Science Research
2026, Vol. 14, No. 2, pp. 22–27
https://doi.org/10.11648/j.sr.20261402.11

Research Article

A Type of Holographic Scalar Field Model and Coincidence Problem

Ri Song Chol[1], Kim Il Guk[2,*], Kim Ryong Gwang[2], Kim Jik Su[2]

[1]Department of Physics, Sariwon Ri Kye Sun University of Education, Sariwon, Democratic People's Republic of Korea
[2]Pyongyang Astronomical Observatory, Pyongyang, Democratic People's Republic of Korea

Abstract

One of the main problems in cosmology is to resolve the problem of accelerating expansion and cosmological coincidence problem of the late universe. Many models, such as the cosmological constant model, the scalar field model, and the holographic model proposed in Einstein's tensor gravity theory, have solved many problems in explaining these problems. The Brans-Dicke (BD) scalar-tensor theory, proposed by Brans and Dicke in 1961, is a generalization of Einstein's tensor theory of gravity. In this theory, the scalar field can be combined with gravity, and many scalar field models have been proposed, since the observational constraints of the coupling parameters are not clear. In general formalism for scalar fields, parameterization may be relevant in some sense. For example, parameterization of parameters such as dark matter-dark energy interaction coefficient, equation of state parameter, and holographic constant, which have been proposed to solve the problem of cosmological coincidence problem, has some significance. Therefore, in this paper, we have applied the well-known Jassal-Bagular-Padmanabhann (JBP) parameterization in cosmology to the scalar field to construct a holographic scalar field model, perform cosmological verification, and consider the problem of late cosmic acceleration expansion and cosmological coincidence problem. The model parameters obtained by minimizing the chi-square function have nonzero values, and the current values of the transition red shift, equation of state parameter, deceleration parameter, and coincidence parameter are in good agreement with previous studies. We also confirmed the validity of the model and finally obtained a limit on the rate of change of the gravitational constant.

Keywords

Holographic Scalar Field, Coincidence Problem, Scalar-Tensor Theory, Parameterization

1. Introduction

One of the unsolved problems in modern cosmology is to explain the late cosmic acceleration expansion predicted from observations such as supernova, cosmic background radiation, and baryon acoustic vibrations, as mentioned earlier [1-9]. According to observations, the current cause of the acceleration expansion of the universe is a strange energy component with a large negative pressure, called dark energy. Hence, previous studies have put the cosmological constants, Quintessence, Phantom, Chaplygin gases, etc., as candidates for such dark energy. The cosmological constant-cold dark matter model, which includes cosmological constants, is recognized as the most natural model. However, this model encounters fine-tuning problems

*Correspondence: Kim Il Guk (cy.kim1992@star-co.net.kp)

Received: 7 November 2025; **Accepted:** 24 November 2025; **Published:** 16 March 2026

and cosmological constant problems [10]. Hence, several dynamic dark energy models have been proposed. Recently, a holographic dark energy model with the superior properties of quantum theory has been proposed to explain the acceleration expansion of the universe. This model provides the best way to explain the accelerated expansion of the universe in comparison with other dynamic models. Although the BD scalar-tensor theory has several advantages, previous studies have shown that the exponential form of the BD scalar field is an inappropriate assumption in the analysis of cosmic evolution by dark energy. [1-5] In other words, this form is an inappropriate assumption to explain the phase transition of the universe in terms of the dark energy. In view of this issue, we propose a new form of the BDscalar field, where the exponent of the power function of the scalar field is not a constant but a variable. Here, the exponent of the scalar field α is parameterized as a function of the red shift by the well-known JBP parameterization as follows. [6].

$$\alpha = \alpha_0 + \alpha_1 \frac{z}{(1+z)^2}$$

Here, α_0 and α_1 are constants without units, respectively. Then the expression for the scalar field is

$$\phi = \phi_0 (1+z)^{\alpha(z)},$$

The slowness of the BD scalar field is higher than that of other parameterizations in satisfying the condition that it must evolve slowly to observe the slow change of the gravitational constant. The new form of scalar fields above satisfies these requirements. We see that the case of $\alpha_1 = 0$ is transformed into the original BD scalar field.

