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Research Article

# Modification of Automobile Shock Absorbers Thickness and Weight Parameters for Adaptation on Poorly Maintained Roads via Mathematical Modelling

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## Abstract

Uneven oscillatory forces applied to an object cause vibration, which can lead to noise, discomfort, and mechanical wear. The optimization of shock absorber thickness and weight is critical for enhancing vehicle performance on poorly maintained roads. In automobiles, vibrations due to irregular road surfaces have been mitigated using shock absorbers to enhance performance, comfort, and control. The main aim of this study was to determine the effect of automotive weights and shock absorber thickness on automobile vibrations. The objectives of this study were; to formulate a mathematical model describing the effect of automobile weight on unsteady vibrations; to generate numerical solutions of the mathematical model equations for the shock absorber modified to reduce vibrations at the mounting points; and to analyze from the numerical solution the effects of increase in velocity and weight on the automobile unsteady spring vibration. A mathematical model was developed, utilizing the central difference scheme for discretization and solved using the Jacobian iterative method, with stability conditions implemented. Through computational simulations, various thickness and weight configurations were tested under diverse road conditions. The results indicate a significant improvement in shock absorber performance, with optimized configurations that increasing shock absorber thickness reduced vibration amplitude by approximately 35%, while increased vehicle weight amplified vibrations by 20%, necessitating thicker shock absorbers for stability. Higher speeds above 80 km/h intensified unsteady vibrations on poorly maintained roads. The study concludes that modifying shock absorber thickness and optimizing weight distribution can effectively minimize automobile vibrations.

## Keywords

Vibration Damping, Shock Absorber, Automotive Weights, Shock Absorber Thickness

## 1. Introduction

Automobile designers and engineers have over the years improved the design of shock absorbers so as to improve automobile performance. The main causes of unsteady vibration affecting the users of automobile are kinematic disturbances resulting from road surface irregularities. Elimination of these vibrations is essential in order to improve both the comfort and the safety of the passenger. When the vehicle is driving across a road with large irregularities (obstacles), its wheels might

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get separated from surface of the road, within turn decrease the efficiency of force transmission of the drive, braking and steering system of the car. an improve driving dynamic and better road traction on curves and bumps can be achieved by using the so-called 'hard suspension'. However, the cost is the reduction of the passengers. The criteria for assessing the quality of shock absorbers should therefore include both the minimization of car body vibration and appropriate wheel road adhesion [4]. Dampers use in the suspension system can be passive, semi-active or active. Dynamical properties of the dampers are usually defined by models with hysteresis characteristics, such as Bingham [14] or Spenscer model [13]. Requirements set for the comfort and safety of driving can be fulfilled by using semi-active suspension systems, introduced by [12]. In comparison to passive ones, the semi-active systems allow the damping force to be adjusted depending on driving conditions. Additionally, they require less power than similar active systems. Several methods of control have been used, as presented by [9] and [1] as well as [3], when analyzing on-off control, assume that the damping force should be high if the product of relative and absolute velocity is more than zero. Analyzed the relation between parameters of the car suspension system and the driving comfort as well as the safety indexes [7]. They defined the comfort index as the effective acceleration value while the safety index as the effective ratio of the dynamic and static response. In the study by [10], the results were presented for the theoretical and experimental half-car model, in which the car suspension was controlled by two separate magneto-rheological dampers (MR damper). Some interesting options for control of a semi-active car suspension were presented by [5]. The most common model analyzed was the quarter-car one. In the study-state case, the response to the harmonic excitation was analyzed, while in the transient one [5], the response to the unit step. To ensure a compromise between the requirements for both comfort and safety, hybrid control with a MR damper is used [2] and a combination of sky-hook and ground-hook control. The objective of the paper is to developed a model that will provide information to engineers and designers for making decisions associated with damping and shock absorbers so as to increase comfort ability and durability of vehicles) and provide information to researchers for making decision associated with damping.

