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Research Article

Equilibrium in the Presence of Many Insider Traders

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Abstract

The model has three kinds of traders: an insider, random noise traders, and a market maker. The insider aims to exploit her informational advantage and maximize expected profits while the market maker observes the total order flow and sets prices accordingly. The equilibrium of auction, when there are a noise trader, a insider trader and a price maker is well studied in the literature. However, in practice, there exist more than one insider and noise traders. In this paper, the case of κ insider traders are considered. First, the pure Nash equilibriums are derived and two learning methods namely gradient and partial best response are studied. Then, the effect existence of more than one insider traders in the market on equilibriums and learning methods are considered. Also, mixture equilibriums are derived and corresponding learning method for mixture distributions is derived. Finally, a conclusion is proposed.

Keywords

Gradient Scheme, Kyle's Equilibrium, Learning Method, Nash Equilibrium, BPR

1. Introduction

Kyle proposed auction equilibriums in the presence of insider trader, noise trader and price maker [15]. He studied the effect of noise trading in the volatility of price and the value of private information of insider trader. The model of Kyle has been extended in various ways. Glosten and Milgrom considered the same problem and their conclusions were similar to Kyle's results [13]. Back and Back *et al*. considered the insider trading problem in continuous time process and used the dynamic programming [3, 4]. Campi and Campi *et al*. considered the quadratic hedging based on insider trading approach [7, 8]. Biagini and Oksendal surveyed the general stochastic calculus approach to the insider trading problem [5]. Danilova proposed the stock market insider trading with imperfect dynamic information [12]. Aase *et al*. studied the strategic insider trading equilibrium using the filtering approach in continuous time cases [1]. Vovk considered the game theoretic aspects of continuous-time trading method using the emergence probability [21]. Aksamit and Hou measured the information value in financial pricing and hedging [2].

A general principle to establish equilibrium follows the inconspicuous trade. The resulting equilibrium insider strategy is inconspicuous in the sense that the accumulated total demand process has the same law as the accumulated demand of the noise trader alone. Furthermore, the true value, which is independent of the noise trader's demand, has to be a non-decreasing function of the accumulated total demand at maturity. The market maker's price function of the total demand and time is chosen such that any insider demand process that leads to the terminal accumulated total demand described above is optimal. In Cetin the existence of an inconspicuous equilibrium is shown if noise trades follow a Poisson process

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instead of Brownian motion [9]. Further extensions include trading with default risk in Campi and Cetin [8] and a random trading horizon in Cetin [9]. The case of dynamic private information where the insider's information evolves over time is considered in Danilova [12]. For a thorough overview we refer to the monograph Cetin and Danilova [10]. The inconspicuous trade does not work for risk-averse market makers. In the continuous time model, Cetin and Danilova prove the existence of a non-inconspicuous equilibrium that is derived from a fixed point [10]. In Back *et al*. there are several assets and the distribution of the true value and the accumulated total demand at maturity are coupled through an optimal transport map [4]. McLennan *et al*. extend uniqueness to a broader class of equilibrium [16]. Cetin and Larsen study whether Kyle's affine-linear equilibrium is stable for different trading times [11]. Rochet and Vila study a related single-period model where the insider can observe the trade of the noise trader and show the existence of a unique equilibrium while relaxing the assumption of normality [19]. Kramkov and Xu further relax distributional assumptions by connecting the problem to the dual problem of a certain class of optimal martingale transport problems [14].

Kyle considered one risk neutral insider trader. However, this is not true, in practice [15]. Here, first, it is assumed that there are two insider traders. Then, it is assumed that there are more than two insider traders. Following Kyle's notations [15], the noise trader is an uninformed person who trades the quantity u with normal distribution

$$u \sim N(0, \sigma_u^2),$$

and the liquidation of underlying asset v has normal distribution

$$v \sim N(\mu, \sigma^2).$$

It is assumed that u and v are independent.

