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Research Article

Richness of Arithmetical Structure to Deliver Alternative Equations of Distance in Uniform Acceleration Motion

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Abstract

The study introduces the arithmetical framework to derive classical kinematic equations without using calculus and provides comparable equations for displacement. The proposed method establishes a direct correspondence between kinematic quantities—velocity, acceleration, displacement, and time—and the elements of arithmetic progressions. Conventional calculus-based methods encounter constraints when applied to systems that suffer from accuracy; thus, it indicates that there are alternate mathematical tools and methodologies. A carefully chosen arithmetic-based model is developed, treating time as continuous when studied as a sequence and as discrete when regarded in a series. Further, acceleration is modeled as a deterministic continuous quantity, while velocity and displacement follow deterministic discrete patterns. Through transforming physical correlations directly to arithmetic patterns, this framework provides alternative distance equations. Based on standard summation, a comparative analysis of a constant-acceleration problem shows that the arithmetic model predicts displacement (47 m) with about 9% more accuracy than the classical calculus result (43 m). This confirms that the method is physically valid, and it not only makes calculations easier and reduces mistakes, but it also helps people understand motion under uniform acceleration better and gives more accurate results than the calculus-based model. The results imply that the arithmetic framework offers a unified way to connect discrete and continuous motion and gives classical mechanics a new mathematical base that makes calculations clearer. Moreover, it offers fresh methods for dealing with increasingly simplifying complex physical phenomena and advancing conceptual clarity.

Keywords

Uniform Acceleration Motion, Distance, Velocity, Discrete, Arithmetical Pattern and Calculus

1. Introduction

The arithmetical structure of kinematics under uniform acceleration is notably rich, enabling the derivation and application of multiple, equivalent equations for displacement. These equations, rooted in arithmetic progressions reasoning, offer. The existence of multiple, interconvertible forms allows for tailored solutions depending on which variables are known, enhancing both analytical and computational richness while, while maintaining conceptual clarity and computational efficiency.

The relationship between qualitative physical laws and their mathematical representations is central to understanding the world through the lens of classical mechanics and beyond.

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Qualitative observations often form the basis for developing mathematical models that describe physical phenomena [1]. However, the transition from qualitative insights to quantitative mathematical models is not always straightforward and faces methodological limitations [2]. The use of mathematical representations in physics raises epistemological questions about how we acquire knowledge and understand the nature of reality [3]. Classical mechanics, with its reliance on calculus, provides a precise framework for describing kinematic and dynamic systems [4]. However, calculus has limitations in modeling real-world systems due to its reliance on continuous functions, which may not accurately represent discrete or stochastic phenomena. Arithmetical patterns and discrete mathematics offer greater accuracy in such cases by providing detailed, discrete representations [5]. Arithmetical patterns and discrete mathematics provide a foundation for accurate modeling and problem-solving in diverse fields by offering precise and well-defined representations [6, 7]. For fast calculation, optimization methods and simplification techniques, including data-driven methods and numerical approximation, are used. These limitations necessitate the exploration of alternative mathematical tools and methods to accurately model and understand such systems [8]. Compatibly, the arithmetic pattern is carefully chosen due to its potentiality of discrete and continuous representative frameworks. In this context, acceleration under uniform acceleration motion follows a deterministic continuous model, while velocity and distance adhere to a deterministic discrete model. This work set in the principle that while time retains its continuous character as a cornerstone of classical kinematics when considered equivalent a sequence [9] and exhibit discrete when it equivalently analyzed as arithmetical series.

2. Methodology

This document outlines a method to derive classical kinematic equations for uniform acceleration using the principles of arithmetic sequences and series, bypassing the need for calculus. It will establish clear correspondences between kinematic variables and components of arithmetic progressions. Compared to the arithmetic pattern, the arithmetic pattern is carefully chosen due to its potential for discrete and continuous representative frameworks. In this context, acceleration under uniform acceleration motion follows a deterministic continuous model, while velocity and distance adhere to a deterministic discrete model. This work is set in the principle that while time retains its continuous character as a cornerstone of classical kinematics when considered equivalent to a sequence [9], and exhibits discrete behavior when it is equivalently analyzed as an arithmetical series.

