					
Wald χ^2	333.610					
Prob >χ^2	0.000***					

***significant at 5% level of significance

Table 4, shows the estimates of the dynamic factors (slope, curvature and level component of the Nelson Siegel Model). The results showed that, all the parameters are significant at the 5% level of significance. The short term component has a coefficient of 1.296 representing the slope factor. The medium term has a coefficient of 1.197 representing the curvature factor and 0.672 of the long term representing the level factor indicating that changes in the level, slope and curvature components directly affects returns. An increased in the slope factor, increases short yield more than long yields. These showed that, the shorter the yield to maturity, the better the stability in the returns; thus the 91 day T–bill has stable returns than the 182 T-bill which also has a stable than the1 year note.

Table 4. Estimates of the Dynamic factors.

	Long term	Short term	Medium term
Level (β_{1t})	0.672		
	0.000**		
Slope (β_{2t})		1.296	
		0.000**	
Curvature (β_{3t})			1.197
			0.000**

** Significant at 5% significance level.

3.3. Yield Curve Interpretation of the Various Maturity

Appendix I shows the quarterly yield curve of the 91 day T-bill, the yield at the beginning of the first quarter of the year 1999 was 28% which dropped to 26% in the third quarter indicating that the market was expecting a drop in future interest rates. The curve grew upwards from the fourth quarter of 1999-2001 (up to second quarter) indicating a rise in interest rates. The downward sloping from the second quarter to the fourth quarter of 2001 indicates a drop in futures returns or yields. The humped shape in the year 2003 indicates that, short term and long term are equal and medium term yields are higher than those of the short term and long term and the flattened nature from 2004-2007 (18%-12%) indicates all maturities had similar yields for three years. There was a bigger humped shape from 2008-2009 indicating that short term volatility outweighing long term; hence higher returns are not stable and more unstable and that investors may not meet their expectations because the curve keeps fluctuating at a flattened shapes. From appendix II, which shows the quarterly yield curve of the 182 day T-bill, the yield grew upwards from the first quarter of 1999 at 27% to 42% in the third quarter of 2000 indicating that the market is anticipating a rise in the risk-free rate. Hence will receive better rates or yield in future. The whole forward rate kept fluctuating from the year 2000 with few humped shapes in the year 2001, 2003 and 2009 meaning that, short term and long term are equal and medium term yields are higher than those of the short term and long term. Between the years 2004 and 2008, the returns were low but stable. In the year 2010, the yield dropped to 13% which was far better than the drop in 2007 which was 10% indicating an increase in yield. From appendix III, which shows the quarterly yield curve of the 1 year note, the returns for the 1 year note was not so high, because of serious interest rate fluctuations for long term investment and the unstable nature of it. The yield started from 18% in the first quarter with drastic and long term drops with the only increase in a flat humped in 2003. There was a drop of 15% - 12% between 2004 and 2008. The yield later grew upwards and dropped again in 2010, this shows that the1 year note is not stable and not favorable for investors even though there is anticipation of higher returns. Appendix IV shows the combine yield curve for the various maturity periods. Still the 91 day T-bill was more persistent compared to the 182 day T-bill and the 1 year note. The persistent nature of the 91 day T-bill makes it a better portfolio for investors to consider.

4. Conclusion

This paper studied the impact of yield curve on Treasury bill returns using Treasury bill data from the Bank of Ghana website. From the analysis, the 91 day T-bill produced better and much stable returns compared with the 182 day T-bill

