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Research Article

Comparing Two-parameter Burr Type (X) and Gamma-Weibull Distributions Using Information Criteria for the Heights of Akwa Ibom State University Students

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Abstract

Overtime, many researchers have introduced different probability distribution functions and fitted them to a given datasets using Information Criteria. Among the distribution introduced are the four-parameter Gamma-Weibull, the beta-Weibull distribution, four-parameter beta-normal distribution provides flexibility in modelling not only symmetric heavy-tailed distributions, but also skewed and bimodal distributions, gamma-Lagrange distribution, the Kumaraswamy-Weibull Geometric distribution, beta-Laplace distribution, Kumaraswamy-generalized Exponential Pareto distribution. The AIC and BIC for the Kumaraswamy-generalized Exponential Pareto distribution are smaller than the Pareto and Exponential Pareto distribution. Thus, making Kumaraswamy-generalized Exponential Pareto distribution very competitive for the fitting an uncensored data set corresponding to 100 observations on breaking stress of carbon fibers (in Gba) using the model selection criteria. This paper compares two-parameter Burr type (X), a special case of the Beta-Weibull distribution and four-parameter Gamma-Weibull distribution using log-likelihood function, Bayesian and Akaike's Information Criteria for fitting heights of Akwa Ibom State University Students. The heights of 617 students were obtained from the medical Centre of the Akwa Ibom State University main Campus. It was observed that the log-likelihood function, Akaike information criterion (AIC) and the Bayesian information criterion (BIC) values of the Gamma-Weibull distribution are less than that of the two-parameter Burr type (X). The Gamma-Weibull distribution has a smaller AIC and BIC than that of the two-parameter Burr Type (X) distribution. Hence, the Gamma-Weibull distribution fits the data better than the two-parameter Burr Type (X) distribution. The graphs of the Gamma-Weibull distribution and the two-parameter distributions are also presented.

Keywords

Two-parameter Burr Type (X) Distribution, Gamma-Weibull Distribution, Bayesian Information Criterion, Akaike's Information Criterion, Log-likelihood Function, Heights of Students

1. Introduction

Overtime, many researchers have introduced different probability distribution functions and fits them to a given datasets using Information Criteria. Eugene et al. introduced a generalized class of distributions based on the logit of the beta random variable with cumulative distribution function (cdf) and probability density function (pdf) [1]. The authors further

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introduced a four-parameter beta-normal distribution and applied the beta-normal model to two empirical data sets and compared the results to that of gamma and gamma-Lagrange. It was observed that the beta-normal distribution provides flexibility in modelling not only symmetric heavy-tailed distributions, but also skewed and bimodal distributions.

Famoye et al. introduced and studied the beta-Weibull distribution [2]. The two-parameter Burr Type (X) [3] is a special case of the beta-Weibull distribution when $\beta = 1$ and $c = 1$. Alzaatreh et al. introduced the Gamma-Weibull distribution with the generalized gamma and gamma distribution as special cases [4].

Cordeiro and Lemonte, introduced, studied the beta-Laplace distribution and compared the fits of the beta-Laplace, Laplace, beta normal, beta standard normal distribution and the normal distribution using a real data set of the national index of consumer prices (INPC) of Brazil corresponding to health and personal care, produced by the Brazil Institute of Geography and Statistics (IBGE) (www.ibge.gov.br) [5]. As noted by [5], the beta-Laplace (BL) distribution yielded the highest value of the log-likelihood and the smallest value of the Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC) statistic, making the BL better than the other distributions for the data set. The AIC is a criterion that balances a model's quality of fit with a penalty for the number of parameters, whereas the BIC is a criterion that balances a model's goodness of fit with a penalty for the number of parameters and sample size [6]. The Akaike Information Criterion has been used by a number of authors, including ([7-9]), among others, to assess the goodness of fit in their chosen models in order to get a dependable and acceptable result.

Rasekhi et al. introduced and studied the Kumaraswamy-Weibull Geometric distribution (Kw-WG) [10]. The Exponential Geometric [11], Kumaraswamy-Rayleigh [12], Weibull-Geometric [13] are some of the special cases of the Kw-WG. The survival and hazard rate function, moment generating function, the cumulative distribution (cdf), Renyi and quantile entropies, mean deviation, mean residual life and mean activity time, order statistics and its moments are discussed. The maximum likelihood estimation of parameters of the Kw-WG have been demonstrated and fisher information criteria. According to [10], the Kw-WG density function contains decreasing, right and left skew uni-bimodal shape and the hazard function of this distribution contain increasing, decreasing, unimodal, bathtube and non-monotone shape. The listed properties signify that Kw-WG is flexible.

