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The Existence and Uniqueness of SDE Solutions Diriged by Multiractional Brownian Motion , Based on Several Developed Methods

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Abstract: To overcome some of the limitations of traditional fractional Brownian motion (fBm), multifractional Brownian motion (mBm) was created. The Holder exponent of mBm can vary along the trajectory, unlike fBm, which is advantageous for modeling processes whose regularity varies over time, like internet traffic or photos. This is the main difference between the two processes. Many continuous observations can be modeled by stochastic differential equations governed by mBm, especially in biology and finance. Using a non-stationary multifractional Brownian motion with a time-dependent Hurst parameter as the noise source and a simply linear drift coefficient, we first show the existence and uniqueness of a solution to stochastic differential equations in this article. Next, we examine multifractional Brownian motion (mBm) driven stochastic differential equation models that allow for the simulation of some discontinuity-containing phenomena.

Keywords: Multifractional Brownian Motion, Girsanov's Theorem, Stochastic Differential Brownian, Riemann-Liouville Multifractional Integral, Multifractional Derivative

1. Introduction

Fractional Brownian motion has some limitations for modeling certain phenomena. The need to model these increasingly diverse natural phenomena has recently led to the introduction of stochastic processes generalizing fractional Brownian motion. This is called multifractional Brownian motion. Multifractional Brownian motion $B_{h(t)}(t)$ was introduced by Benassi, Jaffar, Levy Vehel, Peltier, and Roux. The Hurst constant parameter of this Gaussian process, which is an extension of fractional Brownian motion, is defined here as a function h(t) and changes with time. This makes it an excellent choice for simulating a wide range of irregular phenomena, including image processing, Internet traffic, hydrology, economics, and finance.

We demonstrate the existence of solutions to specific SDEs using Girsanov's theorem and a number of established techniques.

$$X_t = X_0 + \int_0^t \alpha(u, X_u)du + B^h(u), \text{ for all t} \in [0, T], \quad (1)$$

The Riemann-Liouville multifractional Brownian motion B^h has a regularity function $h : [0, T] \mapsto [a, b] \subset (0, \frac{1}{2})$. For a well-chosen β and α, we only need $t \mapsto \int_0^t \alpha(u, X_u)du$ to be locally β-Holder continuing to build answers.

2. Notation and Preliminaries

This section contains some ideas that we will utilize to support our findings.

Definition 2.1. Let $h : [0, +\infty) \mapsto (0, 1)$ be the $H\ddot{o}lder$ function with exponent $\beta > 0$. Reduced multifractional Brownian motion with functional parameter h for $t \geq 0$ is represented by the random function $B^h(t)$.

$$B^h = \frac{1}{\Gamma(h(t)+\frac{1}{2})}\left\{\int_{-\infty}^{0}\left[(t-u)^{h(t)-\frac{1}{2}}-(-u)^{h(t)-\frac{1}{2}}\right]dB(u)+\int_{0}^{t}(t-u)^{h(t)-\frac{1}{2}}dB(u)\right\} \tag{2}$$

where mean square integration is employed and B denotes ordinary Brownian motion. A Riemann-Liouville fractional integral is represented by the second term on the right side of equation (2). Similar to fBM with constant Hurst parameter, the multifractional Brownian motion has an integral formulation in terms of the Riemann-Liouville fractional integral with its natural filtration $\mathcal{F} = \{\mathcal{F}_t, t \in [0,T]\}$ that is formally given by

$$B_t^h = \frac{1}{\Gamma(h(t)+\frac{1}{2})}\int_0^t K(t,u)dB(u) \tag{3}$$

with $K(t,u) = (t-u)^{\varphi_t}$ and $\varphi_t = h - \frac{1}{2}$.
We shall call it multifractional calculus as the concept of fractional calculus of variable order is compatible with the concept of multifractional stochastic processes.

