Mathematics Letters
2025; 11(3): 60-65
http://www.sciencepublishinggroup.com/j/ml
doi: 10.11648/j.ml.20251103.11

Coherent Filters of Pseudocomplemented 1-Distributive Lattices

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To cite this article:

Chandrani Nag, Syed Md Omar Faruk. (2025). Coherent Filters of Pseudocomplemented 1-Distributive Lattices. *Mathematics Letters, 11*(3), 60-65. https://doi.org/10.11648/j.ml.20251103.11

Received: 25 August 2025; **Accepted:** 3 September 2025; **Published:** 25 September 2025

Abstract: This work explores coherent filters in the framework of pseudocomplemented 1-distributive lattices. After reviewing the basic properties of such lattices and their pseudocomplements, we introduce the notion of coherent filters and establish conditions under which a filter is coherent. The study further examines the relationships between coherent, strongly coherent, and τ-closed filters, showing how these concepts interact with classical structures such as p-filters and D-filters. Several equivalent characterizations are derived, linking coherence with closure, pseudocomplements, and annihilators. In addition, we investigate semi Stone and Stone lattices, proving that a pseudocomplemented 1-distributive lattice is semi Stone precisely when every τ-closed filter is strongly coherent. This provides a new structural perspective on the role of coherence in lattice theory. By generalizing results previously known in distributive lattices, the paper offers a unified approach to understanding filter behavior in broader algebraic settings, with potential implications for further developments in lattice theory and related algebraic systems.

Keywords: 1-Distributive Lattice, Pseudocomplemented Lattice, Ideal, Filter, Coherent Filter

1. Introduction

The theory of ideals and filters in pseudocomplemented semilattices has been investigated by T. S. Blyth [1]. Extensions of the notion of pseudocomplementation were studied by J. C. Varlet [2]. T. Katriňák has also explored pseudocomplemented algebras, offering valuable characterizations of p-algebras [6, 7]. The works of Nag et al. [8–10] address p-algebras, along with the study of their ideals and congruences. M. S. Rao [11] has contributed results concerning coherent ideals and median prime ideals in distributive pseudocomplemented lattices. In this paper, we extend some of these ideas to the context of 1-distributive lattices.

A lattice L with a top element 1 is said to be 1-*distributive* if, for any $a, b, c \in L$,

$$a \vee b = 1 = a \vee c \quad \Rightarrow \quad a \vee (b \wedge c) = 1.$$

The pentagon lattice P_5 (Figure 1) serves as an example of a 1-distributive lattice that is not distributive, while the diamond lattice M_3 (Figure 1) fails to be 1-distributive.

In a 1-distributive lattice L, for any $a \in L$,

$$x \leq a^* \quad \text{if and only} \quad x \wedge a = 0,$$

the element a^* is called the *pseudocomplement* of a.

In Section 2, we discuss about some basic definitions and properties of 1-distributive lattices. We also gives some characterizations of 1-distributive lattices.

In Section 3, we give the definition of coherent Filter. We prove some conditions for a filter to be a coherent filter. We discuss about Stone lattices and semi Stone lattices. We prove some conditions for 1-distributive lattice to be semi Stone lattice.

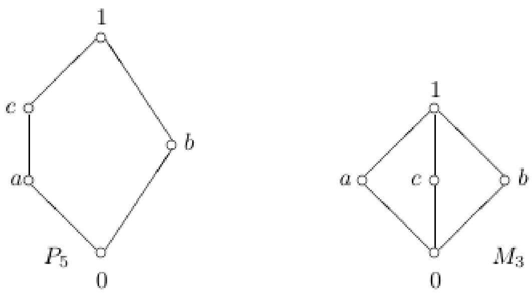

Figure 1. *The pentagonal lattice and the diamond lattice.*

2. Preliminaries

We recall the following well-known identities for pseudocomplemented lattices: [1, 3–7]:

1. $a \le b$ implies $b^* \le a^*$.
2. $a \le a^{**}$.
3. $a = a^{***}$.
4. $(a \vee b)^* = a^* \wedge b^*$.
5. $(a \wedge b)^{**} = a^{**} \wedge b^{**}$.
6. $a \wedge (a \wedge b)^* = a \wedge b^*$.
7. $(a \vee b)^{**} = (a^{**} \vee b^{**})^{**}$.

