

Research Article

On the Reduction of Classical Spin in One-dimensional Heisenberg Magnets with Spin $S > 1/2$

Farhod Rahimi* 

National Academy of Sciences of Tajikistan, Dushanbe, Tajikistan

Abstract

This paper develops a semiclassical framework for one-dimensional Heisenberg magnets with spin $S > 1/2$, where classical approaches are reliable only as $S \rightarrow \infty$ and can miss quantum contributions from multipole moments. The aim is to construct a workable mathematical apparatus that connects the quantum lattice Hamiltonian to an effective classical field description while incorporating an effective reduction of the classical spin length. The analysis starts from an anisotropic Heisenberg chain with nearest-neighbour exchange and single-ion anisotropy. After outlining the continuum (long-wavelength) reduction, including factorization of slowly varying fields and replacement of commutators by Poisson brackets, the semiclassical limit is formulated using a Hartree product ansatz and generalized $SU(N)$ coherent states for each lattice site. For spin $S=1$ the coherent state is built on the $SU(3)/[SU(2) \times U(1)]$ manifold. Two Euler angles define the orientation of the classical spin vector, an additional angle describes rotation of the quadrupole tensor about this vector, and a real parameter g controls the redistribution between dipolar and quadrupolar sectors. Averaging the spin operators yields explicit classical components and shows that the standard constraint $|S|^2=1$ is violated. Instead, an identity relates the reduced spin length to a combination of double correlators, demonstrating that quadrupolar degrees of freedom quantitatively produce a measurable renormalization of the effective classical spin. For spin $S=3/2$ the construction is generalized to $SU(4)$ coherent states, where both quadrupole and octupole moments arise naturally. The averaged spin operators again violate spin-length conservation and, moreover, simple projection sum rules. The corresponding $SU(4)$ identities involve combinations of triple correlators and introduce parameters g and k that quantify reductions due to quadrupolar and octupolar sectors, respectively; the $SU(3)$ case is recovered when $k=0$. The framework provides a transparent route to include multipolar physics in semiclassical dynamics and is relevant for interpreting nonlinear collective excitations, including soliton-like modes, in quantum spin chains.

Keywords

One-dimensional Heisenberg Magnet, Quantum Spin Chain, Semiclassical Approximation, Coherent States, $SU(3)$ and $SU(4)$ Symmetry, Multipole Moments (Quadrupole, Octupole), Effective Classical Spin Reduction, Classical Field Model

*Correspondence: Farhod Rahimi (frahimi2002@mail.ru)

Received: 5 February 2026; **Accepted:** 14 February 2026; **Published:** 27 February 2026



1. Introduction

Over the last few years, interest in the investigation of magnetic systems [1-5, 29-33] has considerably increased. Particular attention is focused on the study of ferromagnetics with spin $S > 1/2$, for which the accurate results were, as a rule, not obtained. As for the theoretical study, it is confined within the classical approach. However, it is impossible to describe entirely the nature of such magnetics by the classical approach, as it is impossible to bring together the contribution of different interactions in behavior to the effective fields – functions of only one vector of magnetization. The classical approach enables obtaining applicable results for magnetics with spin $S > 1/2$ and within limit of $S \rightarrow \infty$. The real situation, when the spin of most magnetics has the finite quantity $S > 1/2$, requires additional investigation, as well as calculation of quantum nature of magnetics.

Particular attention is also focused on investigation of a new type of collective excitations in magneto-cosmic media, so-called particle-like or soliton-like excitations. Usually, they appear as localized solutions of classical equations, such as SG, Schrödinger Non-linear Equation, Landau – Lifshitz Equation, etc. On the other hand, Heisenberg models [6-8] are the fundamentals for microscopic studies of the large class of magnetics. Therefore, a question arises concerning the relation of the collective non-linear effects in classical and quantum models [9], i.e. concerning formulation of “sufficiently consecutive procedure of leading the Heisenberg quantum lattice models to the classical field models”. It is necessary for more complete calculation of quantum nature of magnetics in the equation received.