Definition 2.2. (Riemann-Liouville multifractional integral). Riemann-Liouville multifractional integral operator of order $\varphi : [c,d] \to [a,b] \subset (0,\infty)$ of a function $f \in L^1([c,d])$ for $0 < c < d$ is defined by

$$(I_{c+}^{\varphi} f) = \frac{1}{\Gamma(\varphi(x))}\int_c^x (x-y)^{\varphi(x)-1} f(y)dy \tag{4}$$

where $\Gamma(.)$ is the definition of the Euler Gamma function.

$$\Gamma(\varphi) = \int_0^{+\infty} t^{\varphi-1}\exp(-t)dt.$$

Next, the space $I_{c+}^{\varphi}L^p([0,T])$ is created using the operator I_{c+}^{φ}, is defined as $L^p([0,T])$'s image.

Definition 2.3. (Multifractional derivative of Riemann-Liouville). The definition of the Riemann-Liouville multifractional derivative of order $\varphi > 0$, $n-1 < \varphi < n$, $n \in \mathbb{N}^*$ is

$$D_{0^+}^{(\varphi)} f(t) = \left(\frac{d}{dt}\right)^n I_{0^+}^{(n-\varphi)} f(t) = \frac{1}{\Gamma(n-\varphi)}\left(\frac{d}{dt}\right)^n \int_0^t (t-u)^{n-\varphi-1} f(s)du,$$

where the derivatives of the function $f(t)$ are completely continuous up to order $(n-1)$.

3. Solutions to Differential Equations Driven by Multifractional Brownian Motion: Existence and Uniqueness

Within this section, we will use multifractional calculus to analyze differential equations generated by multifractional Brownian motion. As a result, we will begin with the definition below.

Definition 3.1. Consider $(B_t)_{t\in[0,T]}$ to be a one-dimensional Brownian motion on a filtered probability space $(\Omega, \mathcal{F}, (\mathcal{F}_t)_{t\in[0,T]}, P)$. $h : [0,T] \mapsto [a,b] \subset (0,1)$ is a C^1 function. The filtration $(\mathcal{F}_t)_{t\in[0,T]}$ was applied to the Riemann Liouville multifractional Brownian motion (RLmBm) $(B_t^h)_{t\in[0,T]}$.

$$B_t^h = \frac{1}{\Gamma(h(t)+\frac{1}{2})}\int_0^t (t-u)^{h(t)=\frac{1}{2}} dB_u, \text{ for all t} \in [0,T] \tag{5}$$

The Gamma function is represented as Γ. The regularity function of B^h is represented by the function h.

The definition of the mBm for $t \in [0,T]$ by

$$\tilde{B}_t^h = c(h(t))\left[\int_{-\infty}^0 (t-u)^{h(t)-\frac{1}{2}} - (-u)^{h(t)-\frac{1}{2}}dB_u + \int_0^t (t-u)^{h(t)-\frac{1}{2}}dB_u\right] = \tilde{B}_t^{(1),h} + \tilde{B}_t^{(2),h}, \tag{6}$$

When a real-valued Brownian motion is represented as $\{B_t\}_{t\in(-\infty,T]}$, and $h : [0,T] \mapsto (0,1)$ is a continuous function. In the aforementioned representation, $\tilde{B}_t^{(1),h}$ is always measurable with respect to the filtration $\tilde{\mathcal{F}}_0$ (produced by the Brownian motion), because the stochastic process only contributes from $-\infty$ to zero. Thus, we can consider $\tilde{B}_t^{(2),h}$ as the only part that contributes to the stochasticity of $\tilde{B}_t^h$ when $t > 0$.

First, we shall demonstrate the multifractional Brownian motion Girsanov theorem, and then use this theorem to create mBm-guided EDS solutions.