Ideals

Let L be a pseudocomplemented 1-distributive lattice. A non-empty subset $I \subseteq L$ is an *ideal* if:

1. For $a \in L$ and $b \in I$, $a \le b$ implies $a \in I$;
2. If $a, b \in I$, then $a \vee b \in I$.

For any pseudocomplemented 1-distributive lattice L, define the dense set:

$$D(L) = \{a \in L \mid a^* = 0\}.$$

Clearly, for each $a \in L$, the element $a \vee a^*$ belongs to $D(L)$.

Filters

A non-empty subset F of a pseudocomplemented 1-distributive lattice L is a *filter* if:

1. For $a \in L$ and $b \in F$, $b \le a$ implies $a \in F$;
2. If $a, b \in F$, then $a \wedge b \in F$.

(1) A *proper filter* is a filter if $F \ne L$.

(2) A *maximal filter* is a proper filter that is not strictly contained in any other proper filter.

(3) A *prime filter* P satisfies: for all $a, b \in L$, $a \vee b \in P$ implies $a \in P$ or $b \in P$.

Special Types of Filters

1. A *p-filter* satisfies: for any $a \in L$, $a^{**} \in F$ implies $a \in F$. The family of all p-filters in a p-algebra L is denoted by $F^*(L)$.
2. The set $D(L)$ is the smallest p-filter in a p-algebra [9].
3. A *D-filter* is a filter F such that $D(L) \subseteq F$.

3. Coherent Filters

Let A be a non-empty subset of a 1-distributive lattices L. Define

$$A^\tau = \{x \in L \mid a^{**} \vee x^{**} = 1 \text{ for all } a \in A\}.$$

It is immediate that $(D(L))^\tau = L$ and $L^\tau = D(L)$. For a singleton $\{a\}$, we write $(a)^\tau$ instead of $\{a\}^\tau$.

M. Sambasiva Rao [11] proved that A^τ is a filter in any distributive pseudo-complemented lattice. We observe that the same holds for the pentagon lattice P_5 (Figure 2), which is 1-distributive but not distributive.

Now we have the following theorem.

Theorem 3.1. Let L be a 1-distributive lattice and $A \subseteq L$ a non empty subset. Then A^τ is D-filter of L.

Proof. Since $d^{**} = 1$ for every $d \in D(L)$, it follows that $D(L) \subseteq A^\tau$.

Let $x, y \in A^\tau$. Then $x^{**} \vee a^{**} = 1$ and $y^{**} \vee a^{**} = 1$ for all $a \in A$.

By 1-distributivity:

$$a^{**} \vee (x^{**} \wedge y^{**}) = a^{**} \vee (x \wedge y)^{**} = 1.$$

Thus $x \wedge y \in A^\tau$.

If $x \in A^\tau$ and $x \le y$ for some $y \in L$, then $x^{**} \le y^{**}$ and hence:

$$1 = x^{**} \vee a^{**} \le y^{**} \vee a^{**}$$

for all $a \in A$, so $y \in A^\tau$.

Finally, since $x^{***} = x^*$, it follows that A^τ is a p-filter.

Remark 3.1. If $A \cap A^\tau \ne \phi$, then $A \cap A^\tau \subseteq D(L)$. Indeed, if $t \in A \cap A^\tau$, then $t^{**} \vee t^{**} = 1$, which implies $t^* = 0$, so $t \in D(L)$.

Now we have the following identities.

Theorem 3.2. Let A, B be two non-empty subsets of a pseudocomplemented 1-distributive lattice L. Then

(1) $A \subseteq B \implies B^\tau \subseteq A^\tau$,

(2) $A \subseteq A^{\tau\tau}$,

(3) $A^\tau = A^{\tau\tau\tau}$,

(4) $A^\tau = L$ if and only if $A = D(L)$

Proof.