Developed a variable shock absorber model by varying the spring constant achieved by using helical spring with variable pitch in coils and varying coil diameter and a variable damper [6]. The governing equations were solved and analysis of various parameters such as damping coefficient, spring constant and damper fluid temperature were considered and there effect on damping. They revealed that if the damper fluid density reduces, the vehicles damper fluid temperature increases. The fluid density increases with time as the vehicle moves and as the damper mass reduces, damping increases. Damper vibration is low and as time increases damping also increases. [3] analyzed a half-car model with linear and nonlinear semi-active dampers performance. Using PYTHON software, a response of the system to a harmonic excitation of variable frequency and to impulse excitation was found.

## 2. Mathematical Model

### 2.1. Vibration Governing Equation

The transverse vibration of damper shock absorber $u(x,t)$ could be described by the following equation which is non dimensional in the spatial variable [11].

The system has inhomogeneous boundary conditions represented by $u(x,t)=0$. The equation of motion for the transverse shock absorber vibration was given by [8]:

$$\rho_p t_p \frac{\partial^2 u}{\partial t^2} + D\left(\frac{\partial^4 u}{\partial x^4} + 2\frac{\partial^4 u}{\partial x^2\,\partial y^2} + \frac{\partial^4 u}{\partial y^4}\right) = f \tag{1}$$

Where $u(x,y,t)$, is the shock absorber displacement in the transverse direction, $\rho_p$ is the density of the shock absorber material, $t_p$ is the shock absorber thickness, $D = \frac{E_p t_p^3}{12(1-v_p^2)}$, where $E_p$ is the Young's modulus of plate material and measuring stiffness of an elasticity material, $t_p^3$ is the plate thickness,$v_p^2$ is the poison's ratio of plate material and measures the deformation in the material in the direction perpendicular to the applied force and $f$ is an external distributed load on the shock absorber (automotive weights).

### 2.2. Modification of the Model

Equation (1) was modified to allow the variation of weight and damper thickness for the purpose of analysis of optimal values to improve the efficiency of the shock absorbers, the equation was modified by introducing velocity; $(v)$; Where $(v)$ denotes velocity of the damper, hence the modified equation becomes;

$$\rho_p t_p v \frac{\partial^2 u}{\partial t^2} + D\left(\frac{\partial^4 u}{\partial x^4} + 2\frac{\partial^4 u}{\partial x^2\,\partial y^2} + \frac{\partial^4 u}{\partial y^4}\right) = f \tag{2}$$

### 2.3. Discretization of the Scheme

Equation (2) would be discretized using the forward difference in time (FD), for the (DS) the value $u_t$,$u_{xxxx}$, $u_{xxyy}$ and $u_{yyyy}$ are replaced by forward difference approximation to obtain the following finite difference equation.

$$\rho_p t_p v\left(\frac{u_{i,j}^{n+1}-2u_{i,j}^{n}+u_{i,j}^{n-1}}{(k)^2}\right)+D[\frac{u_{i+2,j}^{n}-4u_{i+1,j}^{n}+6u_{i,j}^{n}-4u_{i-1,j}^{n}+u_{i-2,j}^{n}}{(h)^4}+\frac{\left[u_{i,j+2}^{n}-4u_{i,j+1}^{n}+6u_{i,j}^{n}-4u_{i,j-1}^{n}+u_{i,j-2}^{n}\right]}{(h)^4}+ 2\left(\frac{u_{i+1,j+1}^{n}-2u_{i+1,j}^{n}+u_{i+1,j-1}^{n}-2u_{i,j+1}^{n}+4u_{i,j}^{n}-2u_{i,j-1}^{n}+u_{i-1,j+1}^{n}-2u_{i-1,j}^{n}+u_{i-1,j-1}^{n}}{(h)^4}\right)]=f \tag{3}$$

