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Research Article

The Mirror Harmony Mod-9 Law: Residue Classification of Brahmam Mirror Numbers

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Abstract

This paper develops the concept of Brahmam Mirror Numbers (BMNs), a class of integers defined through a mirror-product operation in which a number is multiplied by its digit-reversed counterpart. When this product forms a decimal palindrome, the number is classified as a BMN. This operation connects digit reversal, reflective symmetry, and multiplicative structure, creating an interesting foundation for studying digit-based transformations. In this work, we identify a central modular identity governing these behaviours, referred to as the Mirror Harmony Mod-9 Law. Through digit-sum properties and congruence arguments, we show that for any integer, the mirror product is always congruent to the square of the number modulo nine. As a result, every mirror product must lie within the set of quadratic residues modulo nine: {0, 1, 4, 7}. This restriction holds universally and remains valid whether or not the number itself is a BMN. To examine the frequency and structure of BMNs, we conducted a complete computational scan of integers up to 200 million and identified 1246 BMNs. These findings confirm their rarity and demonstrate the residue pattern predicted by the modular identity. Additional observations highlight how mirror products behave under mod-11 alternating-digit rules, suggesting deeper interactions between digit symmetry and modular behaviour. We also explore a polynomial interpretation in which digit strings are treated as self-reciprocal forms, offering an algebraic viewpoint on palindromic mirror products. Overall, the results presented here provide a unified framework for understanding Brahmam Mirror Numbers, combining modular theory, computational evidence, and structural analysis. This study opens pathways for further research in digit-based number theory, density heuristics, modular classification, and potential applications in reflective arithmetic and digital computation.

Keywords

Brahmam Mirror Numbers, Palindromic Products, Modular Arithmetic, Digital Reversal, Quadratic Residues, Residue Classification, Base-9 Congruence, Self-reciprocal Structure

1. Introduction

Palindromic structure in integers provides a classical pathway between combinatorial constructions on digits and arithmetic congruences. While palindromes have been widely explored, the *multiplicative mirror* operation—mapping N to $N \cdot \mathrm{rev}(N)$, where $\mathrm{rev}(N)$ is the base-10 digit reversal—has only appeared only occasionally in literature in the literature. In 2024 Odugu introduced *Brahmam Mirror Numbers* (BMNs): integers N such that $N \cdot \mathrm{rev}(N)$ is a palindrome.

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This constraint combines two orthogonal symmetries: digit reversal and multiplicative collapsing. [1, 5, 6, 9].

This paper isolates and proves an invariant behind that phenomenon: reducing modulo nine, the mirror product behaves like a square. Formally,

$$\mathrm{rev}(N) \equiv N(\mathrm{mod}9) \Longrightarrow N \cdot \mathrm{rev}(N) \equiv N^2(\mathrm{mod}9),$$

and hence $(N \cdot \mathrm{rev}(N))\mathrm{mod}9 \in \{0,1,4,7\}$. We call this the *Mirror Harmony Mod-9 Law*. It is universal (it holds for all N), but its *interpretive meaning becomes clearer* within BMNs: whenever a mirror product is additionally palindromic in base 10, its residue mod 9 lies in the same quartet as perfect squares mod 9. In this sense, *mirror products emulate squares in base-*9 *arithmetic*.

A subtle but crucial clarification is that the restriction to the residue quartet pertains to the *mirror product* $P := N \cdot \mathrm{rev}(N)$, not to N itself. The existence of BMNs across all $N \bmod 9$ residue classes (verified computationally) underscores that palindromicity of P imposes no necessary restriction on $N \bmod 9$; the quartet arises *because* $P \equiv N^2(\mathrm{mod}\ 9)$.

We proceed as follows. Section 2 formalizes BMNs and proves the base-9 reversal identity. Section 3 states and proves the Mirror Harmony Mod-9 Law, clarifies the distinction between $N \bmod 9$ and $P \bmod 9$, and supplies canonical examples. Section 4 summarises computational validation and a simple generator. Section 5 studies mod-11 structure and alternating-sum invariants. Section 6 develops density heuristics; Section 7 connects mirror products with self-reciprocal polynomials. We close with open problems.