(1) If $A \subseteq B$ and $x \in B^\tau$, then $x^{**} \vee b^{**} = 1$ for all $b \in B$. This implies $x^{**} \vee a^{**} = 1$ for all $a \in A$ and hence $x \in A^\tau$.

(2) Let $x \in A$ and $a \in A^\tau$, then $x^{**} \vee a^{**} = 1$, so $x \in A^{\tau\tau}$.

(3) From (ii), $A^\tau \subseteq A^{\tau\tau\tau}$. If $t \in A^{\tau\tau\tau}$, then $t^{**} \vee a^{**} = 1$ for all $a \in A^{\tau\tau}$, so $t \in A^\tau$.

(4) If $A^\tau = L$, then $x^{**} \vee a^{**} = 1$ for all $x \in L$, $a \in A$, implying $a^{**} = 1$ for all $a \in A$, hence $A = D(L)$. The reverse inclusion is clear.

Now we have the following theorem.

Theorem 3.3. Let F and G be filters of a pseudocomplemented 1-distributive lattice L. Then $(F \vee G)^\tau = F^\tau \cap G^\tau$.

Proof. Since $F \subseteq F \vee G$ and $G \subseteq F \vee G$, Theorem 3.2 gives

$$(F \vee G)^\tau \subseteq F^\tau \cap G^\tau.$$

For the reverse inclusion, take $x \in F^\tau \cap G^\tau$ and any $t \in F \vee G$. There exist $f \in F$ and $g \in G$ such that $t = f \wedge g$. From $x \in F^\tau \cap G^\tau$, we know:

$$x^{**} \vee f^{**} = 1, x^{**} \vee g^{**} = 1.$$

By 1-distributivity, for all $f \in F$ and $g \in G$:

$$x^{**} \vee (f^{**} \wedge g^{**}) = x^{**} \vee (f \wedge g)^{**} = 1$$

which gives

$$x^{**} \vee t^{**} = x^{**} \vee (f \wedge g)^{**} = 1.$$

Thus $x \in (F \vee G)^\tau$.

Now we have the following corollary.

Corollary 3.1. Let L be a pseudocomplemented 1-distributive lattice and $a, b \in L$. Then:

(1) $a \leq b \implies (a)^\tau \subseteq (b)^\tau$;

(2) $(a \wedge b)^\tau = (a)^\tau \cap (b)^\tau$;

(3) $(a)^\tau = L$ if and only if $a \in D(L)$;

(4) $a \in (b)^\tau \implies a \vee b \in D(L)$;

(5) $a^* = b^* \implies (a)^\tau = (b)^\tau$;

Proof.

(1) If $a \leq b$ and $x \in (a)^\tau$, then $a^{**} \leq b^{**}$. From $x^{**} \vee a^{**} = 1$, we deduce $x^{**} \vee b^{**} = 1$, so $x \in (b)^\tau$.

(2) Since $(a \wedge b) \leq a$ and $(a \wedge b) \leq b$, (i) yields $(a \wedge b)^\tau \subseteq (a)^\tau \cap (b)^\tau$.

Conversely, if $x \in (a)^\tau \cap (b)^\tau$, then $x^{**} \vee a^{**} = 1$ and $x^{**} \vee b^{**} = 1$. By 1-distributivity:

$$x^{**} \vee (a^{**} \wedge b^{**}) = x^{**} \vee (a \wedge b)^{**} = 1,$$

so $x \in (a \wedge b)^\tau$.

(3) Follows directly from Theorem 3.2.

(4) If $a \in (b)^\tau$, then $a^{**} \vee b^{**} = 1$, which implies $(a^{**} \vee b^{**})^{**} = 1$ and $(a \vee b)^{**} = 1$, hence $a \vee b \in D(L)$.

(5) If $a^* = b^*$, then $a^{**} = b^{**}$, so $(a)^\tau = (b)^\tau$.

Closed and Stone Elements

An element $a \in L$ is called *closed* if $a = a^{**}$. The set of all closed elements is:

$$B(L) = \{a \in L \mid a = a^{**}\}.$$

Clearly $0, 1 \in B(L)$.