Rearrange equation (3) in terms of $u_{i,j}^{n+1}+u_{i,j}^{n-1}=u_{i,j}^{n}+f$

$$u_{i,j}^{n+1}+u_{i,j}^{n-1}=\rho_p t_p v(2u_{i,j}^{n})-\left[D\frac{k^2}{h^4}\left[u_{i+2,j}^{n}+2u_{i+1,j+1}^{n}-8u_{i+1,j}^{n}+2u_{i+1,j-1}^{n}-8u_{i,j+1}^{n}+u_{i,j+2}^{n}+20u_{i,j}^{n}+u_{i,j-2}^{n}- 8u_{i,j-1}^{n}+2u_{i-1,j+1}^{n}-8u_{i-1,j}^{n}+2u_{i-1,j-1}^{n}+u_{i-2,j}^{n}\right]\right]+f \tag{4}$$

## 2.4. Using Crank-Nicolson Method in Discretization

Substituting these approximations into crank-Nicolson scheme;

$$\frac{u_i^{n+1}-u_i^{n}}{\Delta t}=\frac{\alpha}{2}\left[\frac{u_{i+1}^{n+1}-2u_i^{n+1}+u_{i-1}^{n+1}}{\Delta x^2}+\frac{u_{i+1}^{n}-2u_i^{n}+u_{i-1}^{n}}{\Delta x^2}\right] \tag{5}$$

Re-arranging equation (4) as;

$$u^{n+1}=u^{n}+u^{n-1}+f \tag{6}$$

$$\left(\frac{\rho_p t_p v}{k^2}+D\frac{1}{2h^4}\left[u_{i+2,j}+2u_{i+1,j+1}-8u_{i+1,j}+u_{i+1,j-1}+u_{i,j+2}-8u_{i,j+1}+20u_{i,j}-8u_{i,j-1}+u_{i,j-2}+8u_{i-1,j}+2u_{i-1,j+1}- 8u_{i-1,j}+2u_{i-1,j-1}+u_{i-2,j}\right]\right)u_{i,j}^{n+1}=f_{i,j}^{n}k^2-\left(\frac{\rho_p t_p v}{k^2}+D\frac{1}{2h^4}\left[u_{i+2,j}+2u_{i+1,j+1}-8u_{i+1,j}+u_{i+1,j-1}+u_{i,j+2}-8u_{i,j+1}+ 20u_{i,j}-8u_{i,j-1}+u_{i,j-2}+8u_{i-1,j}+2u_{i-1,j+1}-8u_{i-1,j}+2u_{i-1,j-1}+u_{i-2,j}\right]\right)u_{i,j}^{n}+\left(\frac{k^2}{\rho_p t_p v}+D\frac{1}{2h^4}\right)u_{i,j}^{n-1} \tag{7}$$

## 2.5. Forming the Matrix Equation

Rearrange the terms to express $u^{n+1}$ in terms of known values from $u^{n}$ and $u^{n-1}$. This results in a linear system:

$$Au^{n+1}=Bu^{n}+Cu^{n-1}+F \tag{8}$$

Where A, B, C are matrices formed from the discretized terms.

$$\rho_p t_p v D\emptyset\begin{bmatrix}20&0&1&0&0\\0&20&-8&2&0\\1&-8&20&-8&1\\0&2&-8&20&0\\0&0&1&0&20\end{bmatrix}u^{n+1}=-D\emptyset\begin{bmatrix}20&0&1&0&0\\0&20&-8&2&0\\1&-8&20&-8&1\\0&2&-8&20&0\\0&0&1&0&20\end{bmatrix}u^{n}+\rho_p t_p v\begin{bmatrix}1&0&0&0&0\\0&1&0&0&0\\0&0&1&0&0\\0&0&0&1&0\\0&0&0&0&1\end{bmatrix}u^{n-1}+f_{i,j}^{n}k^2 \tag{9}$$

by taking i to represent shock absorber transverse vibration along y axis, j to represent thickness of the shock absorber. Taking $i=1,2,3,4\ldots\ldots10, j=1, n=1, (\Delta x)=(\Delta y)=0.02, (\Delta t)=0.0001,$ and $\emptyset=0.0625$

$\rho_p=7000kg/m^3$

$D=80kg/m^2$

We would now apply the following set of boundary conditions and initial conditions;

Boundary conditions

$$\left.\begin{matrix}u(u,y,t), x=0\ and\ x=L\\ u(x,y,t), y=0\ and\ y=L\\ t=0\end{matrix}\right\} \tag{10}$$

Initial conditions;

For the initial displacement;

$$u(x,y,0)=A\sin(\pi L)\sin(\pi L)\ \text{ when }\ t=0 \tag{11}$$

Initial velocity;

$$\frac{\partial u}{\partial t}(x,y,0)=B\sin(\pi L)\sin(\pi L) \tag{12}$$

These conditions ensures that the initial displacements and velocity are spatially dependent on $x$ and $y$ in a sinusoidal pattern.