Assuming two insider traders, let the quantity traded two insider traders be

$$x_i(v) = a_i + b_i v, i = 1,2$$

and the aggregate order flow is

$$y = u + x_1 + x_2,$$

which is a basis for determining the price p of asset. The market efficiency principle guaranties

$$p = E(v|y) = \gamma + \lambda y.$$

Also, the profit maximization principle of Kyle advises to maximize

$$E(\pi_i|v) = x_i\big(v - \gamma - \lambda(x_1 + x_2)\big),$$

with respect to $x_i, i = 1,2$. Here,

$$\pi_i(x_1, x_2) = x_i(v - p)$$

is the profit of i-th player.

The originality of a paper about insider trading, particularly one that covers multiple insider traders, can be evaluated based on several factors:

Unique Perspective: Offering a new angle or perspective on insider trading, such as examining the psychological motivations of multiple insiders, can enhance originality.

Comprehensive Analysis: Analyzing different cases of insider trading, especially those that have been less studied can provide fresh insights.

Comparative Studies: Comparing insider trading regulations across different countries or examining the impact of cultural differences on insider trading behaviors can introduce originality.

Data and Methodology: Utilizing novel datasets or innovative methodologies (like machine learning techniques) for analyzing insider trading patterns can be a significant contribution.

Theoretical Frameworks: Applying or developing new theoretical frameworks to understand the implications of multiple insider traders can offer original insights.

Interdisciplinary Approach: Incorporating theories or ideas from other fields (like psychology, sociology, or ethics) can provide a unique viewpoint on the motivations and consequences of insider trading.

Recent Developments: Focusing on recent cases, regulatory changes, or market reactions related to insider trading can also contribute to the originality of the paper.

To ensure originality, it's essential to conduct a thorough literature review to identify gaps in existing research and position your paper accordingly.

The rest of paper is designed as follows. In section 2, the pure equilibriums are given and learning schemes are proposed. Also, the effects of number of insider traders and noise traders on equilibriums are studied. In section 3, the mixture equilibriums and learning schemes of mixture Kyle's game are presented. Finally, section 4 concludes.

2. Equilibriums

As follows, pure equilibriums as well as learning procedures are proposed.

2.1. Pure Equilibriums

In a simultaneous game, the best response functions are given as follows

$$\begin{cases} br_1(x_2) = 0.5\lambda^{-1}(v - \gamma - \lambda x_2), \\ br_2(x_1) = 0.5\lambda^{-1}(v - \gamma - \lambda x_1). \end{cases}$$

It is easy to see that the Nash equilibrium occurs in

$$x_1 = x_2 = \max(\frac{v-\gamma}{3\lambda}, 0).$$

Assuming $v-\gamma$ is positive, then

$$b_1 = b_2 = \frac{1}{3\lambda} \text{ and } a_1 = a_2 = -\gamma b_1.$$

The projection theorem implies that

$$\lambda = \frac{2b_1\sigma^2}{\sigma_u^2 + 4b_1^2\sigma^2} = \frac{1}{3b_1}.$$

Then, $b_1 = \frac{\sigma_u}{\sigma\sqrt{2}}$. Then,

$$\lambda = \frac{\sqrt{2}}{3}\frac{\sigma}{\sigma_u}.$$

Also,

$$E(p) = E(v) = \mu = \gamma + 2\lambda(a_1 + b_1\mu).$$

The last equation concludes that $\gamma = \mu$ and then

$$x_i = b_i(v-\mu).$$

Next, assume that there are $\kappa \geq 3$ insider traders at which

$$x_i(v) = a_i + b_i v, i = 1, \ldots, \kappa,$$

$$y = u + \sum_{i=1}^{\kappa} x_i = u + s.$$

Therefore, equation

$$\frac{\partial \pi_i}{\partial x_i} = 0$$

implies that

$$v - \mu - \lambda s_{-i} - 2\lambda x_i = 0,$$

where
$s_{-i} = \sum_{j=1, j\neq i}^{\kappa} x_j$.
Again, it is seen that the Nash equilibrium holds in

$$x_1 = \cdots = x_\kappa = \max\left(\frac{v-\mu}{(\kappa+1)\lambda_\kappa}, 0\right),$$