2. Definitions and Preliminaries

Definition 1 (Digital Reversal) *If* $N = \sum_{i=0}^{k-1} a_i 10^i$ *with digits* $a_i \in \{0,1,\ldots,9\}$ *and* $a_{k-1} \neq 0$, *define*

$$\mathrm{rev}(N) = \sum_{i=0}^{k-1} a_i 10^{k-1-i}.$$

Definition 2 (Brahmam Mirror Number (BMN)) *An integer* $N \geq 1$ *is a Brahmam Mirror Number if the mirror product* $P := N \cdot \mathrm{rev}(N)$ is palindromic in base ten.

Lemma 1 (Base-9 Reversal Identity). For every integer n, the digit reversal preserves the residue modulo nine; that is,

$$R(n) \equiv n(\mathrm{mod}9).$$

Proof. Write n in decimal form as

$$n = a_k 10^k + a_{k-1}10^{k-1} + \cdots + a_1 10 + a_0,$$

where a_i are the digits of n. Reducing each power of ten modulo nine gives $10 \equiv 1(\mathrm{mod}9)$, and hence $10^m \equiv 1^m \equiv 1(\mathrm{mod}9)$ for all integers $m \geq 0$. Therefore,

$$n \equiv a_k + a_{k-1} + \cdots + a_1 + a_0(\mathrm{mod}9).$$

Now consider the reversal

$$R(n) = a_0 10^k + a_1 10^{k-1} + \cdots + a_{k-1}10 + a_k.$$

Applying the same reduction $10^m \equiv 1(\mathrm{mod}9)$ yields

$$R(n) \equiv a_0 + a_1 + \cdots + a_{k-1} + a_k(\mathrm{mod}9).$$

The right-hand sides of the last two expressions are identical. Thus

$$R(n) \equiv n(\mathrm{mod}9),$$

which completes the proof.

Remark 1 *Lemma 1 is a restatement of the standard digit-sum rule under modulo operations* 9*: reversal preserves the sum of digits and thus the residue class.[5, 7, 13].*

3. The Mirror Harmony Mod-9 Law

Theorem 1 (Mirror Harmony Mod-9 Law). For every integer n, the mirror product satisfies

$$n \cdot R(n) \equiv n^2(\mathrm{mod}9),$$

and therefore

$$n \cdot R(n) \in \{0,1,4,7\}(\mathrm{mod}9),$$

the set of quadratic residues modulo nine.

Proof. By Lemma 1, we have $R(n) \equiv n(\mathrm{mod}9)$. Multiplying both sides of this congruence by n gives

$$n \cdot R(n) \equiv n \cdot n \equiv n^2(\mathrm{mod}9).$$

Since the quadratic residues modulo nine are exactly

$$0^2, 1^2, 2^2, 4^2(\mathrm{mod}9) = \{0,1,4,7\},$$

the mirror product must lie in this set.

It is important to note that this restriction applies only to $n \cdot R(n)$ and not to n itself. Indeed, n may assume any of the nine residue classes modulo nine, but its mirror product is always confined to the square-residue quartet. This distinction completes the proof. [8, 10].

Corollary 1 *If* N *is a BMN, then the residue of its mirror product* $P = N \cdot rev(N)$ *modulo* 9 *is in* $\{0,1,4,7\}$.

Remark 2 (What is implied) *Theorem 1 does not imply* $N mod 9 \in \{0,1,4,7\}$. *Indeed,* $N mod 9$ *may be any of* $0,\ldots,8$, *but* $N^2 mod\ 9$*—and hence the mirror product modulo* 9*—is confined to the quartet. This distinction corrects a common misreading.*

Example 1 (Small Canonical BMNs)

1). $N = 12$, $\mathrm{rev}(N) = 21$, $P = 252$ (palindrome). We have $P \bmod 9 = 252 \bmod 9 = 0$.

2). $N = 22$, $\text{rev}(N) = 22$, $P = 484$ (palindrome). $P \bmod 9 = 484 \bmod 9 = 7$.

3). $N = 101$, $\text{rev}(N) = 101$, $P = 10201$ (palindrome). $P \bmod 9 = 10201 \bmod 9 = 1$.

In each case $P \bmod 9 \in \{0,1,4,7\}$, illustrating Corollary 1.

4. Computational Verification and a Simple Generator

A direct test for the BMN property proceeds as follows: for $N \in \mathbb{N}$, compute $R := \text{rev}(N)$, $P := N \cdot R$, and check whether P is palindromic. The complexity is dominated by decimal reversal and multiplication; both are $O(\log N)$ digit operations, so the incremental scan is light. The BMN density is empirically sparse but steady.

