

Research Article

Effect of Parabolically Varying Non-homogeneity on Thermally Induced Vibration of Orthotropic Trapezoidal Plate with Thickness Varies Parabolically in Both Directions

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Abstract

The aim of present paper is to find the frequencies using the mathematical model: Effect of parabolically varying non-homogeneity on thermally induced vibration of orthotropic trapezoidal plate with thickness varies parabolically in both directions. In the above model both thickness and density varies parabolically. Rayleigh-Ritz method is used to solve the governing differential equation for maximum strain energy and maximum kinetic energy for orthotropic trapezoidal plate. Two term deflection function corresponding to clamped-simply supported clamped-simply supported (C-S-C-S) boundary condition is defined by the product of the equation of the prescribed continuous piecewise boundary shape. The effect of frequencies for first and second mode investigated with the variations in structural parameters such as taper constant, non-homogeneity constant, aspect ratio and thermal gradient respectively. Differential equations for maximum strain energy and maximum kinetic energy for orthotropic trapezoidal plate are solved using the mathematica software. All the results are calculated with great accuracy and are displayed graphically. To validate the model all the results are also compared with the preexisting literature and fit well. So by developing such type of model we increase sustainability and also reduce the environmental hazards.

Keywords

Non Homogeneity, Aspect Ratio, Vibration, Density, Thickness Etc

1. Introduction

Plates are the main part in various engineering structure and plays an important role in carrying applied loads. Vibration of plates plays an important role in modern systems like aircrafts, space vehicle, satellites, ships, submarines, turbines, bridges, automobiles etc. Enormous amount of literature exists dealing various aspects of elastic plates like shape (Trapezoidal, Rectangular,

Circular, Polygon or elliptic etc.), material (isotropic, anisotropic, orthotropic etc.), thickness (thick, thin), deflections (small, large), loadings (static or dynamic, mechanical, thermal, hydrostatic etc.), boundary conditions (clamped, simply supported or free) etc. But in this research paper we are analyzing the thermal effect on orthotropic trapezoidal thick plate with small deflection using

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clamped simply supported clamped simply supported (C-S-C-S) boundary conditions. Sharma *et al.* [1] studied the vibration of orthotropic trapezoidal plate with thickness varies linearly in both directions and density also varies linearly. Sharma *et al.* [2] studied the vibration of orthotropic trapezoidal plate with thickness varies parabolically in both directions and density varies linearly. Sharma *et al.* [3] purposed a mathematical model for vibration of non-homogeneous orthotropic trapezoidal plate with linear variation in density. Sharma *et al.* [4] studied the effect of linearly varying non-homogeneity on orthotropic trapezoidal plate with thickness varies linearly in one direction and parabolically in other direction. Gupta and Sharma [5] studied the vibration of trapezoidal plate with varying thickness and density. Gupta and Sharma [6] studied the vibration of trapezoidal plate whose thickness varies parabolically. Gupta and Sharma [7] studied the thermal effect on frequencies of non-homogeneous trapezoidal plate with linearly varying thickness and density. Gupta and Sharma [8] studied the thermally induced vibration of orthotropic trapezoidal plate with linearly varying thickness. Gupta and Sharma [9] studied the free vibration of orthotropic trapezoidal plate with parabolically varying thickness. Kavita *et al.* [10] studied the vibration of trapezoidal plate of varying thickness and density under thermal gradient. Kavita *et al.* [11] studied the vibration of symmetric non-homogeneous trapezoidal plate whose thickness varies bilinearly and density varies parabolically. Leissa [12] purposes a new model to design the aircraft and provide the platform to researcher towards vibration of plates. Leissa [13] studied the vibration of plates for complicating effects. Leissa [14] studied the vibration of plates for classical theory. Au and Wang [15] studied the sound radiation from forced vibration of rectangular orthotropic plate under moving loads. Avalos *et al.* [16] analyzed the vibrating rectangular anisotropic plate with free edge holes. Avalos *et al.* [17] studied the transverse vibrations of anisotropic plate with an oblique cutout. Avalos *et al.* [18] studied the transverse vibrations of rectangular plate with two rectangular cutout. Arenas and Jorge [19] studied the vibrational analysis of rectangular plate with virtual work principle. Tomer and Gupta [20] studied the vibration of orthotropic elliptic plate of non-uniform thickness and temperature. Tomer and Gupta [21] studied the thermal effect on frequencies of an orthotropic rectangular plate of linearly varying thickness. Tomer and Gupta [22] studied the harmonic temperature effect on orthotropic rectangular plate of linearly varying thickness. Tomer and Gupta [23] studied the exponential temperature effect on orthotropic rectangular plate of an exponentially varying thickness. Tomer and Gupta [24] studied

the thermal effect on axisymmetric vibrations of an orthotropic circular plate of variable thickness. Tomer and Gupta [25] studied the thermal effect on axisymmetric vibrations of an orthotropic circular plate of parabolically varying thickness. Tomer and Gupta [26] studied the axisymmetric vibrations of a circular plate of linearly varying thickness using Medlin theory. Tomer and Tiwari [27] studied the thermal effect on frequencies of a circular plate of variable thickness. Tomer *et al.* [28] studied the transverse vibration of non-uniform rectangular orthotropic plates. Vega *et al.* [29] studied the transverse vibration of an annular circular plate with free edge and an intermediate concentric circular support. Sharma *et al.* [30] studied the effect of parabolically varying non-homogeneity on orthotropic trapezoidal plate with thickness varies linearly in one direction and parabolically in other direction.