## 2.6. Jacobian Iterative Method

The Jacobian method is typically used to solve systems of linear equations of the form $Au = B$, where $A$ is a matrix, $\boldsymbol{u}$ is the unknown vector, and $B$ is the right-hand side vector. The basic idea of the method was to isolate the diagonal part of the matrix $A$ and use it to update the solution iteratively.

Given the penta-diagonal nature of the matrix $A$ in our system ($u_{i,j}^{n+1} = Au^n + Bu^{n-1} + F$), we need to solve for $u_{i,j}^{n+1}$ iteratively starting from an initial guess and refining the solution at each iteration.

## 2.7. Setting the Iteration Scheme

Using the Jacobian method, we can now write the iterative update rule for $u^{n+1}$. At each iteration $n+1$, we update the solution as follows:

$$u^{n+1} = (-(A+B)u^n + Cu^{n-1} + F) \tag{13}$$

This formula updates the vector $u^{n+1}$based on the previous iteration, the current state $u^n$ and the previous time step $u^{n-1}$.

In component form, this becomes:

$$u_{i,j}^{n+1} = \frac{1}{2}\left(-\sum_{j\neq i} B_{ij}u_j^n + \sum_j C_{ij}u_j^{n-1} + F_i\right) \tag{14}$$

The diagonal element of $B$ at point $i$, and the summation over $j\neq i$ represents the contributions from the off-diagonal elements.

## 2.8. The Process of Iteration

1) *Initialization:* Start with an initial guess $u^{n+1}$
2) *Iteration:* At each iteration $n+1$, update the solution using the Jacobian iteration formula giving equation (13).
3) *Convergence Check:* Continue with the iterations until the solution converges, i.e., the difference between successive iterations $\left\|u_{(i+1)}^{n+1} - u_{(i)}^{n}\right\|$ is below a given tolerance.

# 3. Results

## 3.1 Effect of Automobile Weight on Vibration Amplitude When Speed and Thickness Are Constant

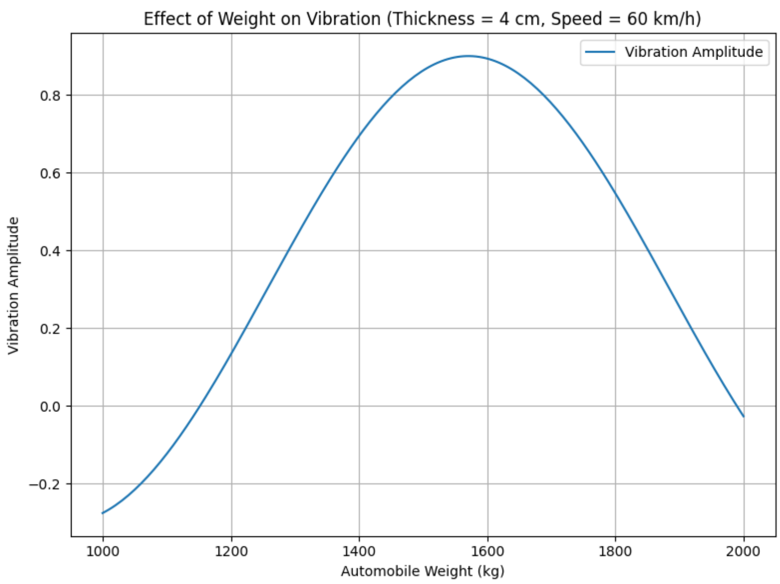


***Figure 1.*** *Vibration Amplitude vs. Automobile Weight.*

This plot emphasizes the importance of matching vehicle weight with suspension characteristics, as under- or over-weighting can significantly impact ride quality. The critical peak near 1600 kg should be carefully considered in suspension system design to avoid excessive vibrations that could compromise comfort or safety.