Haq [14], applied the Kumaraswamy exponentiated inverse Rayleigh distribution (KEIR) on real data set of strength measured in GPa for single carbon fibers and impregnated 1000-carbon fiber tows which were originally reported by [15]. Compared the result obtained from Kumaraswamy exponentiated inverse Rayleigh distribution (KEIR) to Inverse Rayleigh distribution (IR) introduced by [16] and Exponentiated Inverse Rayleigh distribution using Akaike Information Criterion, Consistent Akaike Information Criterion (CAIC), Bayesian information criterion, Hannan-Quinn information criterion (HQIC), Kolmogorov Smirnov test (KS) and graph the fitted densities. From the graph and results of the information criteria, it was observed that the KEIR model is a better model than EIR and IR.

Shams, fitted the Kumaraswamy-Generalized Exponential Pareto distribution, Pareto, Exponential-Pareto distributions to an uncensored data set corresponding to 100 observations on breaking stress of carbon fibers (in Gba) using the model selection criteria namely; the AIC, BIC and CAIC [17]. The results showed that the values of the AIC, BIC and CAIC for the Kumaraswamy-generalized Exponential Pareto distribution are smaller than the Pareto and Exponential Pareto distribution. Thus, making Kumaraswamy-generalized Exponential Pareto distribution very competitive for the data set.

Nassar fitted the Kumaraswamy-Laplace (KL) [18], the beta-Laplace [5] to a given data set using the AIC and the BIC. The KL model had a minimum value of the AIC and BIC, implying that the KL model provides consistently better fit than the BL for the given data set.

Bourguignon et al fitted the Kw-P to the 72 exceedances of flood peaks $(in\, M^3/s)$ of the Wheaton River near Carcross in Yukon Territory, Canada [19]. It was observed that the value of the AIC, BIC, and CAIC were smaller for the Kw-P distribution compared to the Pareto and Exponentiated Pareto, Beta-Pareto distributions.

Pascoa et al., fitted the Kumaraswamy Generalised gamma to a data set on the time to serum reversal of 148 children exposed to human immunodeficiency virus (HIV) by vertical transmission, born at Hospital das Clinicas from1995-2001 [20], where the mothers were not treated [21, 22], using the AIC, BIC and CAIC statistics. The results indicated that the KumGG model had the lowest AIC, BIC and CAIC statistics values among the fitted models, and as such chosen to be the best model. Pascoa et al. also used the likelihood ratio (LR) to compare the KumGG with some of its sub-models and the p-values suggested that the KumGG model yields a better fit to these data than the other three distributions namely; The generalized gamma distribution (GG) introduced by [23], Exponentiated Generalized gamma (EGG) and Weibull distribution [20].

Haq compared the fitting of KEIR, Inverse Rayl [16] and Exponentiated Inverse Rayleigh (EIR), using the AIC and BIC [14]. The author fitted the three models using the real data set of strength measured in GPA for single carbon fibers and impregnated 1000-carbon fiber tows which were originally reported by [15]. According to [14], the KEIR distribution gave the smallest values of the goodness of fit measures for the AIC and BIC when compared to EIR and IR. As such, the KEIR is better comparatively to the IR and EIR distribution. Selim and Badr compared Weibull, generalized power Weibull (GPW) and the Kumaraswamy Generalized Power Weibull Distribution (KGPW) using the AIC. KGPW had the smallest value of the AIC when compared to Weibull and

GPW distributions [24]. As such, the KGPW distribution performs better than the Weibull and the GPW distributio.

Several authors [25-29] have used the maxLik package in R introduced by [30] for maximum likelihood estimation of a model's parameters of the Normal, Gamma, Log-normal and Logistic distributions.

This research work compares the Beta-Exponential and two-parameter Burr-Type (X) distributions [2] using the likelihood function, the Akaike information criterion, the Bayesian information criterion, and its application to heights of 617 students of Akwa Ibom State University. The heights of 617 students of Akwa Ibom State University were obtained from the Medical Centre of the University. The maxLik package will be used for parameters estimation.