2. Basic Equation

The modified Einstein-Hilbert action in the BDscalar-tensor theory of gravity is given by

$$S = \int d^4x\sqrt{-g}[\frac{1}{2}(-\phi R + \frac{\omega}{\phi} g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi) + L_m], \quad (1)$$

where R is the Ricci scalar curvature and ϕ is the BD scalar field, which is time dependent and coupled with gravity. [7] And ω is the coupling parameter between the scalar field and gravity, and L_m is the material Lagrangian. Here we use the metric of the Friedman-Robertson-Walker universe.

We assume that the universe is filled with a hidden substance without pressure, baryon matter, and holographic dark energy. Then the variation of the action with the metric tensor yields the field equation as follows:

$$\rho_H + \rho_m = 3\phi H^2 + 3H\dot{\phi} - \frac{\omega}{2\phi}\dot{\phi}^2 \quad (2)$$

where ρ_m and ρ_H are the dark matter and holographic dark energy density, respectively. The advantage here is the differentiation with space time. Many researchers have considered the interaction between the dark energy and the dark matter. [8-12] Recent observations also suggest this possibility of interaction. [13] Considering the interaction between the dark matter and holographic dark energy, the energy conservation equation for the dark matter and holographic dark energy is

$$\dot{\rho}_H + 3H(1+w)\rho_H = -\xi\dot{\rho}_H \quad (3)$$

$$\dot{\rho}_m + 3H\rho_m = \xi\dot{\rho}_H \quad (4)$$

The spaceships factor can then be written as

$$a / a_0 = 1/(1+z), \quad (5)$$

where $a_0 = 1$ is the current value of the spacecraft distance scale factor. From the equations, we can obtain the following expression: Using $\frac{\partial}{\partial t} = -(1+z)H\frac{\partial}{\partial z}$

$$\rho_H' = -\frac{3(1+w)}{(1+z)(1+\xi)}\rho_H, \quad (6)$$

Here, the upper diagonal represents the derivative with the red shift z.

Thus, we can obtain an analytical expression for the holographic scalar field density as follows:

$$\rho_H = \rho_{H_0}(1+z)^{\frac{3(1+w)}{1+\xi}} \quad (7)$$

Considering that $\rho_{H_0} = 3c^2\phi_0 H_0{}^2$, the energy density of the holographic scalar field is

$$\rho_H = 3c^2\phi_0 H_0{}^2(1+z)^{\frac{3(1+w)}{1+\xi}} \quad (8)$$

On the other hand, the analytical expression for the time derivative for the scalar field and the derivative with the red shift can be written as

$$\dot{\phi} = -(1+z)H\phi' \quad (9)$$

$$\phi' = \alpha\alpha'(1+z)^{-1}\phi \quad (10)$$

From the above equation, the Friedman equation with the red shift of the scalar field can be rewritten as

$$\rho_H+\rho_m=H^2(3-3\alpha\alpha'(1+z)^{\alpha-1}-\frac{\omega}{2}\alpha^2\alpha'^2)\phi \qquad (11)$$

Now, by substituting the variables with $A=3-3\alpha\alpha'(1+z)^{\alpha-1}-\frac{\omega}{2}\alpha^2\alpha'^2$, the expression for the energy density of the dark matter is

$$\rho_m=(\mathrm{A}-3c^2)\phi H^2 \qquad (12)$$

Then, from the definition of the consistency parameter and the expression for the energy density of the holographic scalar field, we can derive the following relation:

$$r\equiv\frac{\rho_m}{\rho_H}=\mathrm{E}^2(\mathrm{z})\frac{(\mathrm{A}-3c^2)}{3c^2(1+z)^{3(1+w)/(1+\xi)}} \qquad (13)$$

Then, the weighting function for verification using the observed data can be obtained as follows (13).

$$\mathrm{E}^2(\mathrm{z})=\frac{3c^2r(1+z)^{-3(1+w)/(1+\xi)}}{A-3c^2} \qquad (14)$$