Girsanov's Theorem

We examine the disturbance that follows.

$$\begin{aligned}
\tilde{B}_t^h &= B_t^h + \int_0^t \xi_u du \\
&= \frac{1}{\Gamma(h(t)+\frac{1}{2})}\int_0^t (t-u)^{h_t-\frac{1}{2}}dB_u + \int_0^t \xi_u du \\
&= \frac{1}{\Gamma(h(t)+\frac{1}{2})}\int_0^t (t-u)^{h(t)-\frac{1}{2}}d\tilde{B}_u,
\end{aligned}$$

This method $\tilde{B}$ is described as

$$\tilde{B}_t = B_t + \int_0^t D_{0+}^{h+\frac{1}{2}} \left(\int_0^{\cdot} \xi_u du \right) (r) dr.$$

It is $D_{0+}^{h+\frac{1}{2}}(\int_0^{\cdot} \xi_u du)$ that is the multifractional derivative.It is evident from the final equation that is an element of $L^2([0,T])$.The condition is that $\int_0^{\cdot} \xi_u du \in I_{0+}^{h+\frac{1}{2}} L^2([0,T])$.

Theorem 3.1. (The Theorem of Girsanov for mBm): Consider the multifractional Brownian motion $(B_t^h)_{t\in[0,T]}$ with the regularity function $h : [0,T] \to [a,b] \subset (0, \frac{1}{2})$ so that $h \in C^1$. Assume the following two hypotheses are true for $\varepsilon > 0$ and $\frac{1}{2} - h(t) > \varepsilon$,$\forall t \in [0,T]$.

(i) $\int_0 \xi_u du$ is suitable, and $D_{0+}^{h+\frac{1}{2}} \left(\int_0^{\cdot} \xi_u du\right)(t)$. p.s.
(ii) $E[Z_T] = 1$ for

$$Z(T) := \exp\left[-\int_0^T D_{0+}^{h+\frac{1}{2}} \left(\int_0^{\cdot} \xi_u du \right) (r) dB_r - \frac{1}{2} \int_0^T \left(D_{0+}^{h+\frac{1}{2}} \left(\int_0^{\cdot} \xi_u du \right) (r) \right)^2 dr. \right]$$

The stochastic process comes next,

$$\tilde{B}_t^h = B_t^h + \int_0^t \xi_u du,$$

is an mBm under the measure $\tilde{P}$ defined by

$$\frac{d\tilde{P}}{dP} = Z(T).$$

Proof of Theorem 3.1. After deriving $\tilde{B}_t^h$, the proof is obtained by using the conventional Girsanov theorem to $\tilde{B}_t^h = B_t + \int_0^t D_{0+}^{h+\frac{1}{2}}(\int_0^{\cdot} \xi_u du)(r)dr$, $C^{h(.)+\frac{1}{2}+\varepsilon}([0,T];\mathbb{R}) \subseteq I_{0+}^{h+\frac{1}{2}} L^2([0,T])$, demonstrating that $\tilde{B}_t$ is a Brownian motion under $\tilde{P}$.

Existence of Multifractionnal Brownian Motion-driven Solutions to SDEs

First, we shall demonstrate that there is a solution. This indicates that two stochastic processes X and B^h, a mBm on a probability space that has been filtered $(\Omega, \mathcal{F}, (\mathcal{F}_t)_{t\in[0,T]}, P)$, exist such that B^h is a $\mathcal{F}_t - mBm$ in accordance with Definition 3.1 and that X and B^h fulfill the following equation:

$$X_t = X_0 + \int_0^t \alpha(u, X_u) du + B^{(h)}(u), \text{ for all t} \in [0,T] \quad (7)$$

Theorem 3.2. Let B^h be an mBm on a probability space that has been filtered $(\Omega, \mathcal{F}, (\mathcal{F}_t)_{t\in[0,T]}, P)$. Assume that α is integrable and has linear growth, which means that there is a constant $C > 0$ such that for every $(t,x) \in [0,T] \times \mathbb{R}$, $|\alpha(t,x)| \leq C(1 + |x|)$. The preceding equation thus has a solution $(X_t)_{t\in[0,T]}$.