2. Mathematical Formulation

2.1. Geometrical Representation

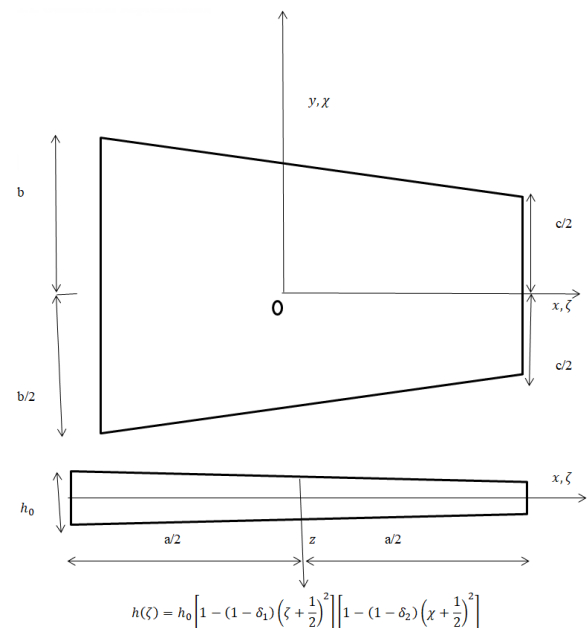


Figure 1. Geometry of orthotropic trapezoidal plate *Figure 1*.

2.2. Governing Equation Becomes

Expression for maximum strain energy and maximum kinetic energy for orthotropic trapezoidal plate is [1] as

$$P = \frac{ab}{2} \iint \left[\mathfrak{D}_\zeta \left(\frac{\partial^2 \psi}{\partial \zeta^2} \right)^2 + \mathfrak{D}_\chi \left(\frac{\partial^2 \psi}{\partial \chi^2} \right)^2 + 2\mathfrak{D}_1 \left(\frac{\partial^2 \psi}{\partial \zeta^2} \right) \left(\frac{\partial^2 \psi}{\partial \chi^2} \right) + 4\mathfrak{D}_{\zeta\chi} \left(\frac{\partial^2 \psi}{\partial \zeta \partial \chi} \right)^2 \right] d\zeta d\chi, \quad (1)$$

and

$$K = \frac{ab}{2} \omega^2 h_0 \iint \rho h(\zeta) \psi^2 d\zeta d\chi, \quad (2)$$

where ω is the angular frequency of vibration.

2.3. Deflection Function and Corresponding Boundary Condition

Two term deflection function with boundary condition clamped simply supported- clamped simply supported (C-S-C-S) for vibrational analysis has been considered and can be written as

$$\psi = P_1 \left\{ \left(\zeta + \frac{1}{2} \right) \left(\zeta - \frac{1}{2} \right) \right\}^2 \left\{ \chi - \left(\frac{b-c}{2} \right) \zeta + \left(\frac{b+c}{4} \right) \right\} \cdot \left\{ \chi + \left(\frac{b-c}{2} \right) \zeta - \left(\frac{b+c}{4} \right) \right\} + P_2 \left\{ \left(\zeta + \frac{1}{2} \right) \left(\zeta - \frac{1}{2} \right) \right\}^3 \cdot \left\{ \chi - \left(\frac{b-c}{2} \right) \zeta + \left(\frac{b+c}{4} \right) \right\}^2 \left\{ \chi + \left(\frac{b-c}{2} \right) \zeta - \left(\frac{b+c}{4} \right) \right\}^2, \quad (3)$$

P_1 and P_2 are two unknown constants to be determined.

Thus, for all the non-homogeneous trapezoidal plate eq. (6) satisfied the following conditions clamped simply supported-clamped simply supported (C-S-C-S) for vibrational analysis such as

$$\begin{aligned} \chi &= \frac{c}{4b} - \frac{\zeta}{2} + \frac{1}{4} + \frac{c\zeta}{2b}, \\ \chi &= -\frac{c}{4b} + \frac{\zeta}{2} - \frac{1}{4} - \frac{c\zeta}{2b}, \\ \zeta &= -\frac{1}{2}, \\ \zeta &= \frac{1}{2}, \end{aligned} \quad (4)$$

introducing the following non-dimensional variables as $\zeta = \frac{x}{a}$ and $\chi = \frac{y}{b}$.

2.4. Temperature

The general equation of temperature with linear temperature distribution in x-direction is

$$\theta = \theta_0 \left(\frac{1}{2} - \zeta \right) \quad (5)$$

where θ and θ_0 denote the temperature excess above the reference temperature on the plate at any point and at the origin, respectively.

For most orthotropic materials, modulus of elasticity is described as a function of temperature as

$$\begin{aligned} \check{E}_\zeta(\theta) &= \check{E}_1(1 - \gamma\theta), \\ \check{E}_\chi(\theta) &= \check{E}_2(1 - \gamma\theta), \\ G_{\zeta\chi}(\theta) &= G_0(1 - \gamma\theta), \end{aligned} \quad (6)$$

where $\check{E}_\zeta$ and $\check{E}_\chi$ are Young's moduli in x-direction and y-direction, respectively, and $G_{\zeta\chi}$ is the shear modulus, γ is slope of vibration of moduli with temperature, and $\check{E}_1, \check{E}_2$ and G_0 are the values of moduli at some reference temperature; that is, $\theta = 0$.