The given paper investigates the magnetic combinations, which are described by the Heisenberg model. First, we briefly dwell on basing the selection of the object of investigation. For this purpose, we will describe those models, by

which the magnetic combinations are described, and we will indicate the advantage and necessity of studying the magnetics of the Heisenberg model.

2. The Main Models of the Described Magnetic Systems

1) Ising Model

One of very few models, admitting accurate solution in a number of cases, is the Ising model, which describes the system of spins, related to exchange interaction, and the dimension of which depends only on values Z – component of interactive spins. This model occupies particular place in statistical physics. The Ising model, which was introduced initially for the description of anisotropic exchange in strong magnetics [10], with its developed mathematical apparatus proved to be efficient means for formulating and solving series of other problems from very far fields of physics, such as phase transition “order – disorder” in binary systems [11, 12] and ferroelectrics [13], conformation of polymeric molecules [14, 15] and many others.

As a result of intensive investigation of the accurately solvable Ising model, it is now possible to determine (at least in principle) any of their balanced parameters, such as thermal capacity, magnetization, including also delicate characteristics of the system such as the balanced spin correlation functions [16-18].

Now, many magnetic combinations are known, particularly the combinations, in which one- or two-dimension Ising situation is realized (see the Table below).

Table 1. A Substance with exchange interaction of Ising type.

Substance	Spin	Dimension	Quantity of exchange integral
H ₃ Fe(CB ₆)	½	1	-0,23
(NH ₄)MnF ₅	½	1	-12
CsCoCl ₃ ·2H ₂ O	½	1	-23
CsCoCl ₃	½	1	-75
CoCl ₂ ·2NC ₅ H ₅	½	1	+9,5
CoCl ₂ ·2H ₂ O	½	1	+9,3
[(CH ₃) ₃ NH]CoCl ₃ ·2H ₂ O	½	1	+9,9
RbFeCl ₃ ·2H ₂ O	½	1	-0,35
CoCs ₃ Rr ₅	½	2	-0,22
Co(HCOO) ₂ ·2H ₂ O	½	2	-4,3

Substance	Spin	Dimension	Quantity of exchange integral
RB ₂ CoF ₄	½	2	-91
K ₂ CoF ₄	½	2	-97
FeCl ₂	½	2	+3,4
CsFeCl ₃ ·2H ₂ O	½	1	-42

$$H = -2J \sum_j S_j^z S_j^z + 1$$

The methods, related to violation of the balanced state of a substance, such as the methods of magnetic resonance, neutron scattering [19, 20], play the increasing role in the study of magnetic combinations. To get information about an object being investigated through these methods, appropriate theories are required, which give the relation between the quantities under review and the parameters of internal structure of the substance. However, constructing the complete microscopic theory of non-equilibrium behavior of Ising model is the problem, which hasn't been solved so far even in the case of one dimension. This is bound up first of all with necessity of calculation in such theory, equally with the spin-spin interactions, interactions of spins with thermostat, and interaction of spins with external fields.

We should note that despite of some simplifications in construction of the theory on the Ising model, considerable difficulties remain at each stage. They have both common character (difficulty in conclusion and solution for the equations of motion of parameters, characterizing the state of the system) and particular character, related to specificity of the model under study, and they are often insurmountable within framework of exact theory. Therefore, the overwhelming majority of results, which have become known up titative estimation. Accurate results are very few.

2) X Y – model

Unfortunately, XY – exchange is rarely realized in nature, and therefore the interesting results of the works [21, 22] cannot find application for experiments on the dynamics of spin sZystems.

3) Heisenberg Model

Heisenberg models [23, 24] are the main (theoretical) instrument for studying the large class of magnetics.