2. Methodology

2.1. Likelihood Function for Multi-parameter Case

Hogg et al. stated the likelihood function for multi-parameter case as follows [31]:

Let $X_1, X_2, \dots, X_n$, be independent and identically distributed with probability density function or probability mass function $f(x;\theta)$, where n is a fixed positive integer, and $\theta \epsilon \Omega \subset R^p$. The likelihood function and its log are given by

$$L(\theta) = \prod_{i=1}^{n} f(x_i, \theta) \tag{1}$$

$$\ell(\theta) = \log L(\theta) = \sum_{i=1}^{n} \log f(x_i, \theta) \tag{2}$$

θ is a vector of parameters and Ω is the parameter space

2.2. Information Criteria

In fitting of statistical models, the goodness of fit test is examined through the model evaluation criteria. In this work, two model evaluation criteria will be used.

According to Jong and Heller [6], the Akaike's Information Criterion is

$$AIC \equiv -2\ell + 2p \tag{3}$$

and the Bayesian Information Criterion (BIC) is

$$BIC \equiv -2\ell + p\ln(n) \tag{4}$$

p is the number of parameters and ℓ is the log-likelihood function of a given probability model and n is the sample size.

3. Models

3.1. The Gamma-Weibull Distribution

The pdf of the gamma-Weibull [4] is given by

$$g(x) = \frac{c}{\gamma^{\alpha c}\Gamma(\alpha)\beta^{\alpha}} x^{\alpha c-1} e^{-\frac{1}{\beta}\left(\frac{x}{\gamma}\right)^{c}}, x > 0; \alpha, \gamma, \beta, c > 0 \tag{5}$$

3.1.1. Likelihood Function of the Gamma-Weibull Distribution

The pdf of the gamma-Weibull distribution is given by

$$g(x) = \frac{c}{\gamma^{\alpha c}\Gamma(\alpha)\beta^{\alpha}} x^{\alpha c-1} e^{-\frac{1}{\beta}\left(\frac{x}{\gamma}\right)^{c}}, x > 0; \alpha, \gamma, \beta, c > 0$$

The likelihood function is defined by

$$L_{GW} = \prod_{i=1}^{n} g(x_i) = \prod_{i=1}^{n}\left(\frac{c}{\gamma^{\alpha c}\Gamma(\alpha)\beta^{\alpha}} {x_i}^{\alpha c-1} e^{-\frac{1}{\beta}\left(\frac{x_i}{\gamma}\right)^{c}}\right)$$

$$L_{GW} = \left(\frac{c}{\gamma^{\alpha c}\Gamma(\alpha)\beta^{\alpha}}\right)^{n} \left(\prod_{i=1}^{n}({x_i}^{\alpha c-1})\right) e^{-\sum_{i=1}^{n}\frac{1}{\beta}\left(\frac{x_i}{\gamma}\right)^{c}}$$

The log-likelihood of the gamma-Weibull distribution denoted ℓ_{GW} is defined by

$$\ell_{GW} = \log\left(\left(\frac{c}{\gamma^{\alpha c}\Gamma(\alpha)\beta^{\alpha}}\right)^{n} \left(\prod_{i=1}^{n}({x_i}^{\alpha c-1})\right) e^{-\sum_{i=1}^{n}\frac{1}{\beta}\left(\frac{x_i}{\gamma}\right)^{c}}\right)$$

$$\ell_{GW} = nlogc - n\alpha clog\gamma - nlog\Gamma(\alpha) - n\alpha log\beta + (\alpha c - 1)\sum_{i=1}^{n}\log(x_i) - \frac{1}{\beta}\sum_{i=1}^{n}\left(\frac{x_i}{\gamma}\right)^{c} \tag{6}$$

3.1.2. Parameters Estimation of the Gamma-Weibull Distribution

The maximum likelihood estimators of the gamma-Weibull distribution parameters α, β, c and γ are obtained by differentiating the log-likelihood function in (6) with respect to the parameters α, β, c and γ and equating to zero, we have

$$\frac{\partial \ell_{GW}}{\partial c} = \frac{n}{c} + n\alpha log\gamma + \alpha\sum_{i=1}^{n}\log(x_i) - \frac{1}{\beta}\sum_{i=1}^{n}\left(\left(\frac{x_i}{\gamma}\right)^{c}\ln\left(\frac{x_i}{\gamma}\right)\right) = 0 \tag{7}$$