Then the following relation must be used:

$$\alpha'=\alpha_1(1+z)^{-2}-2\alpha_1 z(1+z)^{-3}$$

Then the analytical expression for the obtained weight function is

$$\begin{aligned}E^2(z)=&(3(1+z)^{-\frac{3(1+w)}{1+\xi}}c^2r)(3(1+z)^{\alpha_0+\frac{\alpha_1\cdot z}{(1+z)^2}}+3(-(1+z)^{-2+\alpha_0+\frac{\alpha_1\cdot z}{(1+z)^2}}((1+z)^2\alpha_0+\alpha_1 z+\\&(\alpha_1-\alpha_1 z)\ln(1+z)))-3c^2(1+z)^{\alpha_0+\frac{\alpha_1\cdot z}{(1+z)^2}}-\frac{1}{2}\omega(1+z)^{-4+\alpha_0+\frac{\alpha_1\cdot z}{(1+z)^2}}((1+z)^2\alpha_0+\alpha_1 z+\\&(\alpha_1-\alpha_1 z)\ln(1+z))^2)^{-1}\end{aligned} \qquad (15)$$

Validation of the model using cosmological observations yields the following results. Figure 1 shows the optimum values and ranges of parameters obtained at different significance levels.

3. Conclusion and Discussion

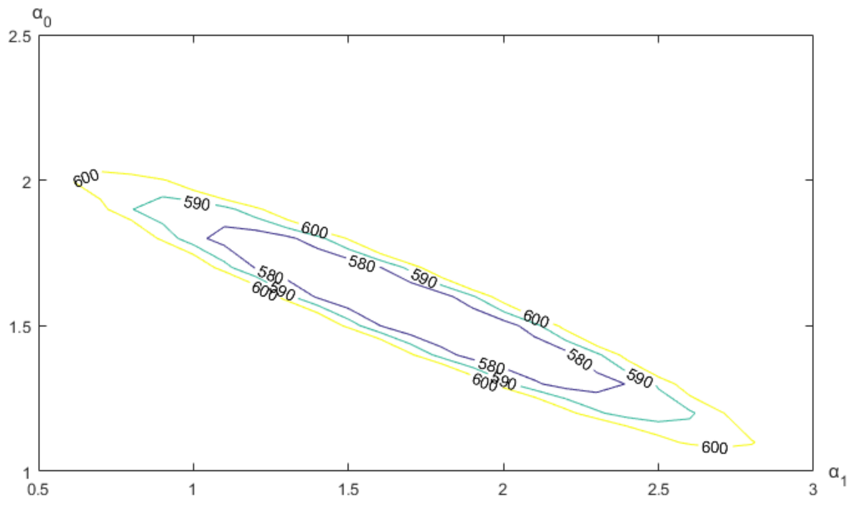

***Figure 1**. The Value Range of the Parameter (α_0, α_1) Obtained at Different Significance Levels ($\Omega_{m_0}=0.267$, $\Omega_{\phi_0}=0.723$).*

The expression for the BD parameter is

$$\omega=-1-\frac{(1+z)^2}{\alpha_0(1+z)^2+\alpha_1 z}. \qquad (16)$$

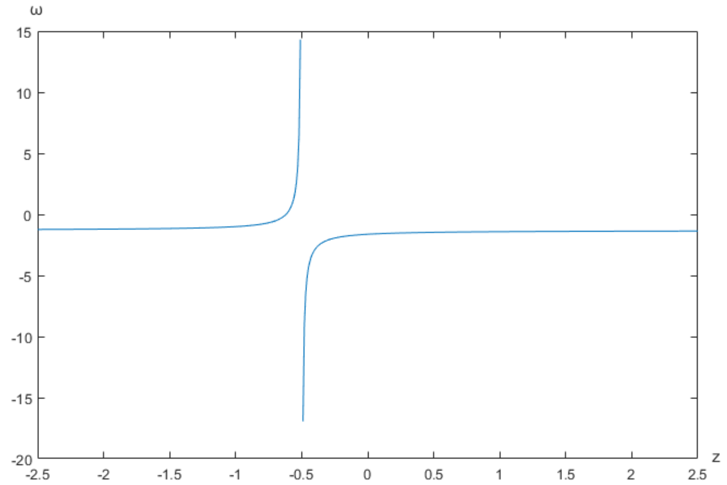

***Figure 2**. Variation of BD Parameters with Red Shift ($\alpha_0 = 0.14$, $\alpha_1 = -0.1$).*

The change with the red shift of BD parameter corresponding to the optimum value of parameter (α_0 , α_1) is shown in Figure 2.