Proof of Theorem 3.2. Let $\xi_u = \alpha(u, B_u^h + x)$, and $\tilde{B}_t^h = B_t^h + \int_0^t \xi_u du$, and

$$k_t = D_{0+}^{h+\frac{1}{2}} \left(\int_0^{\cdot} \xi_u du \right) (t).$$

The process $\int_0^{\cdot} \xi_u du$ must satisfy the requirements (i) and (ii) of Theorem 3.1. First, we will demonstrate (i), which asserts that $k \in L^2([0,T])$ and $\int_0^{\cdot} \xi_u du$ is adapted.$\alpha(u, B_u^h + x)$, is adapted, and $D_{0+}^{h+\frac{1}{2}}$ is a deterministic operator. According to Theorem 2.1 [4.2].

$$|k_t| \leq \| \int_0^{\cdot} \xi_u du \|_{h(.)+\frac{1}{2}+\varepsilon;[0,T]} C(T, \varepsilon, h + \frac{1}{2}; t)$$

where $t \mapsto C(T, \varepsilon, h + \frac{1}{2}; t) \in L^2([0,T])$ since $2h_* < 1$ $\varepsilon > 0$.

$$\| \int_0^{\cdot} \xi_u du \|_{h(.)+\frac{1}{2}+\varepsilon;[0,T]} \leq \tilde{C}(1, |B^h|_\infty + |x|) T^{(\frac{1}{2} - h^* - \varepsilon)}.$$

The theorem of Fernique [4.2] $|B^h|$ has finite exponential moments of all orders, according to (or [4.2, Theorem. 2.7]), suggesting that

$$t \mapsto k_t \in L^2([0,T]) \qquad P - a.s$$

We then look at condition (ii). It is sufficient to demonstrate that Novikov's condition is met [refKaratzas, Cor. 5.13] as we have previously established that k is adapted. That is, it is sufficient to demonstrate that there exists $\lambda > 0$ such that

$$sup_{0\leq u\leq T} E[\exp(\lambda k_u^2)] < \infty.$$

Equation (11) It is evident from the Fernique theorem [refFernique] that the aforementioned requirement is met. A solution is implied by Girsanov Theorem 3.1.

Solutions' Uniqueness

Theorem 3.3. In the associated filtered probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\in[0,T]}, P)$, let (X, B^h) be a solution to the differential equation (7). Legally speaking, the answer is most likely unique.

Proof of Theorem 3.3. Start again with

$$\tilde{\xi}_t = D_{0+}^{h+\frac{1}{2}} \left(\int_0^{\cdot} \alpha(u, X_u du \right) (t),$$

define a new measure of probability. applying the the derivative of Radon and Nikodym to $\tilde{P}$.

$$\frac{d\tilde{P}}{dP} = \exp\left(-\int_0^T \tilde{u}_r dB_r - \frac{1}{2}\int_0^T \tilde{u}_r^2 dr\right)$$

B represents a normal Brownian motion. The derivative of Radon and Nikodym is identical to the last one, but this time the differential equation is solved within the function α. We shall demonstrate that the new process $\tilde{\xi}$ still meets Girsanov Theorem 3.1's conditions (i) and (ii). Since we understand that,

$$|X_t| \leq |x| + |\int_0^t \alpha(u, X_u)du| + |B_t^h| \leq |x| + \int_0^t \tilde{C}(1 + |X_s|)ds + |B_t^h|,$$

Gronwall's inequality is one of our

$$|X.|_{\infty,[0,T]} \leq \tilde{C}\left(|x| + |B_.^h|_\infty + T\right)\exp(\tilde{C}T) \qquad P - a.s$$

Additionally, using the estimates on Theorem 2.1's multifractional derivative [4.2], we can ascertain that for each $\varepsilon > 0$ such that $h_* - \frac{1}{2} > \varepsilon$,

$$|\tilde{\xi}_t| \leq \| \int_0^. \alpha(., u, X_u)du\|_{h(.)+\frac{1}{2}+\varepsilon;[0,T]} C(T, \varepsilon, h + \frac{1}{2}; t),$$

and since

$$\| \int_0^. \alpha(., u, X_u)du\|_{h(.)+\frac{1}{2}+\varepsilon;[0,T]} \leq C(1 + |X.|_{\infty;[0,T]})T^{(\frac{1}{2}-h^*-\varepsilon)},$$

We acquire the approximation

$$|\tilde{\xi}_t| \leq C(1 + |X.|_{\infty;[0,T]})T^{(\frac{1}{2}-h^*-\varepsilon)}C(T, \varepsilon, h + \frac{1}{2}; t),$$

where $t \mapsto C(T, \varepsilon, h + \frac{1}{2}; t) \in L^2([0, T])$.