An element $a \in L$ is a *Stone element* if:

$$a^* \vee a^{**} = 1$$

The set of all Stone elements is:

$$S(L) = \{a \in L \mid a^* \vee a^{**} = 1\}.$$

A pseudocomplemented 1-distributive lattice L is a *Stone lattice* if every element satisfies the Stone identity $a^* \vee a^{**} = 1$ for all $a \in L$.

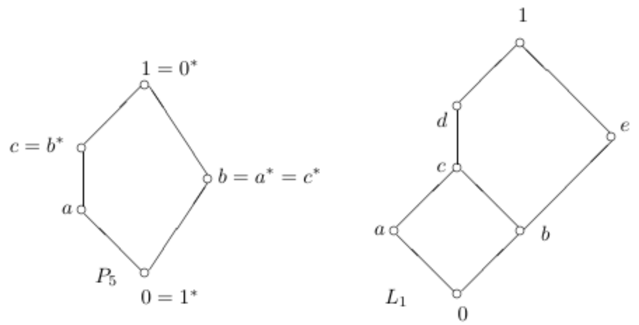

Figure 2. *Two p-algebras.*

Annihilators and ∘-operation

For $A \subseteq L$ non-empty, define the *annihilator*:

$$A^\perp = \{x \in L \mid x \wedge a = 0 \text{ for all } a \in A\}.$$

For a singleton $\{a\}$, write:

$$a^\perp = \{x \in L \mid x \wedge a = 0\}.$$

Define:

$$A^\circ = \{x \in L \mid x \vee a \in D(L) \text{ for all } a \in A\}.$$

Now we have the following result.

Theorem 3.4. For a pseudocomplemented 1-distributive lattice L, the following are equivalent:

(1) L is Stone lattice;

(2) $F^\tau = F^\circ$ for every filter F of L;

(3) $(a)^\tau = (a)^\circ$ for $a \in L$;

(4) for any two filters F, G of L, $F \cap G \subseteq D(L)$ if and only if $F \subseteq G^\tau$;

(5) $a \vee b \in D(L) \implies a^{**} \vee b^{**} = 1$ for all $a, b \in L$;

(6) $(a)^{\tau\tau} = (a^*)^\tau$ for all $a \in L$.

Proof. $(i) \Rightarrow (ii)$: Let F be an ideal of a pseudocomplemented 1-distributive Stone lattice L. Let $x \in F^\tau$. Then $x^{**} \vee f^{**} = 1$ for all $f \in F$. This implies $x^* \wedge f^* = (x \vee f)^* = 0$. So $(x \vee f) \in D(L)$. Hence $F^\tau \subseteq F^\circ$.

To prove the converse part, let $x \in F^\circ$. Then $x \vee f \in D(L)$ for all $f \in F$. This implies $x^* \wedge f^* = 0$. So $x^* \leq f^{**}$. Now since L is Stone, we have $1 = x^* \vee x^{**} \leq x^* \vee f^{**}$. Hence $x \in F^\tau$.

$(ii) \Rightarrow (iii)$: It is obvious.

$(iii) \Rightarrow (iv)$: Let F, G be two filters of L. Let $F \cap G \subseteq D(L)$ and let $x \in F$. Then for any $y \in G$, we have $x \vee y \in F \cap G \subseteq D(L)$. This implies $x \in (y)^\circ$ for all $y \in G$. By condition (iii), $x \in (y)^\tau$ for all $y \in G$. So $x^{**} \vee y^{**} = 1$ for all $y \in G$. Thus $x \in G^\tau$.

Conversely, let $F \subseteq G^\tau$ and $x \in F \cap G$. Since $F \subseteq G^\tau$, $x \in G^\tau$. So $x \in G \cap G^\tau = D(L)$. Hence $F \cap G \subseteq D(L)$.

$(iv) \Rightarrow (v)$: Let $a, b \in L$ with $a \vee b \in D(L)$. This implies $[a) \cap [b) \subseteq D(L)$ and so $(a] \subseteq (b]^\tau$. Hence $a \in (b]^\tau$ and thus $a^{**} \vee b^{**} = 1$.