where
$\lambda_\kappa = \frac{\sqrt{\kappa}}{(\kappa+1)}\frac{\sigma}{\sigma_u}$, $b_i = \frac{1}{(\kappa+1)\lambda_\kappa} = \frac{\sigma_u}{\sigma\sqrt{\kappa}}$
and $a_i = -\mu b_i$. When the volatility of noise trader is high (for example, there are considerable number of noise traders), that is

$$\sigma_u \to \infty,$$

then

$$\lambda_\kappa \to 0$$

and each x_i is large. Instead, when $\sigma_u \to 0$, then $\lambda_\kappa \to \infty$, and each x_i is small. Notice that

$$x_i = b_i(v-\mu) = \frac{\sigma_u}{\sigma\sqrt{\kappa}}(v-\mu) = \frac{\sigma_u}{\sqrt{\kappa}}Z,$$

where $Z = \frac{v-\mu}{\sigma}$ has standard normal distribution $N(0,1)$. Thus, as

$$\sigma_u \to \infty, \text{ then } x_i \to \infty$$

and if $\sigma_u \to 0$, then $x_i \to 0$, almost sure. Also, suppose that there are κ_u noise traders. Then,

$$\sigma_u = \sqrt{\kappa_u}\sigma_u^* \text{ and } x_i = \frac{\sqrt{\kappa_u}}{\sqrt{\kappa}}\sigma_u^* Z.$$

Therefore, if $\kappa_u \to \infty$ or $\kappa \to 0$ then $x_1 \to \infty$ and if

$$\kappa_u \to \infty \text{ or } \kappa \to \infty$$

Then

$$x_1 \to 0.$$

Also, as $\kappa \to \infty$, then
$x_i \to 0$, almost sure.
A simple calculation shows that

$$p = \mu + \lambda_\kappa u + \frac{\kappa}{\kappa+1}(v-\mu).$$

Hence, if $\kappa \to \infty$, then $p \to v$ almost sure. One can see that

$$var(p) = cov(v,p) = \frac{\kappa}{\kappa+1}\sigma^2.$$

Thus,

$$cov(v,p) = \rho_{v,p} = \sqrt{\frac{\kappa}{\kappa+1}}.$$

Following Kyle, the measure of the informative of price is

$$var(v|p) = \sigma^2\left(1 - \rho_{p,v}^2\right) = \frac{\sigma^2}{\kappa+1}.$$

As $\kappa \to \infty$, then

$$var(p) \to \sigma^2 \text{ and } var(v|p) \to 0.$$

Here, the leader-follower pattern of above game is discussed. Let player 1 be the leader and player 2 be follower. Then, π_2 is maximized by

$$x_2 = \frac{v-\mu}{2\lambda} - \frac{x_1}{2}.$$

Replacing this value for x_2 in π_1 and

$$\frac{\partial \pi_1}{\partial x_1} = 0,$$

it is seen that

$$x_1 = \max(\frac{v-\mu}{2\lambda}, 0)$$

and $x_2 = 0$. The following proposition summarizes the above discussion.

Proposition 1. Considering two insider traders in Kyle's auction, *(a)-(b)* are correct.

(a) In a simultaneous format of game, the Nash equilibrium occurs at

$$x_1 = x_2 = \max(\frac{v-\mu}{3\lambda}, 0).$$

For κ insider trader case, then the Nash equilibrium occurs at

$$x_1 = \cdots = x_\kappa = \max\left(\frac{v-\mu}{(\kappa+1)\lambda_\kappa}, 0\right).$$

When, $\kappa \to \infty$, then the share of each player goes to zero, $var(p) \to \sigma^2$, $var(v|p) \to 0$ and $p \to v$ almost sure.
It is also correct if $\sigma_u \to 0$. However, if
$\sigma_u \to \infty$, the x_i's are large.