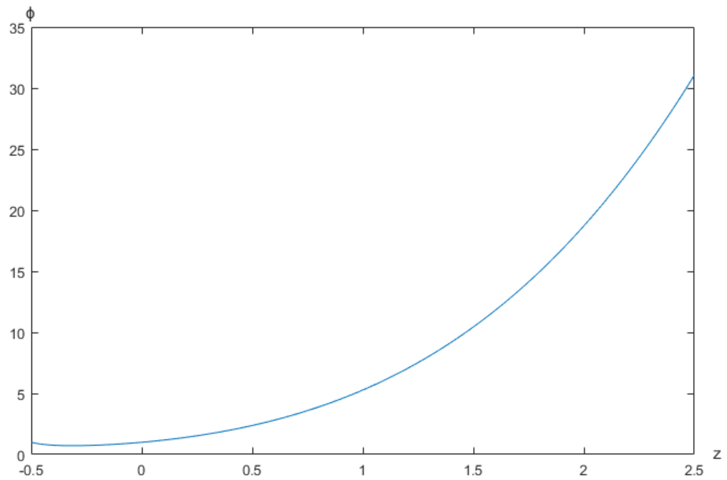

***Figure 3**. Variation of Scalar Fields with Red Shift ($\alpha_0 = 0.14$, $\alpha_1 = -0.1$).*

The variation of the scalar field $0.11^{+0.05+0.03+0.04}_{-0.05-0.03-0.012}$ with the red shift corresponding to the optimum value of the parameter (h , c) is shown in Figure 3. The variation of the scalar field $0.11^{+0.05+0.03+0.04}_{-0.05-0.03-0.012}$ with the red shift corresponding to the optimum value of the parameter (h , c) is shown in Figure 3. From Figure 3, we can see that the scalar field decreases from the past to the present. It is also found that the BD scalar field increases rapidly from past to present.

This means that the scalar field plays an important role in cosmic evolution [14-17].

The optimum values of the model parameters obtained at different significance levels are given in the table below.

***Table 1**. The Optimum Values of Parameters and Tolerance Ranges of Parameters Obtained at the Significance Level of 68.3%, 95.5%, and 99.7%.*

α_0	α_1	w	h	c
$0.14^{+0.06+0.02+0.04}_{-0.05-0.01-0.012}$	$-0.1^{+0.03+0.006+0.023}_{-0.13-0.005-0.032}$	$-1.102^{+0.042+0.079+0.134}_{-0.031-0.068-0.136}$	$0.72^{+0.03+0.035+0.035}_{-0.03-0.035-0.046}$	$0.9^{+0.077+0.024+0.030}_{-0.017-0.053-0.029}$

We now consider the change with the redshift z of the deceleration parameter q. The variation of the deceleration parameter with the red shift corresponding to the optimum values of the model parameters is shown in the figure below.

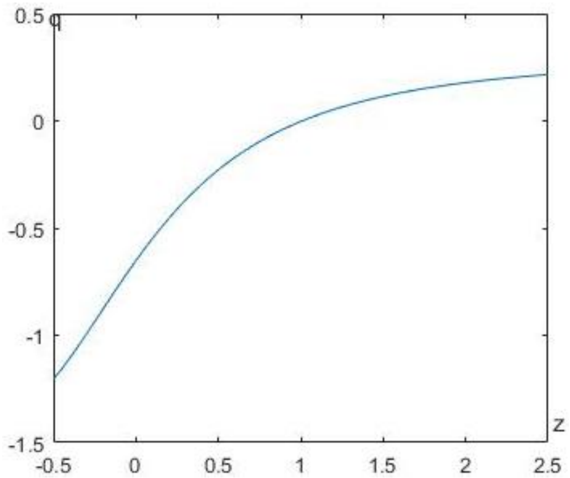

Figure 4. *Variation of the Deceleration Parameter with Red Shift ($\alpha_0 = 0.14$, $\alpha_1 = -0.1$).*

From the above figure, we can see that the current value of the transition red shift and the deceleration parameter is z_t=-0.6, q_0=-0.63. This is consistent with the results obtained in the literature. [18-27] The above figure shows the variation of the consistency parameter r with the red shift z, corresponding to the optimum values of the model parameters. Figure 5 shows the variation of the consistency parameter r with the red shift z, corresponding to the optimum values of the model parameters.