2.1. Transformation of Velocity and Acceleration to Arithmetical Pattern

To derive the transformed terms, by apply the notion of the arithmetic progression of n th term: $A_n = A_1 + (n-1)\ d = \mathrm{A}_1 + \mathrm{n}d - 1d$ [10] is the equation of arithmetic sequence and represent numbers. As illustrated in equation (11) velocity is in a direct correlation to an arithmetic progression as explained further. In a rather similar yet distinct matter, physics in this context defines time, velocity and acceleration relations, and maps them onto the structure of an arithmetic sequence [9].

$$v_n = v_i + (t_n - t_i)a = v_i + at_n - at_i = at_n \tag{1}$$

At point times t_i and t_n the average velocity (v_{av}) is given by the arithmetic mean of the initial (v_i) and final (v_n) velocities. At times t_1 and t_n, the average velocity and acceleration can be expressed as:

$$v_{av} = \frac{1}{2}(v_1 + v_n) \tag{2}$$

and the average acceleration retains its constant value:

$$a = \frac{v_n - v_1}{t_n - t_1} = \frac{v_n - v_i}{t_n - t_i} \tag{3}$$

2.2. Transformed Distance Equations from Arithmetical Pattern

The displacement is given as the product time taken and average velocity [11]. Thus, total time can be: t_{tot}=t_n used to find distance while given initial and final velocities or alternatively total time can be summed in similar arithmetical pattern that is $T_n = \frac{1}{2}t_n(t_1 + t_i)$ during a constant acceleration is used to calculate distance. In this context the following alternative equations are derived [9].

$$s_n = s_0 + \frac{1}{2}(t_1 + t_n)at_n + v_0 t_n \tag{4}$$

$$s_n = \frac{1}{2}(t_1 + t_n)v_n \tag{5}$$

$$s_n = \frac{1}{2}(v_1 + v_n)t_n \tag{6}$$

$$s_n = \frac{1}{2}(v_1 + at_n)t_n \tag{7}$$

$$s_n = \frac{1}{2}(v_1 + at_n)\frac{v_n}{a} \tag{8}$$

$$s_n = \frac{1}{2}(v_1 + v_n)\frac{v_n}{a} \tag{9}$$

3. Result and Discussion

The kinematic equations describe the motion of an object under constant acceleration, allowing you to relate displacement, velocity, acceleration, and time. These equations are fundamental in physics and are widely used in various applications, including mechanics and engineering [12].

3.1. Standard Kinematic Equations

These four equations are commonly used to solve problems involving constant acceleration:

Equations of Motion (for constant acceleration):

$$v = u + at \tag{10}$$

$$s = ut + \frac{1}{2}at^2 \tag{11}$$

$$s = \frac{v^2 - u^2}{2a} \tag{12}$$

(where u = initial velocity, v = final velocity, a = acceleration, s = displacement, t = time) [12, 13].

3.2. Determination of Distance and Velocity Based Existing (Calculus)

Consider Example 2.4 from university physics, fifteenth edition [14]: for comparison of magnitude of distance. A motorcyclist heading east through a small town accelerates at a constant $4.0\ m/s^2$ after he leaves the city limits. At time $t = 0$ he is 5.0 m east of the city-limits signpost while he moves east at $15\ m/s$. (a) Find his position and velocity at t = 2.0 s.

Identify and Set Up: The x-acceleration is constant, so we can use the constant-acceleration equations. We take the signpost as the origin of coordinates ($x = 0$) and choose the positive x-axis to point east. The known variables are the initial position and velocity, $x_0 = 5.0$ m and $v_{0x} = 15\ m/s$, and the acceleration, $a_x = 4.0\ m/s^2$. The unknown target variables in part (a) are the values of the position x and the x-velocity v_x at $t = 2.0\ s$.

EXECUTE (a) Since we know the values of x_0, v_{0x}, and ax, Table 1 tells us that we can find the position x at $t = 2.0\ s$ by using Eq. (17) and the $x-$velocity v_x at this time by using Eq. (15):

$$x = x_o + v_{0_x} t + \frac{1}{2}at^2 = 5.0\ m + \left(15\frac{m}{s}\right)(2.0s) + \frac{1}{2}\left(4\,{}^{m}/{}_{s^2}\right)(2.0s)^2 = 5m + 38\ m = 43m$$

$$v_x = v_{0_x} + a_x t = 15\ m/s + (4.0\ m/s^2)(2.0s) = 23\ m/s$$

3.3. Determination of Distance and Velocity Based New (Arithmetical) Model

The same example is considered for comparison between calculus based and new arithmetical one.