$(v) \Rightarrow (vi)$: Let $a \in L$. Since $a \vee a^* \in D(L)$, we have by condition (v), $a^{***} \vee a^{**} = a^* \vee a^{**} = 1$. So $a^* \in (a)^\tau$. Hence by Theorem 3.2, we have $(a)^{\tau\tau} \subseteq (a^*)^\tau$.

Conversely, let $x \in (a^*)^\tau$ and $t \in (a)^\tau$. So we have $t^{**} \vee a^{**} = 1$ and so $t^* \wedge a^* = 0$. This implies $t^* \leq a^{**}$. Now $x \in (a^*)^\tau$ implies $a^{***} \vee x^{**} = 1$. So $0 = a^{**} \wedge x^* \geq t^* \wedge x^*$ as $t^* \leq a^{**}$.

Thus, we get $x^* \wedge t^* = (x \vee t)^* = 0$. This implies $t \vee x \in D(L)$ and from condition (v), we have $t^{**} \vee x^{**} = 1$ for all $t \in (a)^\tau$. So $x \in (a)^{\tau\tau}$.

$(vi) \Rightarrow (i)$: Let $a \in L$. Since $a \in (a)^{\tau\tau} = (a^*)^\tau$ we have $a^* \vee a^{**} = 1$.

Coherent Filters

A filter F in a pseudocomplemented 1-distributive lattice L is called *coherent* if, for all $x, y \in L$,

$$(x)^\tau = (y)^\tau \text{and } x \in F \implies y \in F.$$

It is straightforward to verify that $(x)^\tau$ is a coherent filter for every $x \in L$.

Lemma 3.1. Let L be a pseudocomplemented 1-distributive lattice and F be a p-filter of L. Then F is a coherent ideal if for any $x \in F$,

$$(x)^{\tau\tau} \subseteq F.$$

Proof. From theorem 3.4, we know that:

$$(x)^{\tau\tau} = (x^*)^\tau.$$

If $x \in F$ and $a \in (x)^{\tau\tau} = (x^*)^\tau$, then $x^{***} \vee a^{**} = 1$, which gives $x^{**} \leq a^{**}$. Since $x \in F$ and F is a p-filter, $x^{**} \in F$ implies $a^{**} \in F$ and hence $a \in F$. Thus $(x)^{\tau\tau} \subseteq F$.

Now we have the following result.

Theorem 3.5. For a pseudocomplemented 1-distributive lattice L, the following statements are equivalent:

(1) Every element of L is closed;

(2) Every filter in L is a p-filter;

(3) Every principal filter is coherent;

(4) Every filter is coherent;

(5) Every prime filter is coherent;

(6) $(a)^\tau = (b)^\tau \implies a = b$ for all $a, b \in L$;

(7) $a^* = b^* \implies a = b$ for all $a, b \in L$.

Proof. $(i) \Rightarrow (ii)$: If each $x \in L$ is closed, then $x = x^{**}$. Hence $x^{**} \in F$ immediately gives $x \in F$, so every filter is a p-filter.

$(ii) \Rightarrow (iii)$: Let $[x)$ be a principal filter. Since $x \vee x^* \in D(L)$ for all $x \in L$, we have $(x \vee x^*)^{**} = 1$. By (ii), $x \vee x^* = 1$. Now if $(a)^\tau = (b)^\tau$ and $a \in [x)$, then $a \geq x$ and so $a \vee x^* \geq x \vee x^* = 1$, giving $a^{**} \vee x^{***} \geq a \vee x^* = 1$. This forces $x \leq b^{**}$, and (ii) then implies $b \in [x)$, so $[x)$ is coherent.

$(iii) \Rightarrow (iv)$: If F is any filter and $a \in F$ with $(a)^\tau = (b)^\tau$, then $b \in [a) \subseteq F$.

$(iv) \Rightarrow (v)$: Clear, as prime filters are particular cases of filters.

$(v) \Rightarrow (vi)$: If $(a)^\tau = (b)^\tau$ but $a \neq b$, pick a prime filter P containing a but not b. Coherence of P forces $b \in P$, a contradiction. So $a = b$.