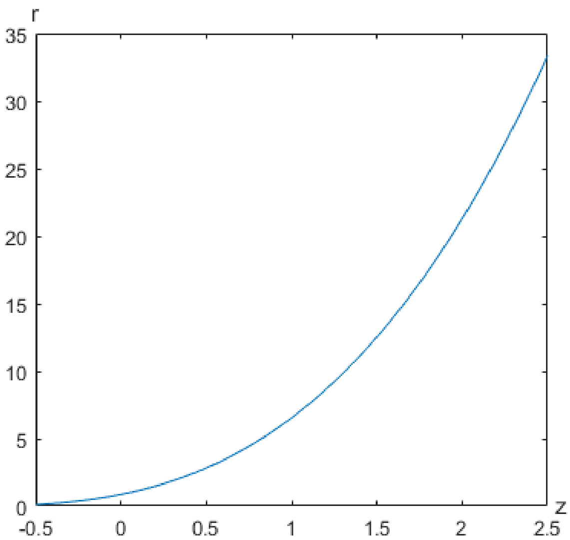

Figure 5. *Variation of the Consistency Parameter with Red Shift ($\alpha_0 = 0.14$, $\alpha_1 = -0.1$).*

From the figure, it can be seen that the consistency parameter is preserved at red shift $z_t = 0.41$, and the current value of the consistency parameter is $r_0 = 0.37$. By our model, a negative value of the interaction coupling strength between the dark matter and the holographic hidden energy is obtained, which is consistent with previous data and defines the direction of energy transport. Meanwhile, the rate of change G in our model is

$$\left|\frac{\dot{G}}{G}\right| \sim \left|\frac{\dot{\phi}}{\phi}\right| = (1+z)[\alpha' \ln(1+z) + \frac{\alpha}{1+z}]H.$$

From the above expression, we can see that the rate of change G at the present time is only dependent on α_0, and using $H_0 = 70km/s \cdot Mpc = 7.15\times10^{-11}yr^{-1}$

$$\left|\frac{\dot{G}}{G}\right| \sim \left|\frac{\dot{\phi}}{\phi}\right|_{z=0} = \alpha_0 H_0 = 1.001\times10^{-11}yr^{-1},$$

Abbreviations

BD Brans-Dicke
JBP Jassal-Bagular-Padmanabhann

Funding

This research received no external funding.

Conflicts of Interest

The authors declare no conflicts of interest.

References

[1] Nojiri, S., et al., The oscillating dark energy: future singularity and coincidence problem. Phys. Lett. B 2006, 637, 139-148. https://doi.org/10.1016/j.physletb.2006.04.026

[2] Yang, W., et al., Novel approach toward the large-scale stable interacting dark-energy models and their astronomical bounds. Phys. Rev. D 2017, 96(12), 123508(16). https://doi.org/10.1103/PhysRevD.96.123508

[3] Banerjee, D., et al., Brans-Dicke brain cosmology, Phy. Lett. 2001, 501, 249-256. https://doi.org/10.1016/s0370-2693(01)00117-4

[4] Huang, P., et al., The Stability of Holographic Dark Energy in Non-flat Universe. Int. J. Theor. Phys. 2014, 53, 10-16. https://doi.org/10.1007/s10773-013-1778-x

[5] Samanta, G. C., Holographic Dark Energy (DE) Cosmological Models with Quintessence in Bianchi Type-V Space Time. Int. J. Theor. Phys. 2013, 52, 4389-4402. https://doi.org/10.1007/s10773-013-1757-z

[6] Raut, V. M., Holographic Dark Energy Cosmological Models in f (g) Theory, New Astronomy, 80, 2020, 101420. https://doi.org/10.1016/j.newast.2020.101420

[7] Yazhou, H., et al., Holographic dark energy with cosmological constant, Journal of Cosmology and Astroparticle Physics, 2015, 12, 2-19. https://doi.org/10.1088/1475-7516/2015/08/012