Heisenberg was the first, who paid attention to the exchange interaction back in 1928 [23]. According to his view, between the spins S_i and S_j of i -th and j -th atoms, there is an interaction with characteristic energy of the following kind:

$$H = -2 J S_i S_j, \quad (1)$$

where J is an exchange integral. Integral J reaches value of the order 10^3 cm^{-1} ($\sim 10 \text{ K}$), exceeding by 10^3 times [24] the dipole-dipole interaction, and therefore, it can be responsible for the formation of spontaneous magnetizedness. In case $J > 0$ the state, possessing lower energy, with parallel spins S_1 and S_2 is settled, and if $J < 0$, antiparallel orientation of spins is settled, which leads to antiferromagnetism. Since the exchange interaction is short-range, then J assumes the most significance, if S_i and S_j belong to the nearest neighbors. Owing to the tendency towards the parallel regulating, all neighbor spins finally draw up in parallel, which results in spontaneous magnetizedness. The Weiss theory [25] assumed that molecular field is in proportion to the medium magnetizedness, and this promotes the assumption that the exchange interaction is the same for any pair of S_i and S_j spins, irrespective of distance between them. However, in reality the exchange forces are closely-active. Therefore, when the paralleledness of spins is considerably upset with approach of the temperature to the Curie point, it remains the same in relatively closer neighbors, which form clusters with parallel spins.

Table 2. Substances of Heisenberg type.

Substance	Spin	Dimension	Quantity of J/K
CuSO ₄ ·5H ₂ O	½	1	-1,45
CuSeO ₄ ·5H ₂ O	½	1	-0,8
Cu(NH ₃) ₄ SO ₄ ·H ₂ O	½	1	-3,5
Cu(NH ₃) ₄ SO ₄ ·H ₂ O	½	1	-2,36
CuCl ₂	½	1	

Substance	Spin	Dimension	Quantity of J/K
CuBr ₂	½	1	
Cu(HCO ₂) ₂ ·(OH) ₂	½	2	
Cu(C ₆ H ₅ COO ₂)·3H ₂ O	½	1	
Cu(NH ₃)(NO ₃) ₂	½	1	1,2
CuCl ₂ ·2NC ₅ H ₅	½	1	-13
KcuF ₃	½	1	-190
CsNiCl ₃	½	1	-13
RbNiCl ₃	1	1	-17
(CH ₃) ₄ NNCl ₃	1	1	+2
CsNiF ₃	1	1	+11,8
KniF ₃	1	1	
TlNiCl ₃	1	1	+2
K ₂ NiF ₄	1	1	
Rb ₂ NiF ₄	1	1	
Tl ₂ NiF ₄	1	1	
Ba ₂ NiF ₆	1	1	
NH ₄ NiCl ₃	1	1	
Ni(HCOO) ₂ ·2H ₂ O	1	1	
CsMnCl ₃ ·2H ₂ O	5/2	1	-6,5
CsMnF ₃	5/2	1	
CsMnI ₃	5/2	1	
RbMnBr ₃	5/2	1	
C ₃ MnBr ₃	5/2	1	
BaMnF ₄	5/2	2	
K ₂ MnF ₄	5/2	2	10,1
Rb ₂ MnF ₄	5/2	2	0,3
Rb ₂ MnCl ₄	5/2	2	9,2
Cs ₂ MnCl ₄	5/2	2	3,0
CsMnCl ₃ ·2H ₂ O	5/2	1	3,2
CsMnBr ₃ ·2H ₂ O	5/2	1	3,0
TMMC	5/2	1	-6,5
VoCE		2	1,48
RbFeCl ₃	2	1	?
FePs ₃	3/2	1	?
CsCuCl ₃	½	1	?

$$H = -J \sum_j (S_j S_{j+1} + \delta S_j S_{j+1})$$

Heisenberg Model [23, 24], to which the overwhelming majority of combinations correspond (see Table 2), allows getting accurate description and statistic and dynamic qualities.