## 3.2. Effect of Weight and Thickness on Vibration Amplitude

The surface plot confirms that optimal vibration reduction occurs when both vehicle weight and shock absorber thickness are appropriately balanced, and that overdesign (excessive thickness for light vehicles) may result in diminishing returns. These findings support the need for customized suspension tuning based on vehicle class and load.

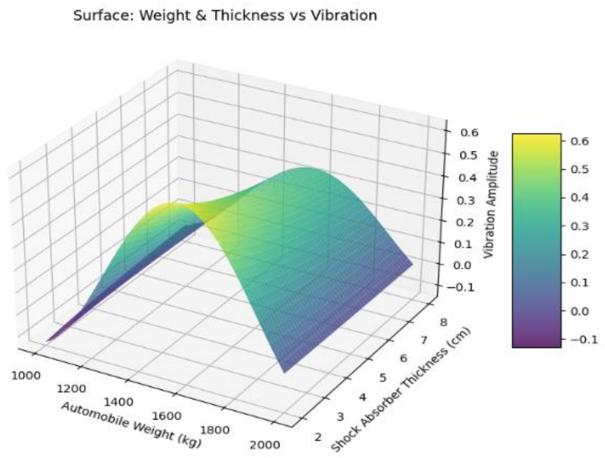


***Figure 2.*** *3D Surface Plot of Vibration Amplitude vs. Weight and Thickness.*

## 3.3. Effect of Speed and Weight on Vibration When the Shock Absorber Thickness Is Kept Constant

As display below, vibration amplitude varies non-linearly with speed and weight. At moderate speeds (around 80–100 km/h) and medium weights (~1600 kg), vibration peaks near 0.9, indicating maximum vibration. In contrast, at lower speeds (20–50 km/h) or very high speeds (>110 km/h), the vibration amplitude drops to below 0.2 or even -0.8, especially for lighter vehicles (1000–1300 kg). This suggests that intermediate speed and weight combinations can amplify vibrations, while extreme values tend to dampen them.

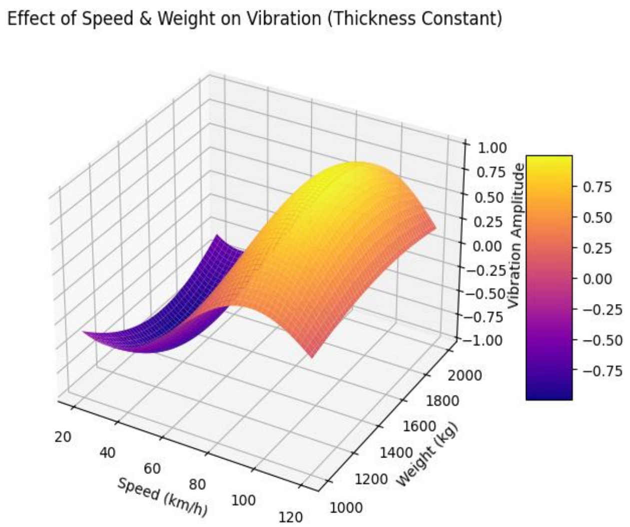


***Figure 3.*** *Vibration Amplitude vs. Speed and Weight.*

# 4. Discussion

## 4.1. Effect of Automobile Weight on Vibration Amplitude

The figure 1 Illustrates the variation in vibration amplitude as a function of automobile weight, with the shock absorber thickness held constant at 4 cm and the speed fixed at 60 km/h. The weight range analyzed spans from 1000 kg to 2000 kg. The plot reveals a clear non-linear relationship between weight and vibration amplitude. As the weight increases from 1000 kg to 1600 kg, the vibration amplitude rises sharply. The amplitude reaches its maximum value of 0.92 units at 1600 kg, indicating a potential resonance point where the system's natural frequency aligns with the external excitation frequency, resulting in maximum oscillation.