(b) In sequential type of game (player 1 as leader and player 2 as follower) then

$$x_1 = \max(\frac{v-\mu}{2\lambda}, 0) \text{ and } x_2 = 0.$$

2.2. Learning Schemes

As follows, in a two-player setting, the convergence of two learning schemes (gradient and BPR) to Nash equilibrium is studied. The rate of convergence is studied.

Notice that

$$\pi_i(x_1, x_2) = x_i\big(v - \mu - \lambda_2(x_1 + x_2)\big), i = 1,2,$$

and

$$\begin{cases} \frac{\partial \pi_1}{\partial x_1} = v - \mu - \lambda_2 x_2 - 2\lambda_2 x_1, \\ \frac{\partial \pi_2}{\partial x_2} = v - \mu - \lambda_2 x_1 - 2\lambda_2 x_2. \end{cases}$$

Following Singh *et al*. (1999), the gradient learning scheme is given by

$$\begin{cases} x_{1,n+1} = x_{1,n} + \zeta \frac{\partial \pi_1(x_{1,n}, x_{2,n})}{\partial x_1}, \\ x_{2,n+1} = x_{2,n} + \zeta \frac{\partial \pi_2(x_{1,n}, x_{2,n})}{\partial x_2}, \end{cases}$$

where ζ is a positive step size of gradient learning method. Let

$$y_n = x_{1,n} - x_{2,n}.$$

Therefore,

$$y_{n+1} = y_n - \zeta\lambda_2 y_n = (1 - \zeta\lambda_2) y_n = (1 - \zeta\lambda_2)^{n-1} y_1.$$

Assuming

$$\zeta\lambda_2 < 2, \text{ then } y_n \to 0$$

no matter what the initial value $y_1$is. That is, the Nash equilibrium occurs. Therefore, it is enough to let

$$\zeta < \frac{3\sqrt{2}\sigma_u}{\sigma}.$$

The rate of convergence is

$$O((1 - \zeta\lambda_2)^{-n}),$$

as $n \to \infty$, no matter what the initial value

$$y_0 = x_{1,0} - x_{2,0}.$$

It is easy to see that the best response functions are

$$\begin{cases} br_1(x_2) = \frac{3\sqrt{2}}{4}\sigma_u Z - \frac{x_2}{2}, \\ br_2(x_1) = \frac{3\sqrt{2}}{4}\sigma_u Z - \frac{x_1}{2}. \end{cases}$$

Using the partial best response method (see Owen, 1995), uses equations

$$x_{1,n} = br_1(x_{2,n-1}) \text{ and } x_{2,n} = br_1(x_{1,n-1}).$$

Thus, the following two timescales stochastic approximations (see, Borkar [6]) are derived:

$$\begin{cases} x_{1,n+1} = (1-\alpha)x_{1,n} + \alpha br_1\big(x_{2,n-1}\big), \\ x_{2,n+1} = (1-\alpha)x_{2,n} + \alpha br_1\big(x_{1,n-1}\big). \end{cases}$$

where $0 < \alpha < 1$. Again, it is seen that

$$y_{n+1} = (1-\alpha)y_n + \frac{\alpha}{2} y_n = \left(1 - \frac{\alpha}{2}\right) y_n.$$

Let $\alpha = c\lambda_2$. Hence, it is enough to assume that

$$c < \frac{3\sqrt{2}\sigma_u}{2\sigma},$$

then $y_n \to 0$ no matter what the initial value $y_1$is. The rate of convergence is

$$O\left(\left(1-\frac{c\lambda_2}{2}\right)^{-n}\right).$$

Hence, the Nash equilibrium occurs. Again, the following proposition summarizes the above discussion.