Using (3), (4) becomes

$$\begin{aligned} \check{E}_\zeta(\theta) &= \check{E}_1 \left(1 - \delta \left(\frac{1}{2} - \zeta \right) \right), \\ \check{E}_\chi(\theta) &= \check{E}_2 \left(1 - \delta \left(\frac{1}{2} - \zeta \right) \right), \\ G_{\zeta\chi}(\theta) &= G_0 \left(1 - \delta \left(\frac{1}{2} - \zeta \right) \right). \end{aligned} \quad (7)$$

where $\delta = \gamma\theta_0$ ($0 \leq \delta < 1$) known as thermal gradient.

2.5. Thickness and Density

For tapering in plate thickness has important role as compared to uniform thickness. The general equation of thickness with parabolic variation in both directions is

$$h(\zeta) = h_0 \left[1 - (1 - \delta_1) \left(\zeta + \frac{1}{2} \right)^2 \right] \left[1 - (1 - \delta_2) \left(\chi + \frac{1}{2} \right)^2 \right] \quad (8)$$

where h_0 is the maximum plate thickness occurring at the left edge, δh_0 is the minimum plate thickness occurring at the right edge and δ_1, δ_2 are the taper constants.

For non-homogeneity of the plate density is assumed parabolic.

$$\rho = \rho_0 \left[1 - (1 - \beta) \left(\zeta + \frac{1}{2} \right)^2 \right] \quad (9)$$

where β is the non-homogeneity constant of the plate, ρ_0 is the mass density at $\zeta = -\frac{1}{2}$.

Also flexural and torsion rigidity is given by [4] as

$$\begin{aligned} D_\zeta &= \frac{\check{E}_\zeta h^3}{12(1-\nu_\chi\nu_\zeta)}, \\ D_\chi &= \frac{\check{E}_\chi h^3}{12(1-\nu_\chi\nu_\zeta)}, \\ D_{\zeta\chi} &= \frac{G_{\zeta\chi} h^3}{12}. \end{aligned} \quad (10)$$

Using (5), (10) becomes

$$\begin{aligned} D_\zeta &= \frac{\check{E}_1 h^3 \left(1 - \delta \left(\frac{1}{2} - \zeta \right) \right)}{12(1-\nu_\chi\nu_\zeta)}, \\ D_\chi &= \frac{\check{E}_2 h^3 \left(1 - \delta \left(\frac{1}{2} - \zeta \right) \right)}{12(1-\nu_\chi\nu_\zeta)}, \end{aligned} \quad (11)$$

$$\mathbb{D}_{\zeta\chi} = \frac{g_0 h^3 (1 - \delta(\frac{1}{2} - \zeta))}{12}$$

Also

$$\mathbb{D}_1 = \nu_\chi \mathbb{D}_\zeta = \nu_\zeta \mathbb{D}_\chi, \quad (12)$$

where h is the thickness of the plate.

After applying boundary conditions (7), (8) and (9) becomes

$$P = \frac{ab}{2} \int_{-\frac{1}{2}}^{\frac{1}{2}} \int_{-\frac{1}{2}}^{\frac{1}{2}} \frac{c}{4b} \frac{\zeta}{2} + \frac{1}{4} + \frac{c\zeta}{2b} \left[\mathbb{D}_\zeta \left(\frac{\partial^2 \psi}{\partial \zeta^2} \right)^2 + \mathbb{D}_\chi \left(\frac{\partial^2 \psi}{\partial \chi^2} \right)^2 + \right. \\ \left. P_1 = \int_{-\frac{1}{2}}^{\frac{1}{2}} \int_{-\frac{1}{2}}^{\frac{1}{2}} \frac{c}{4b} \frac{\zeta}{2} + \frac{1}{4} + \frac{c\zeta}{2b} \left[1 - (1 - \delta_1) \left(\zeta + \frac{1}{2} \right)^2 \right] \left[1 - (1 - \delta_2) \left(\chi + \frac{1}{2} \right)^2 \right]^3 \left(1 - \delta \left(\frac{1}{2} - \zeta \right) \right) \left[\left(\frac{\partial^2 \psi}{\partial \zeta^2} \right)^2 + \frac{E_2}{E_1} \left(\frac{\partial^2 \psi}{\partial \chi^2} \right)^2 + \right. \right. \\ \left. \left. 2\nu_\zeta \left(\frac{\partial^2 \psi}{\partial \zeta^2} \right) \left(\frac{\partial^2 \psi}{\partial \chi^2} \right) + 4 \frac{g_0 (1 - \nu_\chi \nu_\zeta)}{E_1} \left(\frac{\partial^2 \psi}{\partial \zeta \partial \chi} \right)^2 \right] d\zeta d\chi \right. \\ \left. K_1 = \int_{-\frac{1}{2}}^{\frac{1}{2}} \int_{-\frac{1}{2}}^{\frac{1}{2}} \frac{c}{4b} \frac{\zeta}{2} + \frac{1}{4} + \frac{c\zeta}{2b} \left[\rho \left[1 - (1 - \delta_1) \left(\zeta + \frac{1}{2} \right)^2 \right] \left[1 - (1 - \delta_2) \left(\chi + \frac{1}{2} \right)^2 \right] \psi^2 \right] d\zeta d\chi \right. \quad (13) \\ K = \frac{ab}{2} \omega^2 h_0 \int_{-\frac{1}{2}}^{\frac{1}{2}} \int_{-\frac{1}{2}}^{\frac{1}{2}} \frac{c}{4b} \frac{\zeta}{2} + \frac{1}{4} + \frac{c\zeta}{2b} [\rho h(\zeta) \psi^2] d\zeta d\chi \quad (14) \\ \delta(P - K) = 0 \text{ implies } \delta(P_1 - \mu^2 K_1) = 0 \quad (15)$$