and the 1 year note. The Nelson Siegel model was able to capture the movement in the term structure of interest rate with the short term component been more vibrant in terms of returns and this was supported by the dynamic factor model. Also the research revealed that, the shapes of a yield curve can help one decide whether to purchase a long, medium or short-term bonds, treasury bills, etc. And those changes in the shape of the entire term structure of interest rates should be considered in the context of asset allocation, hedging, and forecasting. Also the research revealed that, staying closer to the short end of the market is better; since returns on short term investments are better and much stable. Therefore the 91 day T-bill is the best in terms of stability and returns.
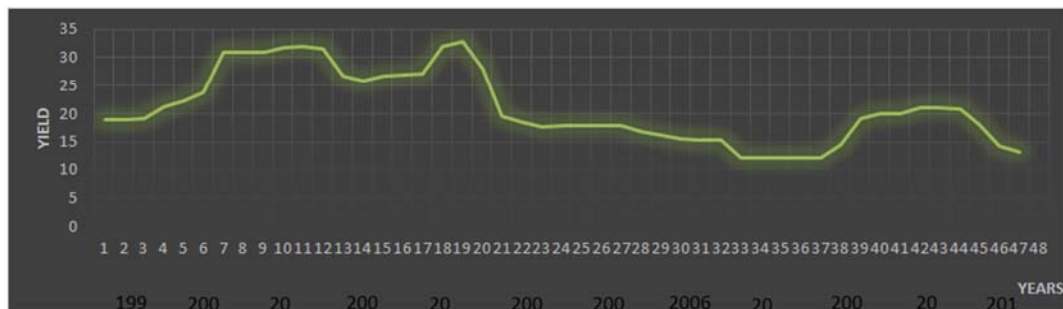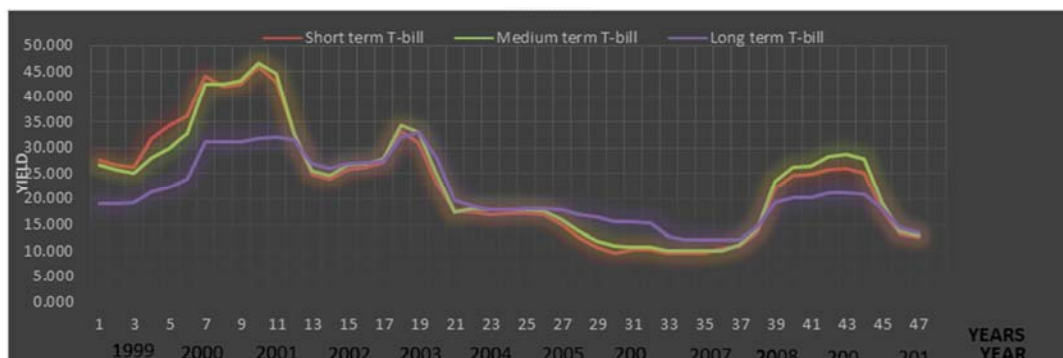
Appendices
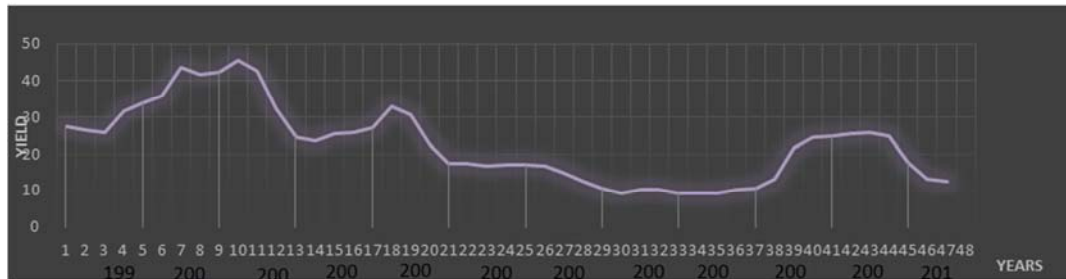
Figure A1. *A quarterly yield curve of the 91 day T-Bill.*
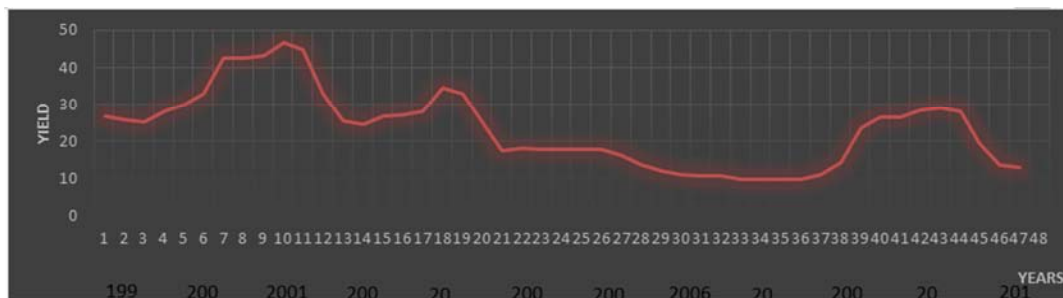
Figure A2. *A quarterly yield curve of 182 day T-Bill.*

Figure A3. *A quarterly yield curve of the 1 year note.*

Figure A4. *A combine yield curve of the various maturity.*

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