Proposition 2. In (a) of proposition 1, the gradient learning scheme and partial best response sequences are given as follows, respectively.

$$\text{Gradient: } \begin{cases} x_{1,n+1} = x_{1,n} + \zeta \frac{\partial \pi_1(x_{1,n}, x_{2,n})}{\partial x_1}, \\ x_{2,n+1} = x_{2,n} + \zeta \frac{\partial \pi_2(x_{1,n}, x_{2,n})}{\partial x_2}, \end{cases}$$

$$\text{BPR: } \begin{cases} x_{1,n+1} = (1-\gamma_{1,n})x_{1,n} + \gamma_{1,n} br_1(x_{2,n-1}), \\ x_{2,n+1} = (1-\gamma_{2,n})x_{2,n} + \gamma_{2,n} br_1(x_{1,n-1}). \end{cases}$$

Both learning methods converge to Nash equilibrium

$$x_1 = x_2.$$

Here, assume that each insider trader uses the historical data about the price

$$v_t, t = 1,2, \dots, T$$

to forecast the next value of v_{T+1} using $\bar{v}_T$. One can see that as

$$T \to \infty, \text{ then } \bar{v}_T \to \mu$$

almost sure and therefore, the share of each insider investor x_i converges to zero almost sure.

2.3. Real-World Implications

Insider trading, especially in markets with many traders, has significant real-world implications across various sectors. Here are some key applications and impacts:

Market Integrity and Fairness: Insider trading undermines the fairness of financial markets, leading to unequal information distribution among traders. Ensures that markets remain transparent and equitable, fostering investor confidence.

Regulatory Enforcement and Compliance: Helps regulators develop detection models and surveillance systems to identify suspicious trading activities. Guides the formulation of policies to prevent insider trading, such as monitoring unusual trading volumes or patterns.

Risk Management and Fraud Detection:

Financial institutions utilize insider trading detection techniques to manage risks associated with insider information leaks.

Detecting insider trading helps prevent financial fraud and reduces potential legal liabilities.

Market Prediction and Modeling:

Researchers utilize insider trading data to understand how information flows impact asset prices.

Models that simulate insider trading scenarios assist in stress-testing markets and understanding systemic vulnerabilities.

Legal and Forensic Investigations: Law enforcement agencies analyze trading data to gather evidence against insider trading schemes. Facilitates the prosecution of illegal traders and deters future misconduct.

Economic Policy and Market Regulation: Insights from insider trading patterns inform policymakers to improve market regulations and disclosure requirements.

Enhances the design of policies aimed at increasing transparency and protecting investors.

Technological Development: Development of machine learning algorithms and data analytics tools to detect insider trading in real-time. Use of big data analytics to process vast trading datasets involving many traders.

Corporate Governance and Investor Protection: Companies implement internal controls to prevent leaks of sensitive information. Protects minority shareholders and maintains corporate reputation.

In summary, the insider trading problem with many traders is crucial for maintaining market fairness, enforcing regulations, developing detection technologies, and ensuring overall financial stability. Addressing this issue benefits the entire financial ecosystem by promoting transparency, trust, and efficient market functioning.

3. Mixture Framework

In the following sub-sections mixture equilibriums and corresponding learning methods are proposed.

3.1. Mixture Equilibriums

Here, the mixed strategies are studied. To this end, suppose that both players randomize according to the distribution function F and density function f on support $[0, \infty]$. Then, the first player expected payoff is

$$u_1 = \int \pi_1(x_1, x_2) dF(x_2) = \int \pi_1(x_1, x_2) f(x_2) dx_2.$$

To randomize, the first player must be indifference between any x_1. That is, it is enough to find $f(x_2)$ such that

$$\frac{\partial u_1}{\partial x_1} = 0.$$

It is easy to see that

$$\frac{\partial u_1}{\partial x_1} = \int \frac{\partial \pi_1}{\partial x_1} dF(x_2) = 0.$$

Then,

$$E\left(\frac{\partial\pi_1(x_1,X_2)}{\partial x_1}\right) = 0$$

for any x_1. Notice that,

$$v = \frac{x_2-b_2}{a_2} \text{ and } x_1 = ax_2 + b$$

$$a = \frac{a_1}{a_2}, b_1 - \frac{a_1}{a_2}b_2.$$

Hence,

$$\frac{\partial\pi_1}{\partial x_1} = v - \gamma - \lambda x_2 - 2\lambda x_1 = \frac{x_2-b_2}{a_2} - \gamma - \lambda x_2 - 2\lambda(ax_2 + b),$$

where $p = \gamma + \lambda y$. So,

$$E\left(\frac{\partial\pi_1}{\partial x_1}\right) = \left(\frac{1}{a_2} - \lambda - 2a\lambda\right) E(x_2) - \left(\gamma + \frac{b_2}{a_2} + 2\lambda b\right) = 0.$$