Given: initial position, $x_0 = 5.0$ m, initial velocity, $v_0 = 15$ m/s, acceleration, a = 4.0 m/s²and time, t = 2.0 s

$$v_n = v_1 + (t_n - t_1)a = v_1 + at_n - at_1 = at_n \tag{13}$$

$$v_n + v_0 = v_i + v_0 + (t_n - t_i)a = v_i + v_0 + at_n - at_i = v_0 + at_n \tag{14}$$

and

$$v_1 = at_1 = 4m/s^2\ (1.0\ s) = 4m/s$$

$$v_i = v_1 + v_0 = 4m/s + 15m/s = 19m/s$$

$$v_n + v_0 = v_f = 4m/s^2\ (2.0s) + 15m/s = 8m/s + 15m/s = 23m/s$$

$$s = s_0 + \frac{1}{2}(v_i + v_f)t_n = t_f = 5m + \frac{1}{2}(19m/s + 23m/s)2.0\ s = 5m + 42m = 47m$$

Table 1. *Equations of Constant Acceleration motion conventional versus arithmetical pattern.*

Existing equation of distance		**Arithmetical patterns of distance**	
Equivalent equations when $v_0 \neq 0$		**Equivalent equation when $v_i \neq v_1$ and $v_n \neq v_f$**	
$v_x = v_{0x} + a_x t = 23$	(15)	$s_n = s_0 + \frac{1}{2}(t_1 + t_n)at_n + v_0 t_n = 42 + 5 = 47m$	(16)
$s = x_n + v_{0x}t + \frac{1}{2}at^2 = 43$	(17)	$s = s_0 + \frac{1}{2}(t_1 + t_n)v_n + v_0 t_n = 42 + 5 = 47m$	(18)
$s = x_0 + \frac{v_x^{\,2} - v_0^{\,2}}{2a}$	(19)	$s = s_0 + \frac{1}{2}(v_i + v_f)t_n = 42 + 5 = 47m$	(20)
		$s = s_o + \frac{1}{2}(v_1 + v_0 at_n + v_0)t_n = 42 + 5 = 47m$	(21)
		$s = s_o + \frac{1}{2}(v_1 + v_0 at_n + v_0)\frac{v_n}{a} = 42 + 5 = 47m$	(22)
		$s = s_0 + \frac{1}{2}(v_i + v_f)\frac{v_n}{a} = 42 + 5 = 47m$	(23)

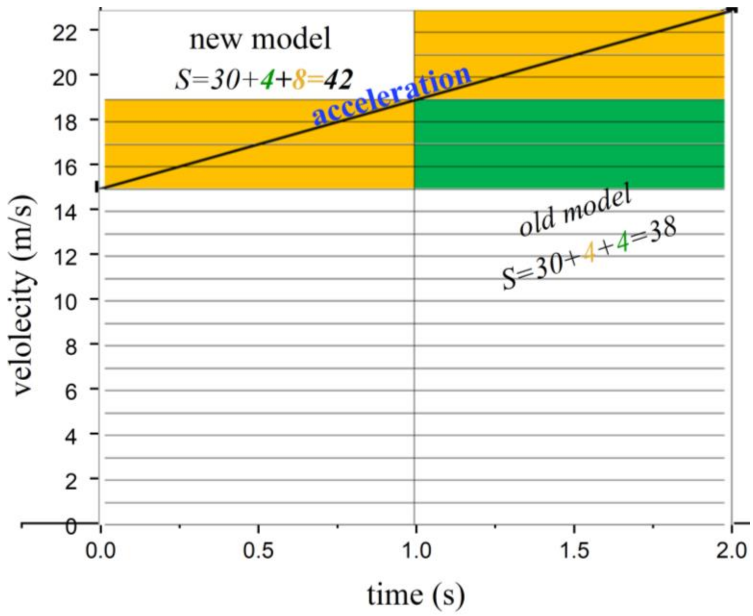

Figure 1. *The v-t graph of the uniform acceleration motion and distance represented by area.*

The results presented in Table 1 compare the equations of motion for constant acceleration using calculus-based equations and a new arithmetical pattern. The key focus is on the determination of distance (S) when *initial velocity* at $t_0 = 0$, $v_0 \neq 0$ and the equivalence or differences when intermediate velocities $v_i \neq v_1 and\ v_n \neq v_f$ are considered. The calculus-based approach (43m, 23m/s) is physically the distance is inaccurate for constant acceleration. On the other hand arithmetical model's position result (47m, 23m/s). The percentage accuracy including the initial distance calculus-based position (43 m) to the arithmetical model position (47m), yielding approximately 91.45%. Without considering the initial distance, the accuracy is the ratio of the displacement from the calculus method (38 m) to that from the arithmetical approach (42 m), resulting in about 90.48%.