Where

$$\mu^2 = \frac{12 \rho_0 \omega^2 a^5 (1 - \nu_\chi \nu_\zeta)}{E_1 h_0^2} \quad (16)$$

Equation (15) involves two constants C_1 and C_2 to be evaluated as follows

$$\frac{\partial(\Omega_1 - \mu^2 \Theta_1)}{\partial C_m} = 0; m=1, 2 \quad (17)$$

On simplifying (16) we get

$$\Theta_{m1} C_1 + \Theta_{m2} C_2 = 0; m=1, 2 \quad (18)$$

2.6. Rayleigh Ritz Technique

To obtain equation of frequency and vibrational frequency Rayleigh Ritz technique is used according to which

In this way the frequency equation can be obtained as

$$\begin{vmatrix} \Theta_{11} & \Theta_{12} \\ \Theta_{21} & \Theta_{22} \end{vmatrix} = 0 \quad (19)$$

and results validated with preexisting literature.

3. Results and Discussion

With the help of Mathematica Software two values of frequency parameter are calculated for different values of thermal gradient, aspect ratio and non-homogeneity constant respectively.

Table 1. In the following table first and second mode values of frequency parameter μ for a orthotropic trapezoidal plate for different values of thermal gradient δ and taper constant $\delta_1 = \delta_2 = 0.0, 0.6$ and non-homogeneity constant $\beta = 0.0, 0.4$ and fixed values of $\frac{a}{b} = 1.0, \frac{c}{b} = 0.5$ are calculated.

δ	$\delta_1 = \delta_2 = 0.0$				$\delta_1 = \delta_2 = 0.6$			
	$\beta = 0.0$		$\beta = 0.4$		$\beta = 0.0$		$\beta = 0.4$	
	First Mode	Second Mode	First Mode	Second Mode	First Mode	Second Mode	First Mode	Second Mode
0.0	30.589	154.064	29.2556	147.332	31.5694	169.634	30.0839	161.416
0.2	27.7991	142.965	26.5868	136.72	28.9422	158.245	27.5798	150.581
0.4	24.6943	130.928	23.6168	125.213	26.0502	145.969	24.8233	138.904
0.6	21.1348	117.668	20.2119	112.535	22.7923	132.562	21.718	126.15

δ	$\delta_1 = \delta_2 = 0.0$				$\delta_1 = \delta_2 = 0.6$			
	$\beta = 0.0$		$\beta = 0.4$		$\beta = 0.0$		$\beta = 0.4$	
	First Mode	Second Mode	First Mode	Second Mode	First Mode	Second Mode	First Mode	Second Mode
0.8	16.831	102.711	16.095	98.2363	18.9788	117.637	18.0832	111.954
1.0	10.9201	85.1694	10.4414	81.4684	14.1616	100.532	13.492	95.6747

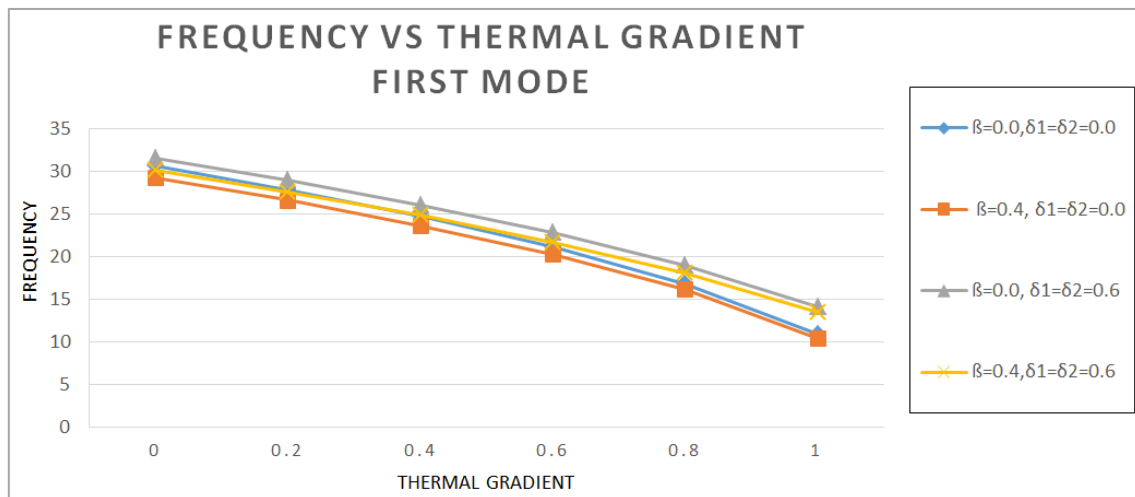


Figure 2. Represents the first mode of frequency parameter μ for a orthotropic trapezoidal plate for different values of thermal gradient δ and constant aspect ratios $\frac{a}{b} = 1.0, \frac{c}{b} = 0.5$, non-homogeneity constant $\beta=0.0, 0.4$ and Taper Constant $\delta_1 = \delta_2 = 0.0, 0.6$.