[8] Pieter, B., et al., Holographic generalised Gukov-Witten defects, SISSA, 2025, 118, 3-41. https://doi.org/10.1007/JHEP03(2025)118

[9] Ryong Gwang Kim et al., Observational constraints on cosmicparameters of holographic dark energy model with varying interaction, Astrophysics and Space Science, 2020, 365, 128-133. https://doi.org/10.1007/s10509-020-03842-1

[10] Chevallier, M., et al., Accelerating Universes with Scaling Dark Matter. Int. J. Mod. Phys. 2001, 10, 213. https://doi.org/10.1142/s0218271801000822

[11] Majeed, B., et al.: Holographic Dark Energy with Varying n2 Parameter in Non-Flat Universe. Int. J. Theor. Phys. 2015, 54, 42-49. https://doi.org/10.1007/s10773-014-2197-3

[12] Wetterich, C., et al.: Phenomenological parameterization of quintessence. Phys. Lett. 2004, 594, 17. https://doi.org/10.1016/j.physletb.2004.05.008

[13] Jahromi, A. S., et al., Generalized entropy formalism and a new holographic dark energy model. Phys. Lett. B 2018, 780, 21-24. https://doi.org/10.1016/j.physletb.2018.02.052

[14] Sarkar, S., Interacting holographic dark energy, cosmic coincidence and the future singularity of the closed FRW universe. New Astronomy. 2015, 34, 144-150. https://doi.org/10.1016/j.newast.2014.07.003

[15] Ryong Gwang Kim et al., A type of holographic dark energy interacting with dark matter and observational constraints, Astrophysics and Space Science, 2021, 366, 33-37. https://doi.org/10.1007/s10509-021-03941-7

[16] Ryong Gwang Kim et al., Generalized holographic dark energy and cosmic coincidence, New Astronomy, 2021, 84, 101540. https://doi.org/10.1016/j.newast.2020.101540

[17] Ryong Gwang Kim et al., Holographic Dark Energy Cosmology with a new coupling function, Indian Journal of Physics, 2021, 367, 567-573. https://doi.org/10.1007/s12648-021-02194-y

[18] Thomas, H., et al., Holographic (eternal) inflation, SISSA, 2025, 061, 1-20. https://doi.org/10.1007/JHEP07(2025)061

[19] Abdollahi, M., Z., et al., Note on sign-changeable holographic dark energy in BDtheory, Can. J. Phys, 2018, 67, 1-13. https://doi.org/10.48550.arXiv.1610.08093

[20] Amornthep, T., Cosmological dynamics of holographic dark energy with non-minimally coupled scalar field, General Relativity and Gravitation, 2025, 57, 106, 1-25. https://doi.org/10.1007/s10714-025-03442-W

[21] Ualikhanova, U., Holographic reconstruction of k-essence model with Tsallis and the most generalized Nojiri-Odintsov version of holographic dark energy, Indian J Phys, 2025, 99, 4433-4441. https://doi.org/10.1007/s12648-025-03606-z

[22] Singh, C. P., Kumar, P., Holographic dark energy in BDtheory with logarithmic form of scalar field, Int. J. Theor. Phys. 2017, 56, 3297-3310. https://doi.org/10.1007/s10773-017-3496-2

[23] Peebles, P. J. E., Principles of physical cosmology, Princeton University Press, Princeton, 1993, 57, 28-43. https://doi.org/10.1515/9780691206721

[24] Peebles, P. J. E., Testing general relativity on the scales of cosmology, in proceedings of the 17th international conference on general relativity and gravitation, astro-ph/, 2004, 0410284, 46. https://doi.org/10.1142/9789812701688_0010

[25] Bak, D., et al., Classical and Quantum Gravitation. Phys. Rev. D, 2000, 62, 084035. https://doi.org/10.1103/PhysRevD.62.084035

[26] Sheykhi, A., et al., New holographic dark energy model inspired by the DGP braneworld. IJMPD, 2016, 25, 1650018. https://doi.org/10.1142/S0218271816500188

[27] Wang, B., et al., Dark Matter and Dark Energy Interactions: Theoretical Challenges, Cosmological Implications and Observational Signatures. Rep. Prog. Phys. 2017, 696, 1. https://doi.org/10.1088/0034-4885/79/9/096901