Elaboration of the theory of dynamic, statistic processes in the Heisenberg model is highly urgent, the more so, as precisely in Heisenberg systems there is hope to combine experimental study of dynamic qualities in variable field with their strict theoretical description.

As we mentioned earlier, Heisenberg models are the main theoretical instrument for the large class of magnetics. The experimental investigations [25-27] of one-dimension magnetics, such as the crystals CsNiF_3 , $(\text{CH}_3)_4\text{NmnCl}_3$, PbFeCl_3 and $[(\text{CH}_3)_4\text{N}][\text{NiCl}_3]$ revealed that they contain excitations that exist in low temperatures. They can be integrated as solitons. The solitons are usually described by solutions of non-linear classical equations. Therefore, a question concerning the relation of collective non-linear effects in classical and quantum models arises, i.e. the problem of formulating sufficiently consecutive "procedure of bringing together" the models of quantum statistical mechanics, in particular, the Heisenberg quantum lattice models with classical field models. Sometimes, such transition is made by formal replacement of spin operator S in the knot of crystal lattice by classical value, equal to magnetic moment per one knot M . Many of scientific works were devoted to justification of such procedure. Among them, the

work of Herring and Kittel [28] (see [29] as well) is very important. This procedure, which is relevant for the spin $S \rightarrow \infty$, leads to the well-known classical models, such as the Landau-Lifshitz Equation, the Sin-Gordon Equation, etc.

3. The Effect of Reduction of the Length of Classical Spin for Heisenberg Magnetics with the Value of Spin $S > 1/2$

Now, we are going to investigate Heisenberg anisotropic magnetic with spin $S=1$, which is described by the Hamiltonian:

$$H = -J_0 \sum_j (S_j S_{j+1} + \bar{\delta} S_j^Z S_{j+1}^Z) \quad (2)$$

where J_0 is the exchange integral, $\bar{\delta}$ - the constant of anisotropy.

To investigate the anisotropic magnetics with spin $S=1$, we will use the spin operators, which have the following formula:

$$S^- = \sqrt{2} \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}, S^+ = \sqrt{2} \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}, S^Z = \sqrt{2} \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}. \quad (3)$$

The simplest plan of transition from the Hamiltonian (2) and the quantum-mechanic motion equation.

$$i\hbar \frac{\partial S}{\partial t} = [H, S] \quad (4)$$

to the energy of magnetic and the equation for classical values consists of the following steps:

- 1) We replace the operator of spin in the knot of crystal lattice by the classical value S , which is equal to magnetic moment per 1 knot:

$$S \rightarrow S = -\frac{\alpha}{2\mu_0} M. \quad (5)$$

- 2) Using continuity of the function M , we factorize:

$$S_{j+1} = S + \alpha_0 S_{jx} + \frac{\alpha_0^2}{2} S_{jxx} + \dots \quad (6)$$

- 3) We replace the equation of motion (4) by the classical equation:

$$i\hbar \frac{\partial S}{\partial t} = \{H, S\} \quad (7)$$

where $\{A, B\}$ is Poisson braces, and as the Hamilton function we assume the following energy:

$$E = \int M \left(M, \frac{\partial M_i}{\partial x_k} \right) d^3 x. \quad (8)$$

We put (5), (6) in (2), confining by quadratic, on α , members of expansion and going over from summing up on knots of crystal lattice to integration on volume of crystal, we get classical Hamiltonian.

Going over to semi-classical description, we will use the Hartree approximation [30], in which the state of spin in separate spin is described by the wave function:

$$|\Psi\rangle = C_{-1}|0\rangle + C_0|1\rangle + C_1|2\rangle, \quad (9)$$

where C_{-1}, C_0, C_1 - are the complex functions, and

$|0\rangle, |1\rangle, |2\rangle$ - are the referent states. In view of arbitrariness of selection of phase $|\Psi\rangle$ and the conditions of rate setting:

$$|C_{-1}|^2 + |C_0|^2 + |C_1|^2 = 1, \quad (10)$$

the quantity of parameters for full description of quasi-classical behavior of the system will reduce to four.