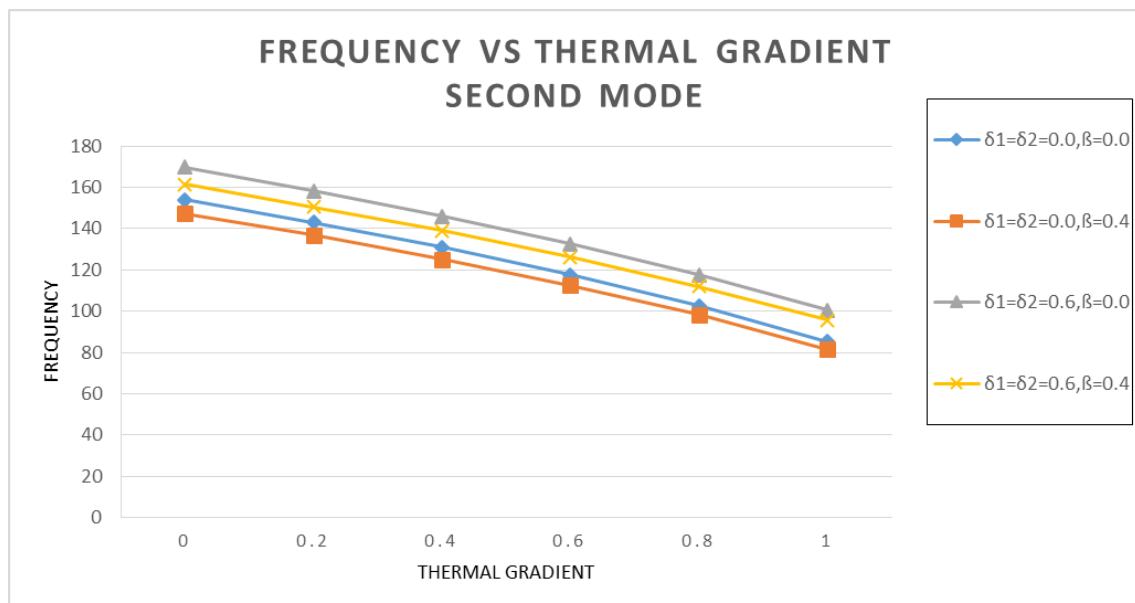


Figure 3. Represents the second mode values of frequency parameter μ for orthotropic trapezoidal plate for different values of thermal gradient δ and constant aspect ratios $\frac{a}{b} = 1.0, \frac{c}{b} = 0.5$, non-homogeneity constant $\beta=0.0, 0.4$ and Taper Constant $\delta_1 = \delta_2 = 0.0, 0.6$.

Table 2. In the following table two different values of frequency parameter μ for a orthotropic trapezoidal plate for different values of aspect ratio $\frac{c}{b}$ and constant aspect ratio $\frac{a}{b} = 0.75$, $\beta = 0.0$, $\delta_1 = \delta_2 = 0.0, 0.6$ and $\delta = 0.0$ are calculated.

$\beta = 0.0$								
$\frac{c}{b}$	$\delta_1 = \delta_2 = 0.0$ $\delta = 0.0$		$\delta_1 = \delta_2 = 0.0$ $\delta = 0.4$		$\delta_1 = \delta_2 = 0.6$ $\delta = 0.0$		$\delta_1 = \delta_2 = 0.6$ $\delta = 0.4$	
	First Mode	Second Mode	First Mode	Second Mode	First Mode	Second Mode	First Mode	Second Mode
0.25	35.6888	145.323	28.3814	120.11	35.7883	151.584	28.7843	126.454
0.50	28.7355	111.436	23.0381	93.0969	28.7925	117.357	23.4843	99.3246
0.75	23.829	86.1293	19.3147	72.7386	24.1977	91.9108	20.1725	79.038
1.0	20.5614	68.414	16.8739	58.408	21.4894	74.601	18.4092	65.348

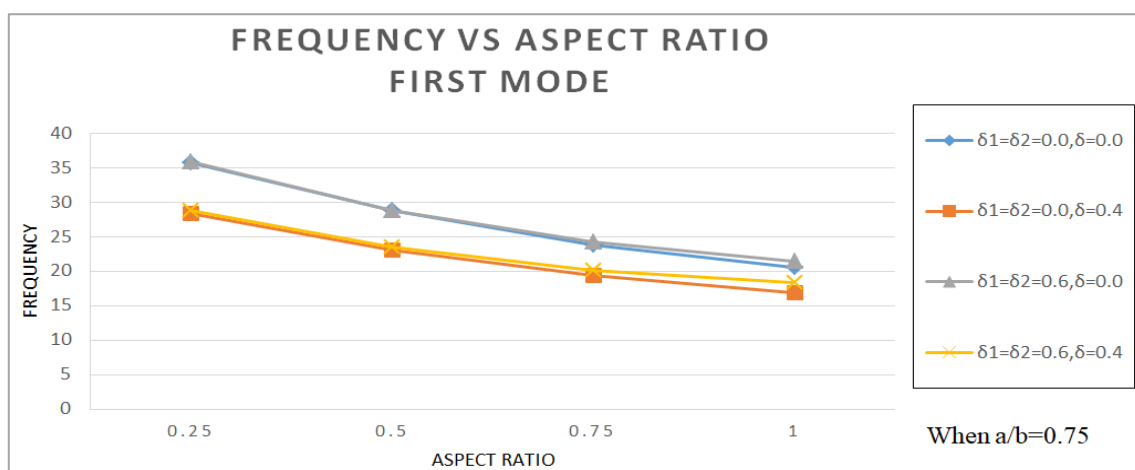


Figure 4. Represents the first mode values of frequency parameter μ for a orthotropic trapezoidal plate for different values of thermal gradient δ and constant aspect ratios $\frac{a}{b} = 0.75$, non-homogeneity constant $\beta = 0.0$, Thermal gradient $\delta = 0.0, 0.4$ and Taper Constant $\delta_1 = \delta_2 = 0.0, 0.6$.