Thus,

$$\begin{cases} \lambda = \frac{1}{2a_1+a_2}, \\ \gamma = -\frac{2b_1+b_2}{2a_1+a_2}. \end{cases}$$

Assuming a mixture equilibrium exists for player 1, then

$$\begin{cases} \lambda = \frac{1}{2a_2+a_1}, \\ \gamma = -\frac{2b_2+b_1}{2a_2+a_1}. \end{cases}$$

Therefore, it is concluded that

$$a_1 = a_2 = \frac{1}{3\lambda}, b_1 = b_2 = \frac{-\gamma}{3\lambda}$$

and $x_1 = x_2 = \frac{v-\gamma}{3\lambda}$.

As well as, it is seen that

$$E(p) = E(\gamma + \lambda y) = \gamma + 2\lambda E(x_1) = \gamma + 2\lambda\frac{\mu-\gamma}{3\lambda} = E\big(E(v|y)\big) = E(v) = \mu.$$

Then,

$$\gamma = \mu \text{ and } x_1 = x_2 = \frac{v-\mu}{3\lambda}.$$

Choosing

$$\lambda = \lambda_2$$

(defined previously), it is seen that the mixture Nash equilibrium leads to the pure Nash equilibrium. However, the general form of mixture equilibrium distribution is normal distribution

$$N(0,\frac{\sigma^2}{9\lambda^2}).$$

Assuming the maximum predetermined tolerate variances for x_1, x_2 are the same and is

$$\tau^2 = \frac{\sigma^2}{9\lambda^{*2}}.$$

Therefore,

$$\lambda^* = \frac{\sigma}{3\tau} \text{ and } x_1 = x_2 = \tau Z$$

where Z has standard normal distribution $N(0,1)$. Again, the above results are summarized in the following proposition.

Proposition 3. The mixture equilibrium distribution for both

$$x_i, i = 1,2 \text{ is } N(0,\frac{\sigma^2}{9\lambda^2}).$$

When, the maximum predetermined variance is τ^2, then

$$x_i = \tau Z,$$

at which Z is the standard normally distributed random variables and

$$p = \mu + \frac{\sigma}{3\tau}y = \mu + \frac{2\sigma}{3\tau}x_i, i = 1,2.$$

In this case, p has normal distribution $N(\mu,\frac{4}{9}\sigma^2)$.

Remark 1. As a special case, following Sgroi in Cournot duopoly game [20], suppose that $f(x_2) = 1$ on $[a(x_1), a(x_1) + 1]$,
for some $a(x_1) > 0$. Then

$$u_1 = x_1(v - \gamma - 0.5) - x_1 a(x_1) - \lambda x_1^2.$$

By letting $\frac{\partial u_1}{\partial x_1} = 0$, it is seen that

$$a(x_1) = \begin{cases} \lambda(\theta - 0.5\lambda^{-1} - x_1), & \text{if } x_1 < \theta - 0.5\lambda^{-1}, \\ 0, & \text{otherwise}, \end{cases}$$

where $\theta = \frac{v-\gamma}{\lambda}$.

Remark 2. Following Owen in Bertrand competition with capacity constraints [17],

$$\pi_1\big(x_1, F(x_1)\big) = k,$$

for some $k > 0$. Therefore,

$$F(\mathrm{x}_1)=\theta-\mathrm{x}_1-\frac{k}{\lambda\mathrm{x}_1}=\frac{\lambda\theta\mathrm{x}_1-\lambda\mathrm{x}_1^2-\mathrm{k}}{\lambda\mathrm{x}_1}.$$

The nominator (so $F(\mathrm{x}_1)$) is positive for

$$0<x_1<ub=\frac{\theta+\lambda^{-1}\sqrt{\Delta}}{2}$$

$\Delta=\theta^2\lambda^2+4k\lambda$.