$$\frac{\partial \ell_{GW}}{\partial \gamma} = -\frac{n\alpha c}{\gamma} + \frac{c}{\beta\gamma}\sum_{i=1}^{n}\left(\frac{x_i}{\gamma}\right)^{c} = 0 \tag{8}$$

$$\frac{\partial \ell_{GW}}{\partial \alpha} = nclog\gamma - \frac{n}{\Gamma(\alpha)}\frac{\partial \Gamma(\alpha)}{\partial \alpha} - nlog\beta + c\sum_{i=1}^{n}\log(x_i) = 0 \tag{9}$$

$$\frac{\partial \ell_{GW}}{\partial \beta} = -\frac{n\alpha}{\beta} + \frac{1}{\beta^2}\sum_{i=1}^{n}\frac{x_i^c}{\gamma^c} = 0 \quad (10)$$

The numerical solution of the nonlinear equations (7)-(10), is the maximum likelihood estimates of the parameters α, c and γ denoted by $\hat{\alpha}, \hat{\beta}, \hat{c}$ *and* $\hat{\gamma}$.

3.1.3. AIC and BIC for the Gamma-Weibull Distribution

The Akaike's Information Criterion (AIC) for the gamma-Weibull distribution will be given by

$$AIC_{GW} = -2\left(nlog\hat{c} - n\hat{\alpha}\hat{c}log\hat{\gamma} - nlog\Gamma(\hat{\alpha}) - n\hat{\alpha}log\hat{\beta} + (\hat{\alpha}\hat{c}-1)\sum_{i=1}^{n}\log(x_i) - \frac{1}{\hat{\beta}}\sum_{i=1}^{n}\left(\frac{x_i}{\hat{\gamma}}\right)^{\hat{c}}\right) + 2p \quad (11)$$

And

The Bayesian Information Criterion (BIC) for the gamma-Weibull distribution is given by

$$BIC_{bW} = -2\left(nlog\hat{c} - n\hat{\alpha}\hat{c}log\hat{\gamma} - nlog\Gamma(\hat{\alpha}) - n\hat{\alpha}log\hat{\beta} + (\hat{\alpha}\hat{c}-1)\sum_{i=1}^{n}\log(x_i) - \frac{1}{\hat{\beta}}\sum_{i=1}^{n}\left(\frac{x_i}{\hat{\gamma}}\right)^{\hat{c}}\right) + 2\ln(n) \quad (12)$$

3.2. The Two Parameters Burr- Type (X) Distribution

When $\beta = 1$ and $c = 2$, the beta-Weibull distribution reduces to the two-parameter Burr-Type(X) distribution [Johnson et al, (1994, page 54)] with density function.

$$f(x) = \frac{2\alpha}{\gamma^2}x\left[1 - e^{-\left(\frac{x}{\gamma}\right)^2}\right]^{\alpha-1}.e^{-\left(\frac{x}{\gamma}\right)^2}$$

3.2.1. Likelihood Function of the Two Parameters Burr- Type (X) Distribution

When $\beta = 1$ and $c = 2$, the beta-Weibull distribution reduces to the two-parameter Burr-Type(X) distribution [Johnson et al, (1994, page 54)] with density function.

$$f(x_i) = \frac{2\alpha}{\gamma^2}x_i\left[1 - e^{-\left(\frac{x_i}{\gamma}\right)^2}\right]^{\alpha-1} e^{-\left(\frac{x_i}{\gamma}\right)^2}$$

The likelihood function of the Two Parameters Burr- Type (X) Distribution

$$L_{BT} = \prod_{i=1}^{n} f(x_i) = \prod_{i=1}^{n}\frac{2\alpha}{\gamma^2}x_i\left[1 - e^{-\left(\frac{x_i}{\gamma}\right)^2}\right]^{\alpha-1} e^{-\left(\frac{x_i}{\gamma}\right)^2}$$

$$L_{BT} = \left(\frac{2\alpha}{\gamma^2}\right)^n \prod_{i=1}^{n}\left(x_i\left[1 - e^{-\left(\frac{x_i}{\gamma}\right)^2}\right]^{\alpha-1}\right).e^{-\sum_{i=1}^{n}\left(\frac{x_i}{\gamma}\right)^2}$$