The work [31] describes the building of coherent state, which has the following formula:

$$|\Psi\rangle = U(\theta, \phi, \gamma)|\eta\rangle, \quad (11)$$

where $U(\theta, \phi, \gamma)$ - is the unitary operator, similar to Winger operator, which ensures transiting self-traveling system of coordinates for each lattice site. Two Euler angle θ and ϕ determine orientation of the vector of classical spin, and the angle γ characterizes rotation of quadrupole moment around the vector of spin. As it was mentioned in [31], the rotation of quadrupole moment is an important element for the non-linear dynamics of anisotropic magnetic. And we assume the trial function $|\eta\rangle$ like:

$$|\eta\rangle = e^{2ig\theta^{xy}}|1\rangle, \quad (12)$$

$$q^2 = \langle S^+ S^Z \rangle \langle S^Z S^- \rangle + \langle S^- S^Z \rangle \langle S^Z S^+ \rangle + \langle S^+ S^+ \rangle \langle S^- S^- \rangle + (1 - \langle S^Z S^Z \rangle). \quad (15)$$

As we can see in (14), for magnetics with spins $S^2=1$, the reductions of the length of classical spin take place within space SU(3) on the following value:

$$S^2 = 1 - q^2, \quad (16)$$

$$S^+ = \begin{pmatrix} 0 & \sqrt{3} & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & \sqrt{3} \\ 0 & 0 & 0 & 0 \end{pmatrix}; S^- = \begin{pmatrix} 0 & 0 & 0 & 0 \\ \sqrt{3} & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & \sqrt{3} & 0 \end{pmatrix}; S^Z = \frac{1}{2} \begin{pmatrix} 3 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 2 & -1 & 0 \\ 0 & 0 & 0 & -3 \end{pmatrix},$$

where $S^+ = S^x + iS^y$, $S^- = S^x - iS^y$ and they are satisfied by commutation correlation:

$$[S^+, S^-] = 2S^Z; [S^Z, S^\pm] = \pm S^\pm.$$

where $\theta^{xy} = \frac{i}{2} \begin{pmatrix} 0 & 0 & -1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$ is the quadrupole moment.

The parameter g characterizes the change of the length of vectors of classical spin and quadrupole moment.

The averaged values of the spin operators on KC(11) have the following obvious form:

$$\begin{aligned} \langle S^+ \rangle &= e^{i\phi} \cos 2g \sin \theta \\ \langle S^- \rangle &= e^{-i\phi} \cos 2g \sin \theta \end{aligned} \quad (13)$$

$$\langle S^Z \rangle = \cos 2g \cos \theta$$

As you can see in (13), the law of reservation of the square of classical spin in such form $S^2=1$ does not take place, and in this case the following identity takes place:

$$S^2 + q^2 = 1, \quad (14)$$

where q is the combination of the double correlators, which has the following form:

which appear owing to quadrupole interaction, and it naturally effects on the dynamic behavior of the magnetic crystal.

Now, we will consider the Heisenberg anisotropic one-dimension magnetic with spin 3/2 within the space SU(4), which is described by Hamiltonian (2). Here, the spin operators are used, which have the following form in the space SU(4):

The scheme of transition remains the same as in the case SU(3), and we build the KC for magnetics with the value of spin $S=3/2$ in the following form:

In the complex parameterization:

$$|\Psi\rangle = \left(1 + |\Psi|^2\right)^{-1/2} \left(|0\rangle + \sum_{i=1}^{2S} \Psi_i |i\rangle\right), |\Psi|^2 = \sum_{i=1}^{2S} |\Psi_i|^2. \quad (17)$$