Using Fernique's theorem [4.2, Theorem. 2.7] and combining the previous estimates, we can determine that Novikov's condition According to the Girsanov theory, the process

$$\tilde{B}_t = \int_0^t \xi_r dr + B_t.$$

is a $\{\mathcal{F}_t\}$-Brownian motion under $\tilde{P}$, from which we may express

$$X_t = x + \int_0^t K_h(t, u)d\tilde{B}_u,$$

where $K_h(t, u) = \frac{1}{\Gamma(h(t)+\frac{1}{2})}(t - u)^{h(t)-\frac{1}{2}}$.Thus, we define $\tilde{B}_t^h = \int_0^t K_h(t, u)d\tilde{B}_u$. $X_t - x$ is a $\mathcal{F}_t$-multifractional Brownian motion with regard to $\tilde{P}$, as we already know. Consequently, we need to make sure that, under probability P, $X_t - x$ and $\tilde{B}_t^h$ have the same distribution. By looking at a measurable functional φ on C ([0, T]), we may show that

$$E_p\left[\varphi(X - x)\right]$$

$$= E_{\tilde{p}}\left[\varphi(X - x)\exp\left(\int_0^T D_{0+}^{h+\frac{1}{2}}\left(\int_0^. \alpha(u, X_u)du\right)(r)dB_r + \frac{1}{2}\int_0^T D_{0+}^{h+\frac{1}{2}}\left(\int_0^. \alpha(u, X_u du\right)^2(r)dr\right)\right]$$

$$= E_{\tilde{p}}\left[\varphi(X - x)\exp\left(\int_0^T D_{0+}^{h+\frac{1}{2}}\left(\int_0^. \alpha(u, X_u)du\right)(r)d\tilde{B}_r + \frac{1}{2}\int_0^T D_{0+}^{h+\frac{1}{2}}\left(\int_0^. \alpha(u, X_u)du\right)^2(r)dr\right)\right]$$

$$= E_p\left[\varphi(B^h)\exp\left(\int_0^T D_{0+}^{h+\frac{1}{2}}\left(\int_0^. \alpha(u, x + B_u^h)du\right)(r)dB_r - \frac{1}{2}\int_0^T D_{0+}^{h+\frac{1}{2}}\left(\int_0^. \alpha(u, x + B_u^h)ds\right)^2(r)dr\right)\right]$$

$$= E_p\left[\varphi\left(\tilde{B}^h\right)\right]$$

.

For the nearly certainly uniqueness, assume there exist two weak solutions. If X^1 and X^2 are in the same probability space $(\Omega, \mathcal{F}, \tilde{P})$, then $\max(X^1, X^2)$ and $\min(X^1, X^2)$ are two solutions that, as previously shown, have the same distribution. What does the halting time mean $\tau_1 = inf\{t \geq 0 | X_t^1 \leq X_t^2\}$. Afterwards, for every $t \in [0, \tau]$, $X_t^1 \geq X_t^2$, $Y_t = \max(X_t^1, X_t^2)$ answers

$$\begin{aligned} Y_t &= X_t^1 \\ &= x + \int_0^t \alpha(X_u^1) du + B_t^h \\ &= x + \int_0^t \alpha(Y_u) du + B_t^h, \qquad t \in [0, \tau_1] \end{aligned}$$