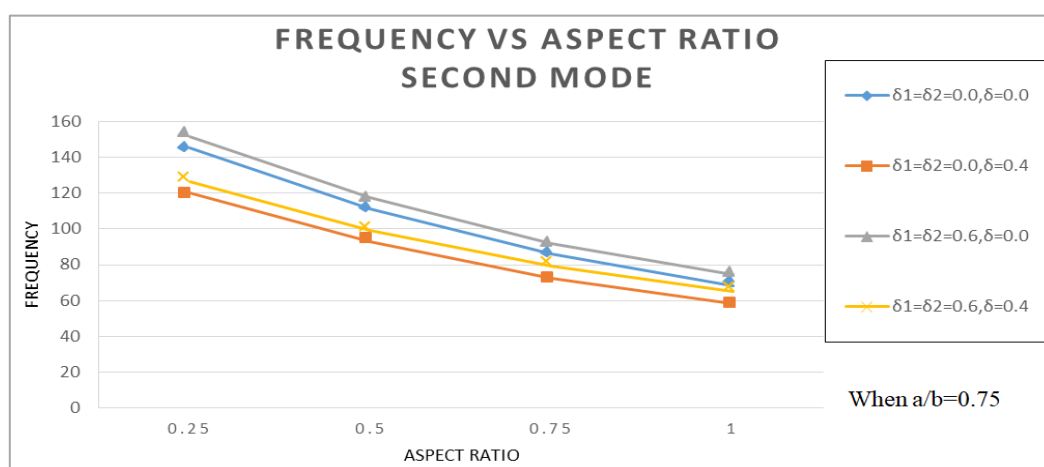


Figure 5. Represents the second mode values of frequency parameter μ for a orthotropic trapezoidal plate for different values of thermal gradient δ and constant aspect ratios $\frac{a}{b} = 0.75$, non-homogeneity constant $\beta = 0.0$, Thermal gradient $\delta = 0.0, 0.4$ and Taper Constant $\delta_1 = \delta_2 = 0.0, 0.6$.

Table 3. In the following table two different values of frequency parameter μ for a orthotropic trapezoidal plate for different values of aspect ratio $\frac{c}{b}$ and constant aspect ratio $\frac{a}{b} = 1.0, \beta = 0.0, \delta_1 = \delta_2 = 0.0$ and $0.6, \delta = 0.0$ are calculated.

$\frac{c}{b}$	$\beta = 0.0$							
	$\delta_1 = \delta_2 = 0.0$ $\delta = 0.0$		$\delta_1 = \delta_2 = 0.0$ $\delta = 0.4$		$\delta_1 = \delta_2 = 0.6$ $\delta = 0.0$		$\delta_1 = \delta_2 = 0.6$ $\delta = 0.4$	
	First Mode	Second Mode	First Mode	Second Mode	First Mode	Second Mode	First Mode	Second Mode
0.25	39.2842	210.645	30.3935	162.397	39.2842	218.645	32.0578	178.783
0.50	30.589	154.064	24.6943	130.928	31.5694	169.634	26.0502	145.969
0.75	25.3956	121.749	20.6833	104.527	26.3826	134.812	22.1526	117.492
1.0	20.5614	88.414	17.9946	84.0116	23.1717	107.673	19.9025	95.0264

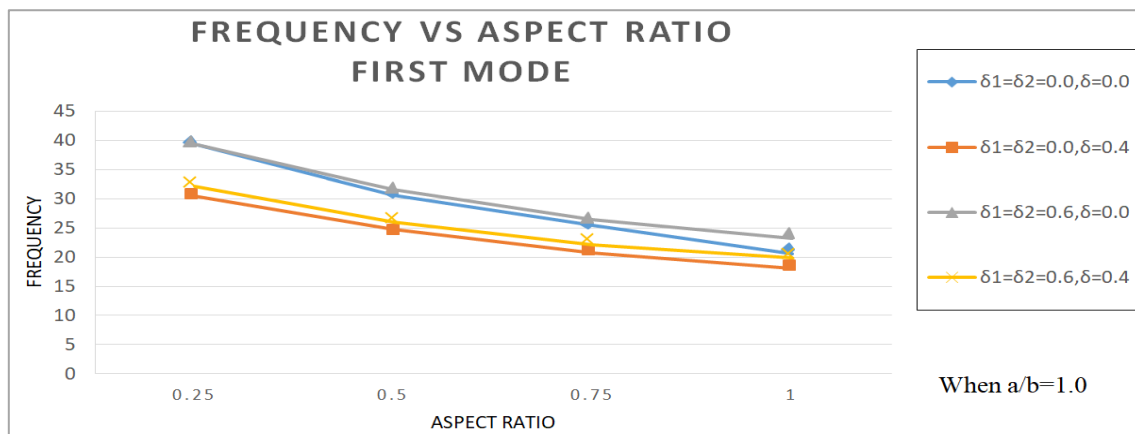


Figure 6. Represents the first mode values of frequency parameter μ for a orthotropic trapezoidal plate for different values of thermal gradient δ and constant aspect ratio $\frac{a}{b} = 1.0$, non-homogeneity constant $\beta = 0.0$, Thermal gradient $\delta = 0.0, 0.4$ and Taper Constant $\delta_1 = \delta_2 = 0.0, 0.6$.