Using a complete brute-force enumeration of all integers up to $200{,}000{,}000$, we identified a total of 1246 Brahmam Mirror Numbers, empirically confirming their extreme sparsity and characteristic residue behaviour.

Regardless of whether N is BMN, Theorem 1 enforces $(N \cdot \text{rev}(N)) \bmod 9 \in \{0,1,4,7\}$. In our BMN scans (hundreds to thousands of instances), the set $\{N \bmod 9\}$ among accepted BMNs typically hits all nine residues. This is consistent with Theorem 1: the restriction belongs to P, not to N.

Several simple filters speed the generator without compromising correctness:

1). If the last digit of N is zero, then $\text{rev}(N)$ begins with zero, so P has trailing zeros; palindromicity then forces leading zeros (disallowed), hence such N can be skipped, except $N = 0$ which we ignore.

2). If the last digit of N is 5, then P ends in $5 \cdot$ (last digit ofR), which is odd; this does not exclude palindromicity, but empirical hit rates are lower—one may deprioritize but not forbid.

3). Residue pruning: although $N \bmod 9$ is unrestricted, $P \bmod 9$ must lie in $\{0,1,4,7\}$. One can quickly compute $N \bmod 9$, deduce $N^2 \bmod 9$, and record the target residue class for P ; a mismatch after multiplication can terminate the check early.

5. Mod-11 Structure

Related residue constraints for palindromic integer transformations were examined by Li and Wang in [11], supporting our residue-based analysis.

Mod- 11 arithmetic provides a distinct invariant: the *alternating-digit sum*. For a base-10 integer

$$N = \sum_{i=0}^{k-1} a_i 10^i, N \equiv \sum_{i=0}^{k-1} (-1)^i a_i (\text{mod} 11).$$

Unlike mod 9, reversal does *not* preserve residues mod 11 in general; rather,

$$\text{rev}(N) \equiv \sum_{i=0}^{k-1} (-1)^{k-1-i} a_i \equiv (-1)^{k-1} \sum_{i=0}^{k-1} (-1)^i a_i (\text{mod} 11).$$

Thus, when k is odd, $\text{rev}(N) \equiv N (\text{mod} 11)$; when k is even, $\text{rev}(N) \equiv -N (\text{mod} 11)$. Consequently:[14].

Proposition 1*Let N have k digits. Then*

$$N \cdot \text{rev}(N) \equiv \begin{cases} N^2 (\text{mod} 11), & k \text{odd}, \\ -N^2 (\text{mod} 11), & k \text{even}. \end{cases}$$

Proof. Substitute $\text{rev}(N) \equiv \pm N (\text{mod} 11)$ by parity of k and multiply.

Remark 3 *Proposition 1 shows a parity-twisted square law mod* 11. *In particular, if k is odd the mirror product behaves again like a square; if k is even it behaves like a negated square. For BMNs, palindromicity adds further constraints that interact with this parity.*

6. Density Heuristics and Growth

Let $B(X)$ denote the number of BMNs not exceeding X. A precise asymptotic is not yet known. We sketch a heuristic:

Heuristic A (independence model). Suppose (very roughly) that for a random k-digit N the product $P = N \cdot \text{rev}(N)$ distributes its central digits enough like a random $2k$-digit number subject to symmetry constraints. Palindromicity enforces $\lfloor 2k/2 \rfloor$ equalities between paired digits, yielding a combinatorial contraction on the order of $10^{\lceil 2k/2 \rceil}$. Among all 10^k choices of N this suggests a sparse but non-negligible proportion—leading to a slowly increasing $B(X)$, potentially on the order of X^θ with small $\theta > 0$.

Heuristic B (residue gate). Because $P \bmod 9$ must lie in $\{0,1,4,7\}$, one can think of a $4/9$ residue "gate" that combines with palindromicity constraints. This gate does not affect N directly but filters P; paired with digit-equality constraints it again suggests polynomially sparse growth.

Proving upper and lower bounds for $B(X)$ is an attractive open problem (see Section 10).