Taking the natural log and denoting it by ℓ_{BT}, we have the log-likelihood function

$$\ell_{BT} = nln(2\alpha) - 2nln(\gamma) + \sum_{i=1}^{n}\ln(x_i) + (\alpha-1)\sum_{i=1}^{n}\ln\left(1 - e^{-\left(\frac{x_i}{\gamma}\right)^2}\right) - \sum_{i=1}^{n}\left(\frac{x_i}{\gamma}\right)^2 \quad (13)$$

3.2.2. Parameters Estimation of the Two Parameters Burr-Type(X) Distribution

The maximum likelihood estimators of the two-parameters Burr-Type (X) Distributionα, *ad* γ are obtained by differentiating the log-likelihood function (13) with respect to the parameters α and γ and equating to zero, we have

$$\frac{\partial \ell_{BT}}{\partial \alpha} = \frac{n}{\alpha} + \sum_{i=1}^{n}\ln\left(1 - e^{-\left(\frac{x_i}{\gamma}\right)^2}\right) = 0 \quad (14)$$

$$\frac{\partial \ell_{BT}}{\partial \gamma} = -\frac{2n}{\gamma} - \frac{2(\alpha-1)}{\gamma}\sum_{i=1}^{n}\left(\frac{\left(\frac{x_i}{\gamma}\right)^2}{1-e^{-\left(\frac{x_i}{\gamma}\right)^2}}\right) + 2\sum_{i=1}^{n}\frac{\left(\frac{x_i}{\gamma}\right)^2}{\gamma} = 0 \quad (15)$$

The numerical solution of the nonlinear equations (14) - (15), are the maximum likelihood estimates of the parameters α and γ denoted by $\hat{\alpha}$ *and* $\hat{\gamma}$.

3.2.3. AIC and BIC for Two Parameters Burr-Type(X) Distribution

The Akaike's Information Criterion (AIC) for two parameters Burr-Type(X) distribution will be given by

$$AIC_{BT} = -2\left(nln(2\hat{\alpha}) - 2nln(\hat{\gamma}) + \sum_{i=1}^{n}\ln(x_i) + (\hat{\alpha}-1)\sum_{i=1}^{n}\ln\left(1 - e^{-\left(\frac{x_i}{\hat{\gamma}}\right)^2}\right) - \sum_{i=1}^{n}\left(\frac{x_i}{\hat{\gamma}}\right)^2\right) + 2p \quad (16)$$

$p = 2$; number of parameters; $\hat{\alpha}$ *and* $\hat{\gamma}$ are the estimated parameters.

The Bayesian Information Criterion (BIC) for two parameters Burr-Type(X) distribution is given by

$$BIC_{BT} = -2\left(nln(2\hat{\alpha}) - 2nln(\hat{\gamma}) + \sum_{i=1}^{n} \ln(x_i) + (\hat{\alpha}-1)\sum_{i=1}^{n} \ln\left(1 - e^{-\left(\frac{x_i}{\hat{\gamma}}\right)^2}\right) - \sum_{i=1}^{n}\left(\frac{x_i}{\hat{\gamma}}\right)^2\right) + 2\ln(n) \quad (17)$$

Table 1. *Maximum Likelihood Estimation of the model parameters for the Heights of Students data and the measures AIC and BIC for Gamma-Weibull and Two parameters Burr-Type(X) distribution.*

MODEL	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\gamma}$	$\hat{c}$	ℓ	AIC	BIC
Beta-exponential	2.138	0.000001367	5.857	9.968	13803.41	-27598.52	-27581.12
Two parameters Burr-Type(X) Distribution	6248.487	-	0.5478	-	593.8144	-1183.629	-1174.779

4. Discussion

The Table 1 provides the Maximum Likelihood Estimates of the model parameters for the Heights of Students data and the log-likelihood function, the measures of AIC and BIC. The

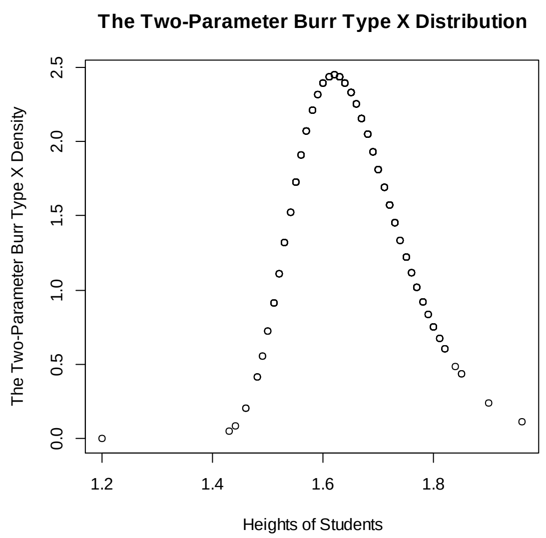

Figure 1. *Plots of the Two-Parameter Burr Type (X) Density and The Heights of Students.*