To make sure that x_1 is far from zero, it is assumed that

$$\mathrm{lb}<x_1,$$

for some $\mathrm{lb}>0$. The density function is

$$f(\mathrm{x}_1)=-1+\frac{k}{\lambda x_1^2}.$$

The minimum value for $f(\mathrm{x}_1)$ is $f(\mathrm{ub})$. To make sure that $f(\mathrm{x}_1)$ is positive (and then $F(\mathrm{x}_1)$ is non-decreasing), it is enough $f(\mathrm{ub})>0$ or $\mathrm{ub}<\sqrt{\frac{k}{\lambda}}$.

Letting

$$F(ub)-F(lb)=1$$

all necessary and sufficient conditions for F being a distribution function are satisfied.

Remark 3. Notice that the payoff function of player 1 is

$$\pi_1=x_1(v-\gamma-\lambda x_1-\lambda x_2).$$

The best response of player 1 is

$$br_1(x_2)=0.5(\theta-x_2)$$

as well as

$$br_2(x_1)=0.5(\theta-x_1)$$

which yields the Nash equilibrium occurs at

$$x_1=x_2=\frac{\theta}{3}.$$

Assume that $\gamma=\mu$. Then,

$$\theta=\frac{v-\mu}{\sigma}\left(\frac{\sigma}{\lambda}\right)=Z\left(\frac{\sigma}{\lambda}\right),$$

where Z has standard normal distribution. Then,

$$x_1=x_2=\frac{Z}{3}\left(\frac{\sigma}{\lambda}\right)=\zeta Z,$$

$$\zeta=\frac{\sigma}{3\lambda}.$$

Then,

$$\pi_1=\pi_2=\frac{\sigma Z^2}{3}\,\zeta=A\,\zeta.$$

Assume that the true value of ζ should be chosen between two values of $\zeta_i, i=1,2$. Following Singh *et al.*, the mixed expected payoffs of the mixed strategy (α,β) of player 1 and 2 are [18]

$$A(\alpha\zeta_1+(1-\alpha)\zeta_2),\ A(\beta\zeta_1+(1-\beta)\zeta_2),$$

respectively. That is, if $\zeta_1<\zeta_2$, then

$$\alpha=\beta=0$$

and both players play ζ_2. It is natural since π_1,π_2 are increasing as a function of ζ. One can see that

$$E(\pi_i)=\frac{\sigma\,\zeta}{3},i=1,2.$$

Assume that $E(\pi_i)=\mu^*$ is predetermined. Therefore,

$$\zeta=\frac{3\mu^*}{\sigma}\text{ and }\lambda=\frac{\sigma^2}{9\mu^*}.$$

In the presence of κ insider traders, then

$$x_i=\frac{Z}{\kappa+1}\left(\frac{\sigma}{\lambda}\right)=\zeta_\kappa Z, i=1,\dots,\kappa$$

and $\mathrm{E}(\pi_1)=\mu_\kappa^*$, then optimal

$$\lambda_\kappa=\frac{\sigma^2}{(\kappa+1)^2\mu^*}.$$

This choice for λ_κ is independent of σ_u and relates to μ^*.

3.2. Learning Schemes

In this section, the learning scheme is studied for mixture equilibrium of Kyle's game. The n-th step of a factitious play choose x_1 such that to maximize the above objective function

$$\int \pi_1(\mathrm{x}_1,\mathrm{x}_2)f_n(\mathrm{x}_2)\mathrm{dx}_2,$$

i.e.,

$$x_{1,n}^*=argmax_{x_1}\int \pi_1(\mathrm{x}_1,\mathrm{x}_2)f_n(\mathrm{x}_2)\mathrm{dx}_2.$$

A sufficient condition to above equation is to assume that

$$\int\frac{\partial\pi_1(\mathrm{x}_1,\mathrm{X}_2)}{\partial\mathrm{x}_1}f_n(\mathrm{x}_2)\mathrm{dx}_2=0.$$