Beyond this point, however, the trend reverses: further increases in weight lead to a progressive decline in vibration amplitude, dropping to below 0 (approximately -0.1 units) at 2000 kg. This suggests that heavier vehicles, beyond a certain threshold, begin to dampen vibrations more effectively, likely due to increased inertial resistance and energy absorption by the suspension system. Notably, vibration amplitudes at weights below 1200 kg remain relatively low (below 0.2 units), indicating that very light vehicles may naturally experience reduced vibrations but could still be susceptible to instability under certain dynamic conditions.

## 4.2. Effect of Weight and Thickness on Vibration Amplitude

The figure 2 presents a 3D surface plot illustrating the relationship between automobile weight (ranging from 1000 kg to 2000 kg), shock absorber thickness (ranging from approximately 2 cm to 8 cm), and the resulting vibration amplitude. This was performed while maintaining a constant vehicle speed of 60 km/h. The surface reveals a non-linear interaction between the two independent parameters. Initially, as both weight and thickness increase, the vibration amplitude also increases, reaching a peak of approximately 0.62 units when the weight is around 1500 kg and the shock absorber thickness is close to 5 cm. This region may represent a point of mechanical resonance, where the system becomes more sensitive to oscillatory inputs from the road.

Beyond this point, however, further increases in either weight or shock absorber thickness result in a decrease in vibration amplitude. For example, at 2000 kg with a shock absorber thickness of 8 cm, the vibration amplitude drops to below 0.1 units, indicating improved damping performance. This suggests that heavier vehicles paired with thicker absorbers are better able to dissipate vibrational energy, likely due to increased inertia and enhanced energy absorption by the suspension system. Interestingly, at lower weights (e.g., 1000 kg), increasing the thickness from 2 cm to 8 cm yields minimal improvement in vibration suppression, as the amplitude remains below 0.2 units throughout. This suggests that lighter vehicles may not benefit significantly from thicker absorbers in terms of vibration control.

## 5. Conclusion

Lighter vehicles should prioritize moderate shock absorber thickness, as thicker shock absorbers show minimal benefit. For medium-to-heavy-duty vehicles use, use shock absorbers with thickness between (6-8 cm) to enhance damping, especially those that transport varying loads, shock absorbers with variable damping capabilities should be developed to adjust dynamically to changes in load and distribution. This could involve using adaptive or semi-active suspension systems that automatically adjust based on real-time weight and road conditions.

Manufacturers should incorporate this into vehicle designs, especially for off-road or rural vehicles, by providing shock absorbers with higher damping coefficients and greater durability to withstand continuous exposure to road irregularities. Government and regulatory body should encourage drivers on rough roads, avoid sustained driving at 80-100 km/h for medium-weight vehicles, as this range maximizes vibrations. Speeds between 40-60 km/h minimizes both vibration amplitude and prediction error**.**

## Abbreviations

| | |
|---|---|
| CDS | Central Difference Scheme |
| VD | Vibration Damping |
| SA | Shock Absorbers |
| AW | Automotive Weights |
| SAT | Shock Absorber Thickness |
| MRD | Magneto-rheological Damper |
| BC | Boundary Conditions |
| IC | Initial Condition |
| D | Flexural Rigidy or Bending Stiffness of the Shock Absorber Material |

## Appendix

### Appendix I: Simulation Parameters

***Table 1****.* *Simulated parameters.*

| Parameter | Range / Value | Notes |
|---|---|---|
| Automobile Weight (kg) | 1000 – 2000 | Key variable in all simulations |
| Shock Absorber Thickness (cm) | 2 – 8 | Constant at 4 cm in some tests |
| Vehicle Speed (km/h) | 20 – 120 | Constant at 60 km/h in some tests |
| Vibration Amplitude Units | Arbitrary (normalized) | Used for comparative analysis |


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## Author Contributions

**Benard Ouma Owino:** Methodology, Writing-original draft

**Cleophas M. Kweyu:** Supervision, Writing-review & editing

**Titus Chebion Rotich:** Supervision, Writing-review & editing


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## Data Availability Statement

The data supporting the outcome of this research work has been reported in this manuscript.