The Lagrangian of this system has the following form:

$$L = i\hbar \frac{\sum_{i,j=1}^{2S} (\Psi_i \bar{\Psi}_j - \Psi_j \bar{\Psi}_i)}{1 + \sum_{i=1}^{2S} |\Psi_i|^2}. \quad (18)$$

Varying the Lagrangian (18), we will get the following equations in the space SU(4):

$$i\Psi_i = \left(1 + \sum_{j=1}^{2S} |\Psi_j|^2\right)^{-1/2} \left[1 + |\Psi_i|^2 \frac{\delta H}{\delta \bar{\Psi}_i} + \sum_{j=1}^{2S} \Psi_i \bar{\Psi}_j \frac{\delta H}{\delta \bar{\Psi}_i} \alpha_{ij}\right], \quad (19)$$

where $\alpha_{ij} = 0$ $i \neq j$, $\alpha_{ij} = 1$ $i = j$.

Averaging the values of spin operators on KC(17), we will make sure that unlike the SU(3) version, here not only the law of preservation of the square of classical spin does not take place, but also the sum does not remain:

$$S^2 + q^2 \neq 1$$

In this case, the following identity takes place:

$$S^2 + q^2 + f^2 = 9/4, \quad (20)$$

where f^2 - is the combination of triple correlators, which has the following form:

$$\begin{aligned} f^2 = & \left(\langle S^Z S^Z S^+ \rangle \langle S^- S^Z S^Z \rangle + \langle S^+ S^Z S^Z \rangle \langle S^Z S^Z S^+ \rangle - \right. \\ & - \langle S^Z S^Z S^+ \rangle \langle S^Z S^Z S^- \rangle + \langle S^+ S^Z S^Z \rangle \langle S^- S^Z S^Z \rangle + \\ & + \langle S^+ S^- S^- \rangle \langle S^+ S^+ S^- \rangle + \langle S^- S^- S^+ \rangle \langle S^- S^+ S^+ \rangle - \\ & \left. - \langle S^- S^+ S^+ \rangle \langle S^+ S^+ S^- \rangle + \langle S^+ S^- S^- \rangle \langle S^- S^+ S^+ \rangle \right), \end{aligned} \quad (21)$$

and q^2 - is the combination of triple correlators, which has the following form:

$$\begin{aligned} q^2 = & 1/4 \left(\langle S^+ S^- \rangle - \langle S^- S^+ \rangle \right)^2 - 5 \left(\langle S^+ S^Z \rangle \langle S^Z S^- \rangle + \right. \\ & + \langle S^+ S^Z \rangle \langle S^- S^Z \rangle + \langle S^Z S^+ \rangle \langle S^Z S^- \rangle + \left. \langle S^+ S^Z \rangle \langle S^- S^Z \rangle \right). \end{aligned} \quad (22)$$

In the real parameterization:

and:

$$|\Psi\rangle = D(\theta, \phi, \gamma) e^{2ig\theta^{xy}} e^{-i\beta S^z} e^{-ikF^{xyz}} |0\rangle, \quad (23)$$

where $D(\theta, \phi, \gamma)$ - is the operator, similar to the Vigner operator, which takes into account revolution of the spin moment,

$$F^{xyz} = \frac{1}{i} \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix} - \text{octopole moment}; \quad (24)$$

$$\theta^{xy} = \frac{1}{2i} \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix} - \text{quadrupole moment. (25)}$$

The spin operators and the operators of multipole moments (24) and (25) are the operators of the group SU(4), and the built KC(23) satisfies all the states.