$(vi) \Rightarrow (vii)$: If $a^* = b^*$, then $a^{**} = b^{**}$, hence $(a)^\tau = (b)^\tau$ and (vi) gives $a = b$.

$(vii) \Rightarrow (i)$: Let condition holds. Let $x \in L$. Since $(x^{**})^* = x^*$, (vii) implies $x = x^{**}$ for all x, so all elements are closed.

Definition of $\pi(F)$

For any filter F in a pseudocomplemented 1-distributive lattice L, define:

$$\pi(F) = \{x \in L \mid (x)^\tau \vee F = L\}.$$

This construction will be used in the next lemmas.

Lemma 3.2. Let F, G, H be filters of a pseudocomplemented 1-distributive lattice L. If $F \vee G = L$ and $F \vee H = L$, then:

$$F \vee (G \cap H) = L.$$

Proof. From $F \vee G = L$, there exist $f_1 \in F$ and $g_1 \in G$ such that $f_1 \wedge g_1 = 0$. Similarly, $F \vee H = L$ gives $f_2 \in F$ and $h_1 \in H$ with $f_2 \wedge h_1 = 0$. Then:

$$f_1 \wedge f_2 \wedge g_1 = 0 \text{ and } f_1 \wedge f_2 \wedge h_1 = 0.$$

Thus $f_1 \wedge f_2 \leq g_1^*$ and $f_1 \wedge f_2 \leq h_1^*$, hence:

$$(f_1 \wedge f_2) \leq g_1^* \wedge h_1^* = (g_1 \vee h_1)^*.$$

It follows that:

$$(f_1 \wedge f_2) \wedge (g_1 \vee h_1) = 0$$

with $f_1 \wedge f_2 \in F$ and $g_1 \vee h_1 \in G \cap H$.

Therefore, $0 \in F \vee (G \cap H)$, so $F \vee (G \cap H) = L$.

Lemma 3.3. Let F, G be filters in L. Then:

(1) If $F \subseteq G$, then $\pi(F) \subseteq \pi(G)$;

(2) $\pi(F \cap G) = \pi(F) \cap \pi(G)$.

Proof. (i) If $x \in \pi(F)$, then $(x)^\tau \vee F = L$. Since $F \subseteq G$, we also have $(x)^\tau \vee G = L$, so $x \in \pi(G)$.

(ii) From (i), $\pi(F \cap G) \subseteq \pi(F) \cap \pi(G)$.

Conversely, if $x \in \pi(F) \cap \pi(G)$, then $(x)^\tau \vee F = L$ and $(x)^\tau \vee G = L$. By Lemma 3.2, $(x)^\tau \vee (F \cap G) = L$, hence $x \in \pi(F \cap G)$.

Lemma 3.4. For any filter F in L, $\pi(F)$ is a D-filter.

Proof. Since $d^{**} = 1$ for all $d \in D(L)$, we have $D(L) \subseteq \pi(F)$. If $a, b \in \pi(F)$, then $(a)^\tau \vee F = L$ and $(b)^\tau \vee F = L$.

By Lemma 3.2 and Corollary 3:

$$((a)^\tau \cap (b)^\tau) \vee F = (a \wedge b)^\tau \vee F = L,$$

so $a \wedge b \in \pi(F)$.

If $a \in \pi(F)$ and $a \leq b$, then $(a)^\tau \subseteq (b)^\tau$ and:

$$L = (a)^\tau \vee F \subseteq (b)^\tau \vee F,$$

so $b \in \pi(F)$. Hence $\pi(F)$ is a D-filter.

Lemma 3.5. A filter F is a D-filter if and only if $\pi(F) \subseteq F$.

Proof. If F is a D-filter and $a \in \pi(F)$, then $(a)^\tau \vee F = L$.

This implies $a = p \wedge q$ with $p \in (a)^\tau$ and $q \in F$. Since $p \in (a)^\tau = (a)^\circ$ by Theorem 3.4, $p \vee a \in D(L) \subseteq F$. Also $q \leq q \vee a \in F$, so:

$$(p \vee a) \wedge (q \vee a) = p \wedge q = a \in F.$$

Conversely, if $\pi(F) \subseteq F$, then $D(L) \subseteq \pi(F) \subseteq F$, so F is a D-filter.