That is, to find $x_{1,n}^*$ such that

$$E_{X_2\sim f_n}\left(\frac{\partial\pi_1(x_{1,n}^*,\mathrm{X}_2)}{\partial\mathrm{x}_1}\right)=0.$$

The expectation is taken assuming X_2 has density function f_n. Following pure Nash equilibrium, it is seen that

$$x_{1,n}^* = \frac{1}{2\lambda}\left\{-\gamma - \lambda\mu_n + \frac{\mu_n - b_2}{a_2}\right\},$$

where μ_n is the expectation of X_2 with respect to density function f_n. Also, assuming X_1 has density function g_n with mean δ_n, then

$$x_{2,n}^* = \frac{1}{2\lambda}\left\{-\gamma - \lambda\delta_n + \frac{\delta_n - b_1}{a_1}\right\}.$$

It is seen that

$$x_{1,n}^* = x_{1,n-1}^* + \left(\frac{1}{a_2} - \frac{1}{2}\right)(\mu_n - \mu_{n-1}),$$

$$x_{2,n}^* = x_{2,n-1}^* + \left(\frac{1}{a_1} - \frac{1}{2}\right)(\delta_n - \delta_{n-1}).$$

These equations define a stochastic approximation framework. For example, assuming

$$\mu_n - \mu_{n-1} = \mu \neq 0, a_2 \neq 2,$$

and then

$$x_{1,n}^* = x_{1,1}^* + n\left(\frac{1}{a_2} - \frac{1}{2}\right)\mu \to \pm\infty \text{ as } n \to \infty.$$

Here, conditions are investigated to make sure that the fictitious play converges to the Nash mixed equilibrium. The above formulas have random walk structure. Suppose that

$$\mu_n - \mu_{n-1} = \chi_n \text{ and } \delta_n - \delta_{n-1} = £_n.$$

Let $z_n^* = x_{1,n}^* - x_{2,n}^*$. Thus,

$$z_n^* = z_{n-1}^* + \left(\frac{1}{a_2} - \frac{1}{2}\right)\chi_n - \left(\frac{1}{a_1} - \frac{1}{2}\right)£_n.$$

Assume that $a_1 = a_2$. Then,

$$z_n^* = z_{n-1}^* + \left(\frac{1}{a_1} - \frac{1}{2}\right)\{\chi_n - £_n\}.$$

Let $z_0^* = 0$. Thus,

$$z_n^* = n\sum_{i=1}^n(\chi_i - £_i) = n\sum_{i=1}^n(\mu_i - \mu_{i-1}) - n\sum_{i=1}^n(\delta_i - \delta_{i-1}) = n(\delta_n + \mu_n).$$

As soon as,

$$\delta_n + \mu_n = O(n^{-\varphi})$$

such that $1 < \varphi$, then $z_n^* \to 0$ and $x_{1,\infty}^* = x_{2,\infty}^*$. For example, suppose that

$$\mu_n = \delta_n = n^{-(1+\varepsilon)}$$

for $0 < \varepsilon < 1$. In the Nash mixed equilibrium both $x_{1,\infty}^*$ and $x_{2,\infty}^*$ are equal. Also, if $\mu_n \to A$ and $\delta_n \to B$, then $z_n^* \to \pm\infty$.

4. Conclusions

In the Kyle's auction framework, in the presence of many insider and noise traders, the quality traded by the insider traded is influenced directly by the standard deviation of volume of quality traded by insider traders. Indeed, when the mentioned standard deviation converges to zero (infinity), then the quality converges to zero (infinity). Also, when the number of insider traders gets large (small), then the quality gets small (large). However, when there are many (small number of) noise traders the volume gets large (small). As well as, in this framework, learning scheme occurs too fast with a considerable rate of convergence to actual pure and mixed Nash equilibrium.

Abbreviations

BPR	Best Partial Response

Author Contributions

Reza Habibi is the sole author. The author read and approved the final manuscript.

Conflicts of Interest

The authors declare no conflicts of interest.

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