Averaging the spin operators on OKC(23), we will get the projection of classical vector of spin in new parameterization:

$$\begin{aligned} \langle S^+ \rangle &= \frac{3}{2} e^{i\phi} \sin \theta (1 - 4 \cos^2 k) \cos 2g \\ \langle S^- \rangle &= \frac{3}{2} e^{-i\phi} \sin \theta (1 - 4 \cos^2 k) \cos 2g \\ \langle S^z \rangle &= \frac{3}{2} \cos \theta (1 - 4 \cos^2 k) \cos 2g \end{aligned} \quad (26)$$

In this case, as in the case of the complex parameterization, the identity takes place (20). It is obvious that parameter g takes into account reduction of spin owing to quadrupole moment, and k - owing to quadrupole moment. If we assume $k=0$, then the parameterization (23) and (17) will come to the case SU(3).

So, here for the first time an effort has been made to build the mathematical apparatus, which would allow taking into account the effective reduction of the length of the classical spin in the magnetics with the spins $S=1$ and $S=3/2$ in the spaces SU(3)/SU(2)×U(1) and SU(2S+1)/SU(2S)×U(1), as owing to the quadrupole moment, as owing to octupole moment.

It should be expected that the reduction by octupole moment become apparent in higher temperatures other than quadrupole one.

Author Contributions

Farhod Rahimi: Conceptualization, Methodology, Formal analysis, Investigation, Resources, Visualization, Writing – original draft, Writing – review & editing.

Conflicts of Interest

The author declares no conflicts of interest.

References

- [1] Kosevich A. M. Kovalev A. S. Introduction to the non-linear physical mechanics. Kiev: Naukova Dumka, 1989, 279 p.
- [2] Akhiezer I. A. Bariakhtar V. G. Pletmenny S. V. Spin waves. Moscow. Publishing House “Nauka”, 1967, 368 p.
- [3] Molozemov A. Slonzueski J. Domain walls in the materials with cylindrical magnetic domain. Moscow. Publishing House “Mir”, 1982, 348 p.
- [4] Abdullaev F. Kh. The dynamics of magnetic solitons under the influence of phonon fluctuations in quasideimensional magnetics. FMN, 1984. 57. № 3. pages 450-456.
- [5] Fedyanin V. K. Dynamical Formfactor of neutron scattering on solitons in quasideimensional magnets. IMM, 1983. 31-34. p. 1237.
- [6] Gochev I. G. Quantum domain wall and coherent states for the Heisenberg-Ising spin $\frac{1}{2}$ chain. Preprint JIHR, E 17-84-253, Dubna, 1984; Phys. Lett. A, 1984, v. 104, n. 1, p. 36-37.
- [7] Abdulloev Kh. O. Makhankov A. V. Quasiclassical behavior of the initial pack of waves within framework of the Heisenberg lightly-plane model. The Proceedings of the Academy of Sciences of the Republic of Tajikistan. 1991. volume III, № 2, pages 170-174.
- [8] de Jongh L. J. Solitons in magnetic chains. J. Appl. Phys. 1982, 53, № 11, pages 8018-8023.
- [9] Perelomov A. M. Generalized coherent states and their annexes. Moscow. Publishing House “Nauka”. 1987. 269 p.
- [10] Ising E. Neitrag sur theorie des ferromagnetismus. Z. Physik, 1925, v. 31, № 1. 253 p.
- [11] Helber P. Experimental investigations of critical Phenomena. – Rep. Prog. Phys. 1967, v. 30 № 2, p. 731-826.
- [12] Zumer S. Pseudo – one – dimensional kinetic Ising model. – Phus. Rev. 1980, v. 21 B№ 3, pages 1298 – 1303.
- [13] Nakamura E. Abe K. Deduchi K. Quasi-one-dimensional behavior of thermal expansion in ferromagnetic Cs H₂ PO₄. J. Phys. Soc. Japan. 1984. v. 53, pages 1614-1616.
- [14] Malakis A. A polymer model equivalent to the Ising model. – J. Phys. 1980, v. A 13 № 2, pages 651-658.
- [15] Flori P. Statistical mechanics of chain molecules. Moscow. Publishing House “Mir”. 1971. 440 p.
- [16] Baxter R. Exactly solvable models in the statistical mechanics. Moscow. Publishing House “Mir”. 1985. 488 p.
- [17] Steiner M. Villian J. Windsor C. G. Theoretical and experimental studies on one-dimensional magnetic system. – Adv. Phys. 1976, v. 25, № 2, pages 87 – 209.
- [18] Fedyanin V. K. Application of the Grin function and the correlation functions in the study of Ising model. In the book “Statistical physics and quantum theory of field”. Moscow. Publishing House “Nauka”. 1997. pages 241-246.
- [19] Zaslavsky O. B. Tsukernik V. M. Relaxation of spin systems in a transversal alternating magnetic field. FNT. 1983. volume 9, № 1, pages 65-73.
- [20] Heisenberg W. Zs. f. Phys. 1928. volume 49. p. 619.
- [21] Heisenberg W. Metallwirtschaft, 1930, v. 9, p. 843.