7. Mirror Products and Self-reciprocal Polynomials

Associate to $N = \sum_{i=0}^{k-1} a_i 10^i$ the *digit polynomial*

$$f_N(x) = \sum_{i=0}^{k-1} a_i x^i, \text{sothat} N = f_N(10), \text{rev}(N) = x^{k-1} f_N(1/x)|_{x=10}.$$

Then

$$N \cdot \text{rev}(N) = f_N(10) \cdot 10^{k-1} f_N(1/10),$$

which resembles the norm of f_N across the involution $x \leftrightarrow 1/x$. A base-10 palindrome corresponds to f_N being

self-reciprocal ($x^{k-1}f_N(1/x) = f_N(x)$); a mirror product being palindromic imposes a self-reciprocity constraint *after* multiplication by $f_N(10)$. Thisviewpoint motivates: [2-4].

Conjecture 1 (Reciprocal Norm Constraint) *If* N *is a BMN, then the digit polynomial* f_N *satisfies a reciprocity relation of the form*

$$f_N(10) \cdot 10^{k-1} f_N(1/10) = g(10),$$

where $g(x)$ is self-reciprocal with controlled coefficient growth.

This conjecture creates a bridge between BMNs and the theory of self-reciprocal polynomials, suggesting tools from algebra and complex analysis (e.g., location of zeros on the unit circle) may be brought to bear.[12].

8. Tables and Worked Examples

Table 1 records small BMNs, their reversals, and the residue of the mirror product modulo 9; the product residue always lies in $\{0,1,4,7\}$.

Table 1. *Small BMNs, reversals, and mirror-product residues mod 9.*

N	rev(N)	P=N·rev(N)	P mod 9
12	21	252	0
22	22	484	7
101	101	10201	1
111	111	12321	0
121	121	14641	1
202	202	40804	4

Example. For $N = 202$, $\text{rev}(N) = 202$, $P = 40804$, a palindrome, and $P \bmod 9 = 4$.

9. Methodological Notes for Data Generation

A minimal generator enumerates N in increasing order, computes $R = \text{rev}(N)$, forms$P = N \cdot R$, and checks whether P equals its reverse. Enhancements include:

1). *Length windows:* iterate over fixed digit-length blocks to exploit parity properties in mod-11.

2). *Early residue gate:* compute $N^2 \bmod 9$ first; if the final $P \bmod 9$ disagrees after multiplication, abort P's palindrome check.

3). *Digit-pattern priors:* emphasize N with symmetric end digits (e.g., $ab\cdots ba$, $a0\cdots 0a$) where the mirror structure plausibly reduces carry turbulence in multiplication.

10. Open Problems and Conclusion

We list problems that emerged in our study.

1. Asymptotics. Determine exponents α, β with $X^{\alpha} \ll B(X) \ll X^{\beta}$. Even coarse, unconditional bounds would be valuable.

2. Mod-11 classification for BMNs. Beyond Proposition 1, describe residue distributions of $P \bmod 11$ for BMNs by digit-length parity and central-digit conditions.

3. Reciprocal polynomial norm. Prove or refute the conjectural reciprocal-norm structure for f_N in the BMN case.

4. Digit-carry model. Develop a probabilistic model for carries in $N \cdot \text{rev}(N)$ that predicts the likelihood of palindromicity as $k \to \infty$.

5. Prime BMNs. Study the intersection with primes (if any) and with near-palindromic forms (e.g., one-perturbation palindromes).

The Mirror Harmony Mod-9 Law provides a universal identity for mirror products: for all integers N, $N \cdot \text{rev}(N) \equiv N^2 (\text{mod} 9)$, and thus the mirror product reduces to one of the four quadratic residues $\{0,1,4,7\}$ modulo nine. For BMNs—integers whose mirror product is additionally palindromic in base ten—this identity functions as a rigid arithmetic backbone: while $N \bmod 9$ remains unrestricted, the mirror product occupies the square-residue quartet. Our analysis clarifies the precise scope of the mod-9 phenomenon, adds a parity-twisted square law modulo 11, and outlines structural links to self-reciprocal polynomials. We close with density heuristics and open directions.

Abbreviations

BMN	Brahmam Mirror Number
R(n)	Decimal Digit Reversal of N
mod	Modulo Operation
QR	Quadratic Residue
BMR	Brahmam Mirror Residue (When Referring to n R(n) Mod 9)

Supplementary Material

The supplementary material can be accessed at https://doi.org/10.11648/j.pamj.20251406.14

Conflicts of Interest

The authors declare no conflicts of interest.

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