Strongly Coherent and τ-Closed Filters

(1) A filter F is called *strongly coherent* if $F = \pi(F)$.

(2) A filter F is called *τ-closed* if $F = F^{\tau\tau}$.

It holds that $F \subseteq D(L)$ if and only if $F^{\tau\tau} = D(L)$.

Observe that the smallest τ-closed filter is $D(L)$, and the largest is L.

Lemma 3.6. Every strongly coherent filter is coherent.

Proof. If F is a strongly, then $F = \pi(F)$. By Lemma 3.5, F is a D-filter. If $(a)^\tau = (b)^\tau$ and $a \in F$, then $(a)^\tau \vee F = L$, so $(b)^\tau \vee F = L$ and $b \in \pi(F) = F$.

Lemma 3.7. Every τ-closed filter is coherent.

Proof. If F is τ-closed and $(x)^\tau = (y)^\tau$ with $x \in F$, then:

$$y \in (y)^{\tau\tau} = (x)^{\tau\tau} \subseteq F^{\tau\tau} = F.$$

Thus F is coherent.

Semi Stone Lattices

A pseudocomplemented 1-distributive lattice L is *semi Stone* if:

$$(x)^\tau \vee (x)^{\tau\tau} = L \text{ for every } x \in L.$$

Theorem 3.6. Every Stone lattice is semi Stone.

Proof. If L is Stone, assume for some $x \in L$ that $(x)^\tau \vee (x)^{\tau\tau} \neq L$. Then there exists a maximal filter M containing both $(x)^\tau$ and $(x)^{\tau\tau}$. Since $x \in (x)^{\tau\tau} \subseteq M$ and M is proper, so $x^* \notin M$. But the Stone property $x^* \vee x^{**} = 1$ implies $x^* \in (x)^\tau \nsubseteq M$, a contradiction. Hence, $(x)^\tau \vee (x)^{\tau\tau} = L$.

M. S. Rao [11] showed that the converse of the above theorem is not true for distributive lattices. So the converse part is also not true for 1-distributive lattices. Now we have the following theorem.

Theorem 3.7. For a pseudocomplemented 1-distributive lattice L, the following are equivalent:

(1) L is semi Stone;

(2) Every τ-closed filter is strongly coherent;

(3) For all $x \in L$, $(x)^{\tau\tau}$ is strongly coherent.

Proof. $(i) \Rightarrow (ii)$: If L is semi Stone and F is τ-closed, then F is a D-filter with $F^{\tau\tau} = F$. By Lemma 3.5, $\pi(F) \subseteq F$.

For $x \in I$, semi Stone implies:

$$L = (x)^\tau \vee (x)^{\tau\tau} \subseteq (x)^\tau \vee F^{\tau\tau},$$

so $x \in \pi(F)$ and thus $F = \pi(F)$, making F strongly coherent.

$(ii) \Rightarrow (iii)$: Immediate, since $(x)^{\tau\tau}$ is τ-closed.

$(iii) \Rightarrow (i)$: If $(x)^{\tau\tau}$ is strongly coherent, then $\pi((x)^{\tau\tau}) = (x)^{\tau\tau}$. Since $x \in (x)^{\tau\tau}$, we have $(x)^\tau \vee (x)^{\tau\tau} = L$.

4. Conclusion

This study has provided a comprehensive investigation into the structure and properties of coherent filters within pseudocomplemented 1-distributive lattices. By establishing rigorous conditions under which a filter can be classified as coherent, we have extended classical results on distributive lattices to a broader algebraic framework. The interplay between coherent, strongly coherent, and τ-closed filters has been clarified, revealing their connections to semi Stone and Stone lattice properties. In particular, we demonstrated equivalence results that characterize when a pseudocomplemented 1-distributive lattice attains the semi Stone property, thereby enriching the understanding of filter behaviour in these settings.

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Conflicts of Interest

The authors declare no conflicts of interest.

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