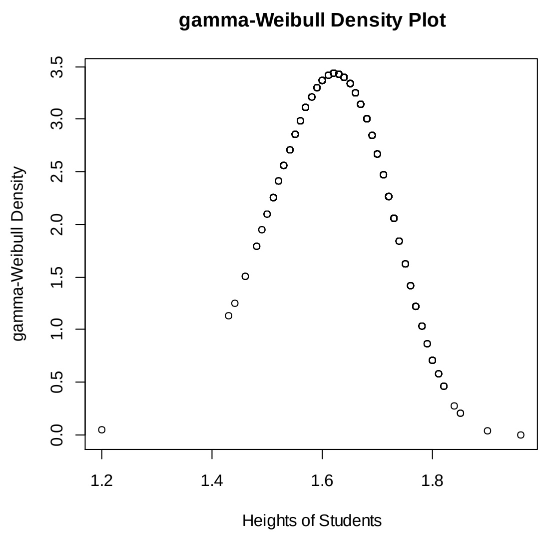

Figure 2. *Plots of the Gamma-Weibull Density and the heights of student.*

Gamma-Weibull distribution (GWD) is a four-parameter probability distribution with parameters; α, β, γ, c. The parameters values for the heights of students are $\hat{\alpha} = 2.138, \hat{\beta} = 0.000001367, \hat{\gamma} = 5.857, \hat{c} = 9.968$, with log-likelihood value $\ell = 13803.41$ with AIC and BIC values of -27598.82 and -27581.12 respectively.

The two-parameter Burr-Type (X) distribution is a two-parameter probability distribution with parameters; α, γ. The parameter values for the heights of students are $\hat{\alpha} = 6248.487, , \hat{\gamma} = 0.5478$ with log-likelihood value $\ell = 593.8144$. The Akaike information Criterion (AIC) is -1183.629 and the Bayesian Information Criterion is -1174.779.

As shown in Eqs. 6 and 13, ℓ_{GW} and ℓ_{TTPBTX} denote the log-likelihood functions of the Gamma-Weibull and the two-parameter Burr-Type(X) distributions respectively.

$$\ell_{GW} > \ell_{TTPBT(X)} \equiv 13803.41 > 593.8144$$

Based on the log-likelihood values for the selected probability distributions, the Gamma-Weibull model is the best fit for the heights of students than the two-parameter Burr Type (X) model.

Based on the Akaike's information criterion (AIC) for distributions under study, AIC(Gamma-Weibull) = -27598.82 < AIC(two-parameter Burr-Type(X)) =-1183.629. The Gamma-Weibull distribution has the lowest Akaike information criterion value for the heights of students than the two-parameter Burr Type (X) distribution. Hence, the gamma-Weibull distribution is better than the two-parameter Burr Type (X) for the heights of Akwa Ibom State University Students.

Based on the Bayesian information criterion (BIC) for distributions under study, BIC (Gamma-Weibull) = -27598.82 < BIC (two-parameter Burr-Type(X)) = -1174.779. the Gamma-Weibull distribution has the lowest Bayesian information criterion value for the heights of students than the two-parameter Burr Type X distribution. The lower the BIC the better the model. Thus, the gamma-Weibull distribution is better than the two-parameter Burr Type X distribution.

5. Conclusion

In order to match a particular dataset (the heights of Akwa Ibom State University students) using information criteria, this study used two distinct probability distribution functions: the gamma-Weibull distribution and the two-parameter Burr Type X distribution. Based on the log-likelihood, Akaike's information criterion, and Bayesian information criteria, the results demonstrate that the four-parameter gamma-Weibull distribution outperforms the two-parameter Burr Type X distribution. Given that height is a good example of a normal distribution, it will be useful to conclude that when analyzing data from a normal distribution, the gamma-Weibull distribution should be prioritized.

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Conflicts of Interest

The authors declare no conflicts of interest.

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