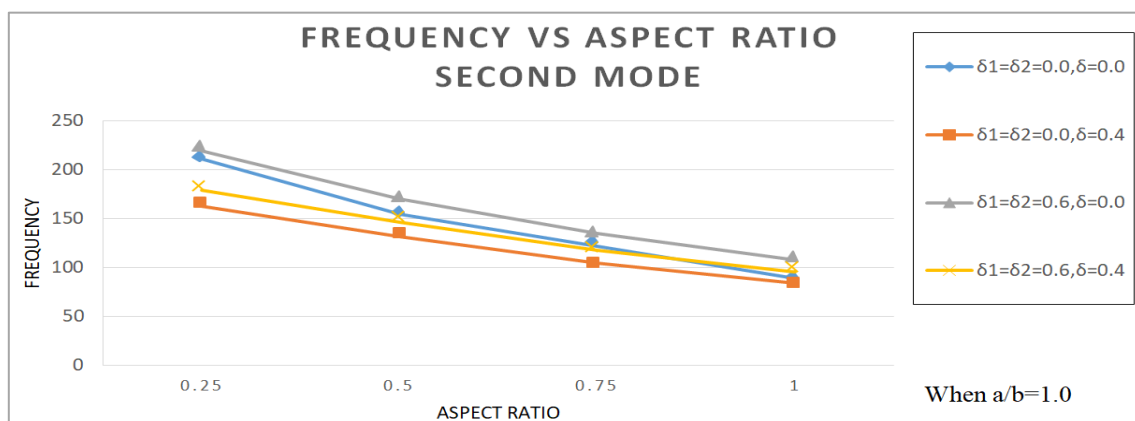


Figure 7. Represents the second mode values of frequency parameter μ for a orthotropic trapezoidal plate for different values of thermal gradient δ and constant aspect ratio $\frac{a}{b} = 1.0$, non-homogeneity constant $\beta = 0.0$, Thermal gradient $\delta = 0.0, 0.4$ and Taper Constant $\delta_1 = \delta_2 = 0.0, 0.6$.

Table 4. In the following table two different values of frequency parameter μ for a orthotropic trapezoidal plate for different values of non-homogeneity constant β and aspect ratios $\frac{a}{b} = 1.0$, $\frac{c}{b} = 0.5$, $\delta = 0.0$ and 0.4 , $\delta_1 = \delta_2 = 0.0$ and $\delta_1 = 0.2, \delta_2 = 0.6$ are calculated.

β	$\delta_1 = \delta_2 = 0.0$				$\delta_1 = 0.2, \delta_2 = 0.6$			
	$\delta = 0.0$		$\delta = 0.4$		$\delta = 0.0$		$\delta = 0.4$	
	First Mode	Second Mode	First Mode	Second Mode	First Mode	Second Mode	First Mode	Second Mode
0.0	30.589	154.064	24.6943	130.928	31.057	160.398	25.2831	137.001
0.2	29.9001	150.581	24.1375	127.971	30.3439	156.631	24.7019	133.787
0.4	29.2556	147.332	23.6168	125.213	29.6776	153.129	24.1589	130.799
0.6	28.6511	144.291	23.1283	122.631	29.0533	149.861	23.6502	128.010
0.8	28.0825	141.437	22.6689	120.207	28.4668	146.801	23.1723	125.398
1.0	27.5464	138.75	22.2359	117.926	27.9143	143.927	22.7223	122.946

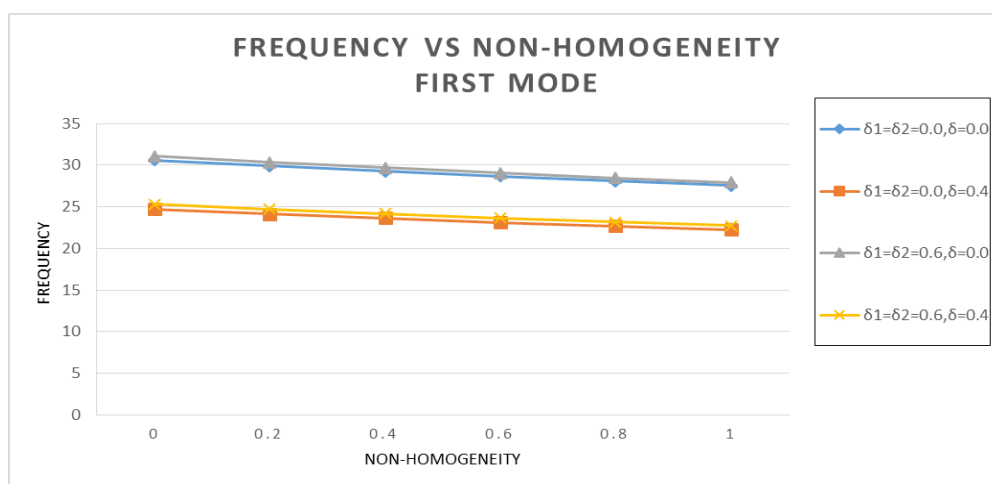


Figure 8. Represents the first mode values of frequency parameter μ for a orthotropic trapezoidal plate for different values of non-homogeneity constant β and aspect ratios $\frac{a}{b} = 1.0$, $\frac{c}{b} = 0.5$, $\delta = 0.0, 0.4$, $\delta_1 = \delta_2 = 0.0$ and $\delta_1 = 0.2, \delta_2 = 0.6$ respectively.