- [22] Tikadzumi S. Physics of ferromagnetism. Magnetic properties of substances. Moscow. Publishing House "Mir". 1983. p. 304.
- [23] Kosevich A. M. Ivanov B. A. Kovalev A. S. Non-linear waves of magnetization. Dynamic and topologic solitons. Kiev. Publishing House "Naukova Dumka". 1983. p. 196.
- [24] Pushkarov K. Primotorova M. Solitary clusters of spin deviations and lattice deformation in an anharmonic ferromagnetic chain. Phys. Stat. Sol. 1984, v. 123, p. 573-584.
- [25] Herring C. Kittel C. On the theory of spin wave in ferromagnetic media. Phys. Rev. 1951, 91, n 5, p. 869-880.
- [26] Perelomov A. M. Comm. Math. Phys. 1972, 26, p. 222.
- [27] Abdulloev Kh. O. Rahimov F. K. Muminov Kh. Kh. Calculation of quadrupole dynamics of the magnetics with spin $S=3/2$. The Proceedings of the Academy of Sciences of the Republic of Tajikistan, 1993, № 1, pages 28-30.
- [28] Rahimov F. K. Abdulloev Kh. O. Semi-classical description of ferromagnetics with spin $S=3/2$ in the space $SU(2S+1)/SU(2S)*U(1)$. DAS RT, 1998, v. XLI, № 3-4, pages 45-51.
- [29] Remund K. Pohle R. Akagi Y. Romhányi J. Shannon N. Semi-classical simulation of spin-1 magnets. Phys. Rev. Research, 2022, v. 4, № 3, 033106.
<https://doi.org/10.1103/PhysRevResearch.4.033106>
- [30] Dahlbom D. Miles C. Zhang H. Batista C. D. Barros K. Langevin dynamics of generalized spins as $SU(N)$ coherent states. Phys. Rev. B, 2022, v. 106, № 23, 235154.
<https://doi.org/10.1103/PhysRevB.106.235154>
- [31] Do S.-H. Zhang H. Dahlbom D. A. Williams T. J. Garlea V. O. Hong T. Jang T.-H. Cheong S.-W. Park J.-H. Barros K. Batista C. D. Christianson A. D. Understanding temperature-dependent $SU(3)$ spin dynamics in the $S=1$ antiferromagnet $Ba_2FeSi_2O_7$. npj Quantum Materials, 2023, v. 8, № 1.
<https://doi.org/10.1038/s41535-022-00526-7>
- [32] Zvyagin A. A. Slavin V. V. Zvyagina G. A. Manifestation of spin nematic ordering in the spin-1 chain system. Phys. Rev. B, 2023, v. 107, 134421.
<https://doi.org/10.1103/PhysRevB.107.134421>
- [33] Wei Z. Du Z. Cao X. You W.-L. Lu Y. Quantum criticality and emergent order in the spin-1 bilinear-biquadratic Kitaev chain. Phys. Rev. B, 2025, v. 112 №, 21, 214434.
<https://doi.org/10.1103/9w4w-7phz>