When an object moves at a constant velocity, the distance it travels over a given time is simply the product of its speed and the time elapsed. For instance, if something moves steadily at 15 m/s for 2 seconds, it covers 30 m. graphically, plotting velocity against time produces a rectangle where the height is the velocity (15 m/s) and the width is the time (2 s), making the area under the curve represent the total distance traveled. This graphical interpretation extends to situations with changing velocity, such as uniformly accelerated motion. In such cases, the graph may consist of triangles and rectangles. Consider a velocity-time plot where, over two 1-second intervals, speed rises and falls—shown by orange and green shaded regions. In this scenario, calculus helps calculate the area under the diagonal line (acceleration) excluding the area above the diagonal line, only combining full rectangles and half-triangles from the changing velocity sections. This yields a total of 38 meters. $s = 30m + orange\frac{1}{2}\left(\left(\frac{19\text{m}}{\text{s}} - \frac{15\text{m}}{\text{s}}\right)\text{x}1.0\text{s} + \left(\frac{21\text{m}}{\text{s}} - \frac{23\text{m}}{\text{s}}\right)\text{x}1.0\text{s}\right)\frac{4\text{m}}{\text{s}^2} + green\ \left(\frac{19\text{m}}{\text{s}} - \frac{15\text{m}}{\text{s}}\right)1.0sx\frac{4\text{m}}{\text{s}^2} = 38m$. However, in arithmetical pattern if one naïvely sums all the regions—including those above the acceleration line that is impossible by calculus [15] it totals 42 m, elongated the distance. $orange\left(\left(\frac{19\text{m}}{\text{s}} - \frac{15\text{m}}{\text{s}}\right)\text{x}1.0\text{s} + \left(\frac{21\text{m}}{\text{s}} - \frac{23\text{m}}{\text{s}}\right)\text{x}1.0\text{s}\right)\frac{4\text{m}}{\text{s}^2} + green\ \left(\frac{19\text{m}}{\text{s}} - \frac{15\text{m}}{\text{s}}\right)1.0sx\frac{4\text{m}}{\text{s}^2} = 42m$. Comparing the two, the precise calculation is 90.5% of the larger overestimate (38 ÷ 42 × 100 ≈ 90.48%). This supports Teferi's assertion that [9] discrete velocity values—when analyzed in steps—better account for the total motion experienced by an object. Furthermore, the arithmetic method effectively reflects the dual nature of time. When modeled as a continuous variable, it aligns with equations (5, 6, 9, 18 and 23); yet, when expressed as arithmetic series, it allows for accurate summation of incremental changes in uniformly accelerated motion as equated in (4, 8, 16, 20 and 21). This dual representation offers a conceptual bridge between the discrete and continuous, positioning arithmetic sequences and series as a foundation for alternative motion modeling strategies. Since mathematical models are naturally occurring yet discoverable representations of reality, the study was carefully assessed to address the inconsistencies between quantitative and qualitative laws. Accordingly, the comparison in calculation highlights important epistemological implications for physics, as similarly noted in the literature [3, 8].

4. Conclusion

Uniform acceleration in kinematics can be described using both calculus and arithmetic sequence approaches. Calculus-based equations are widely accepted for modeling continuous motion, but they rely on idealized assumptions that may not always capture the full complexity of real-world scenarios. In contrast, the arithmetic method models motion in discrete model for velocity and distance; though treating time as either a sequence (continuous) or a series (discrete), which can provide a clearer and sometimes more accurate calculation of displacement in constant acceleration motion. While both methods yield the same results for velocity, the arithmetic pattern approach often leads to more precise displacement values under a given condition. The arithmetic framework maintains the essential physical meaning of classical kinematics, but also introduces a more transparent and structured way to perform calculations, making it easier to understand and apply. Moreover, experimental tests for the viability of the method and the arithmetic framework, refinement of their theoretical basis, and exploration of their implications for classical kinematics are among the future directions. Therefore, I encourage physicists to investigate this arithmetic framework further as a potential bridge between qualitative classical kinematics and its quantitative framework.

Abbreviations

t_n	Last Term of Time
s_n	Distance at Time t_n
v_1	Velocity at t_1
v_n	Velocity at t_n

Author Contributions

Teferi Mekit Dillo is the sole author. The author read and approved the final manuscript.

Conflicts of Interest

The author declares that there are no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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