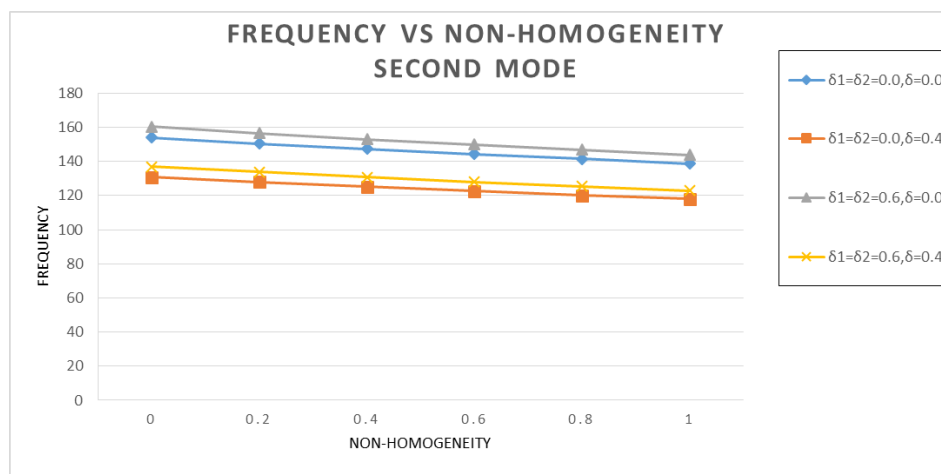


Figure 9. Represents the second mode values of frequency parameter μ for a orthotropic trapezoidal plate for different values of non-homogeneity constant β and aspect ratios $\frac{a}{b} = 1.0$, $\frac{c}{b} = 0.5$, $\delta = 0.0$ and 0.4 , $\delta_1 = \delta_2 = 0.0$ and $\delta_1 = 0.2, \delta_2 = 0.6$ respectively.

4. Conclusion

From Table 1 and Figures 2 & 3 it is clear that in all the four cases:

$$\delta_1 = \delta_2 = 0.0, \beta = 0.0$$

$$\delta_1 = \delta_2 = 0.0, \beta = 0.4$$

$$\delta_1 = \delta_2 = 0.6, \beta = 0.0$$

$$\delta_1 = \delta_2 = 0.6, \beta = 0.4$$

Frequency decreases in both the modes of vibration with increase in thermal gradient.

From Table 2 and Figures 4 & 5 it is clear that in all the four cases:

$$\delta_1 = \delta_2 = 0.0, \delta = 0.0$$

$$\delta_1 = \delta_2 = 0.0, \delta = 0.4$$

$$\delta_1 = \delta_2 = 0.6, \delta = 0.0$$

$$\delta_1 = \delta_2 = 0.6, \delta = 0.4$$

Frequency decreases in both the modes of vibration with increase in aspect ratio.

From Table 3 and Figures 6 & 7 it is clear that in all the four cases:

$$\delta_1 = \delta_2 = 0.0, \delta = 0.0$$

$$\delta_1 = \delta_2 = 0.0, \delta = 0.4$$

$$\delta_1 = \delta_2 = 0.6, \delta = 0.0$$

$$\delta_1 = \delta_2 = 0.6, \delta = 0.4$$

Frequency decreases in both the modes of vibration with increase in aspect ratio.

From Table 4 and Figures 8 & 9 it is clear that in all the four cases:

$$\delta_1 = \delta_2 = 0.0, \delta = 0.0$$

$$\delta_1 = \delta_2 = 0.0, \delta = 0.4$$

$$\delta_1 = \delta_2 = 0.6, \delta = 0.0$$

$$\delta_1 = \delta_2 = 0.6, \delta = 0.4$$

Frequency decreases in both the modes of vibration with increase in non-homogeneity. Results are verified with the preexisting literature [6, 7].

These types of mathematical models are very useful in engineering structures. The main aim to develop such type of mathematical model is to attract the mind of mechanical engineers in that direction. By choosing such type of mathematical models we increase the efficiency and minimizing the cost of manufacturing. Also developing new models to grow the technology towards excellence.

Abbreviations

C-S	Clamped Simply Supported
Temp	Temperature
et. al	And Others
i.e.	That Is
etc.	Et Cetera
Eqn	Equation
Vol.	Volume
1(2)	Volume (Issue No.)
Pp.	Page Number
Fig.	Figure
Id	Identity Document
Email	Electronic Mail
https	Hypertext Transfer Protocol Secure

Conflicts of Interest

The authors declare no conflicts of interest.

Appendix

ρ	Density
$\frac{a}{b}, \frac{c}{b}$	Aspect Ratio
δ	Thermal Gradient
δ_1, δ_2	Taper Constant
β	Non-Homogeneity
μ	Frequency Parameter
$\zeta = \frac{x}{a}$ and $\chi = \frac{y}{b}$	Non-Dimensional Variables
$\check{E}_\zeta$ and $\check{E}_\chi$	Young's Moduli
$h(\zeta)$	Thickness
θ and θ_0	Temperature Excess Above the Reference Temperature and Origin
γ	Slope of Vibration of Moduli with Temperature
ω	Angular Frequency
δh_0	Minimum Plate Thickness
P	Maximum Strain Energy
K	Maximum Kinetic Energy
D_ζ	Flexural Rigidity
D_χ	Torsion Rigidity

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