On $[0, \tau]$, Y solves (10). Next, define $\tau_2 = \inf\{t \geq \tau_1 | X_t^1 \geq X_t^2\}$. This implies that $X_t^2 \geq X_t^1$ on $[\tau_1, \tau_2]$. Likewise,

$$\begin{aligned} Y_t &= X_t^2 \\ &= x + \int_0^t \alpha(X_u^2) du + B_t^h \\ &= x + \int_0^{\tau_1} \alpha(X_u^2) du + B_{\tau_1} + \int_{\tau_1}^t \alpha(X_u^2) du + B_t - B_{\tau_1} \\ &= X_{\tau_1}^1 + \int_{\tau_1}^t \alpha(Y_u) du + B_t - B_{\tau_1} \\ &= x + \int_0^t \alpha(Y_u) du + B_t^h, \qquad t \in [\tau_1, \tau_2] \end{aligned}$$

The formula $X_{\tau_1}^1 = X_{\tau_1}^2 = Y_{\tau_1}$ was applied.Therefore, for both $[\tau_1, \tau_2]$ Y solves (7).$[0, \tau_2]$. The foregoing argument may be extended to show that Y is a solution to (10) on [0,T]. A similar argument demonstrates that the solution to (7) is $\min(X^1, X^2)$.The $\max(x, y) = \frac{1}{2}(x + y + |x - y|)$ might be rewritten and $\min(x, y) = \frac{1}{2}(x + y - |x - y|)$, and so

$$E\left[\max(X^1, X^2)\right] = E\left[\min(X^1, X^2)\right] .,$$

suggests that

$$E_{\tilde{P}}\left[|X^1 - X^2|\right] = -E_{\tilde{P}}\left[|X^1 - X^2|\right],$$

However, this applies only if $X^1 = X^2$ a.s.

4. Some Stochastic Differential Equations Involving the MBM Without Jump

We consider a fixed C^1 deterministic funtion $h : \mathbb{R} \to (0, 1)$. We also consider $(B_t)_{t \in \mathbb{R}}$ a standard Brownian motion, and $(B_t^{(h)})_{t \in \mathbb{R}}$ a mutifractional Brownian motion, independent of B, of funtional parameter h. The resolutions of stochastic differential equation models driven by a multifractional Brownian motion without jumps are provided.

4.1. Multifractional Ornstein-Uhlenbeck Model

Examine the differential equation that follows:

$$dX_t = \alpha(a - X_t) dt + \sigma d^{\diamond} B_t^h \text{ and } X_0 = x_0 \in \mathbb{R}, \tag{8}$$

where $d^{\diamond} B_t^h$ designates the differential taken in the sense of the white noise theory, $B^h = \{B^h\}_{t \geq 0}$ where $\alpha > 0$, a, and $\sigma > 0$ are parameters, has a functional parameter h and is a multifractional Brownian motion. The process $(X_t)_{t \in \mathbb{R}}$ with values in $\mathbb{L}^2(\Omega)$ defined by

$$X_t = X_0 e^{-\alpha t} + a\left(1 - e^{-\alpha t}\right) + \sigma \int_0^t e^{\alpha(u-t)} d^{\diamond} B_u^h \tag{9}$$

is the differential equation's unique solution (8).

4.2. Black-Scholes Multifractional Model

The Black-Scholes model is examined below

$$dX_t = \alpha X_t dt + \sigma X_t dB_t \tag{10}$$

where B_t represents typical Brownian motion and α and σ are constants. By replacing B_t by a multifractional Brownian motion $B_t^{h(t)}$, we obtain

$$dX_t = \alpha X_t dt + \sigma X_t dB_t^{h(t)} \tag{11}$$

It is assumed that the Holderian function of the mBm $B_t^{h(t)}$, or $h(t)$, is known and predictable in time (see, for instance, ref. [4.2] for the situation of a sinusoidal function that varies with time). By replacing $x_t = \ln X_t - bt$, the multifractional Ito lemma [14] leads to:

$$dx_t = -\sigma^2 t^{2h(t)-1}\left[h' t \ln t + h(t)\right] dt + \sigma B^{h(t)} \tag{12}$$

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Abbreviations

fBm	Fractional Brownian Motion
mBm	Multifractional Brownian Motion
SDE	Stochastic Differantial Equations
B	Brownian Motion
RLmBm	Riemann Liouville multifractional Brownian Motion

Conflicts of Interest

The authors declare no conflicts of interest.

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