## Conflicts of Interest

The authors declare no conflict of interest.

## Appendix II: Figures and Graphical Representations

1) Figure 1: Vibration Amplitude vs. Automobile Weight
2) Figure 2: 3D Surface Plot of Vibration Amplitude vs. Weight and Thickness
3) Figure 3: Vibration Amplitude vs. Speed and Weight

## References

[1] Abdelkareem, M. A., Xu, L., Ali, M. K. A., Elagouz, A., Mi, J., Guo, S., Liu, Y., & Zuo, L. (2018). Vibration energy harvesting in automotive suspension system: A detailed review. Applied Energy, 229, 672–699: https://www.sciencedirect.com/science/article/pii/S0306261918311851

[2] Chen, Q., Xu, Z., Wu, M., Xiao, Y., & Shao, H. (2021). Study on dynamic characteristic analysis of vehicle shock absorbers based on bidirectional fluid–solid coupling. Engineering Applications of Computational Fluid Mechanics, 15(1), 426–436: https://doi.org/10.1080/19942060.2021.1878059

[3] Dobre, A. (2021). Modelling the dynamic behaviour of car hydraulic dampers. 1091(1), 012018: https://doi.org/10.1088/1757-899X/1091/1/012018

[4] Ferdek, U., & Dukała, M. (2021). Experimental analysis of nonlinear characteristics of absorbers with wire rope isolators. Open Engineering, 11(1), 1170–1179: https://doi.org/10.1515/eng-2021-0111

[5] Goldstein, L., Ilssar, D., & Gat, A. (2020). On non-Newtonian effects in fluidic shock-absorbers. Applied Physics Letters, 117(15), 153701: https://doi.org/10.1063/5.0023938

[6] Guan, D., Jing, X., Shen, H., Jing, L., & Gong, J. (2019). Test and simulation the failure characteristics of twin tube shock absorber. Mechanical Systems and Signal Processing, 122, 707–719: https://doi.org/10.106/j.ymssp.2018.12.052

[7] Isermann, R. (2022). Suspension Control Systems. In Automotive Control (pp. 445–479). Springer: https://link.springer.com/chapter/10.1007/978-3-642-39440-9_15

[8] Oude Nijhuis, M. H. (2003a). Analysis tools for the design of active structural acoustic control systems: thesis.

[9] Potapenko, O., Gorbunov, N., Mogyla, V., Shcherbina, Y., & Hauser, V. (2019). Function Evaluation of Common and Proposed Friction Shock Absorbers for Open Box Wagon 12-7019 KRVZ. Manufacturing Technology, 19(2), 303–307: https://doi.org/10.21062/ujep/287.2019/a/1213-2489/MT/19/2/303

[10] Rosół, M., & Martynowicz, P. (2019). Identification of the wind turbine model with MR damper based tuned vibration absorber. 1–7: ieeexplore.ieee.org

[11] Sciulli, D. (1997). Dynamics and control for vibration isolation design. Virginia Polytechnic Institute and State University: theses 9733816.

[12] Surblys, V., Žuraulis, V., & Sokolovskij, E. (2019). The influence of semi-active suspension adjustment on vehicle body pitch oscillations. Transport and Telecommunication, 20(2), 107–113. https://doi.org/10.2478/ttj-2019-0009

[13] Tuhta, S., Günday, F., & Abrar, O. (2019). Experimental Study on Effect of Seismic Damper to Reduce the Dynamic Response of Bench Scale Steel Structure Model. İnternational Journal of Advance Research and Innovative İdeas in Education, 5(5), 901–911: https://www.researchgate.net/profile/Furkan-Guenday/publication/337758741

[14] Wen-xue, X., & Zhen-hua, L. (2020). 3D FSI FEA simulation and analyses of the dynamic characteristics of a displacement-sensitive liquid damper. 37(9), 217–229: 217-229: https://doi.org/10.6052/j.issn.1000-4750.2019.10.0584