

Research Article

Mathematical Modeling of the Caspian Sea Geometry Under Water Level Decline: Evidence from Iran's Mazandaran Coast

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Abstract

The Caspian Sea is Earth's largest endorheic water body and a natural laboratory for studying coupled hydroclimatic forcing and coastal geomorphic response. Over the last century, the basin has undergone multi-decadal oscillations and an accelerated decline since the mid-1990s, with strong signals projected to continue through the twenty-first century. This meta-analysis synthesizes peer-reviewed evidence on horizontal shoreline migration, vertical (level) change, and areal transformation, and evaluates how these changes reconfigure the basin's large-scale geometry—well approximated locally by parabolic or saddle-shaped (hyperbolic-paraboloid) surfaces—while cascading into socioeconomic risk. We combine quantitative shoreline metrics (Digital Shoreline Analysis System, DSAS), spectral water delineation (NDWI/MNDWI), sediment-transport theory (Exner equation), and water-balance diagnostics to: (i) characterize recent and projected Caspian water-level trajectories; (ii) resolve planform curvature and alongshore variability along Iran's Mazandaran coast; (iii) contrast Caspian responses to those observed in China's coastal systems (South China Sea littoral and the Yangtze River delta); and (iv) assess risk pathways for agriculture, shipping, fisheries, and wetlands. Results from the literature indicate a long-term negative water balance dominated by increased evaporation relative to precipitation and inflows, superimposed on high interannual variability; the Volga—regulated by reservoirs including Volgograd—remains the principal control on riverine supply. Shoreline retreat and shallow-water expansion are already disrupting Mazandaran's port operations (e.g., Amir-Abad), accelerating maintenance dredging needs, and exposing wetlands such as Gorgan Bay to desiccation and dust-storm hazards. Comparative analysis shows that while China's open-coast margins are influenced by marine processes, the Caspian's closed-basin geometry transmits water-level anomalies more uniformly, amplifying parabolic/saddle-like morphodynamic adjustments. We conclude with actionable adaptation options for Mazandaran (channel realignment, dynamic zoning, nature-based buffers, and flexible port layouts) aligned with realistic twenty-first-century water-level scenarios.

Keywords

Caspian Sea, Mazandaran Coast, Shoreline Change, DSAS, NDWI, Exner Equation, Volga Discharge, Water Balance

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1. Introduction

The Caspian Sea (CS) exhibits pronounced multi-decadal level swings driven by the closed basin's water balance—river inflow plus precipitation minus evaporation and minor losses to Kara-Bogaz-Gol—modulated by climate variability and anthropogenic regulation [1, 4, 5]. Since the mid-1990s, observations reveal a persistent decline linked to warming-driven evaporation increases that outpace precipitation gains over the catchment; models project further losses through 2100 under multiple forcing scenarios [2]. Recent work emphasizes that even moderate declines of 5–10 m could critically disrupt ecosystems, protected areas, and coastal infrastructure [3]. Superimposed on these vertical changes are horizontal shoreline adjustments: delta progradation/retreat, beach profile translation, and lagoonal cut-off or desiccation, which together reshape the basin's planform geometry [7, 8].

The southern CS littoral (Mazandaran Province, Iran) is an acute hotspot where small vertical changes drive outsized horizontal responses due to low gradients, broad shallow shelves, and engineered navigation channels. In recent years, ports (e.g., Amir-Abad) report sedimentation and draft limitations tied to falling levels and altered littoral transport, requiring frequent dredging and adaptive operations [7]. Neighboring wetlands (e.g., Gorgan Bay/Miankaleh) display sensitivity to level fluctuations, with evidence of circulation weakening, habitat loss, and dust-source emergence during low stands [8]. As the Volga River contributes the majority of CS inflow and is strongly regulated by a cascade including the Volgograd Reservoir, changes in upstream hydrology propagate into the basin's water balance and shoreline evolution [6].

Understanding the geometric transformation of the CS therefore needs an integrated lens: (i) a vertical balance framed by a diagnostic equation for water storage; (ii) horizontal shoreline metrics derived from consistent, multi-decadal satellite records (e.g., Landsat, Sentinel) using robust spectral indices; (iii) areal change tracked with DSAS transect statistics; and (iv) morphodynamic process models constrained by sediment-continuity (Exner equation). This paper performs a meta-analysis of these components with explicit focus on Mazandaran's coastal band while drawing comparative insights from two instructive Chinese systems: the Yangtze River (Changjiang)—a river-dominated delta that has undergone post-dam sediment starvation and delta front erosion—and China's open coasts bordering the South China Sea and adjacent shelves, where shoreline migration is heavily modulated by marine forcing and intensive reclamation [9-13]. Contrasting these contexts clarifies how the CS's endorheic setting and basin-scale geometry produce distinctive “parabolic” or “saddle-like” planform responses and risk profiles not easily extrapolated from open-ocean coasts.

Beyond scientific synthesis, we target practical implications: Mazandaran's irrigated agriculture (notably rice), coastal fisheries (including sturgeon supply chains dependent

on nearshore habitats), and commercial navigation all face risk from regression (shoreline retreat lakeward) and shoaling. The combination of falling water levels, steeper nearshore gradients, and shifting wave climates can increase the frequency of channel closures, expand mudflats, and degrade nursery grounds, thereby elevating costs and ecological losses [3, 7, 8, 12]. We close by distilling adaptation pathways grounded in the best-available projections and process mechanics.

2. Methodology

2.1. Water-Balance and Vertical Change

We frame CS vertical change by the storage (volume) balance:

$$\frac{dV}{dt} = Q_R + P A - E A - Q_K - Q_G,$$

$$\frac{d\mu}{dt} = \frac{Q_R + P A - E A - Q_K - Q_G}{\frac{dV}{d\mu}}$$

2.2. Shoreline Extraction and Horizontal Change

For water-land boundary extraction, we also apply Otsu's (1979) thresholding method to enhance spectral separation [18].

We apply a standard spectral workflow suitable for Landsat/Sentinel time series: compute NDWI and MNDWI to delineate water–land boundaries,

$$NDWI = \frac{G - NIR}{G + NIR}, \quad MNDWI = \frac{G - SWIR}{G + SWIR}$$

$$\Delta x \approx \frac{\Delta \mu}{\beta}$$

2.3. Areal Change and Basin-scale Geometry

Areal response is approximated from published hypsometry and thermohaline budgets [4, 5]. Locally, we approximate nearshore bathymetry as a quadratic surface; at broader scales, sections of the CS planform are well represented by parabolic or hyperbolic-paraboloid (saddle) patches:

$$z(x,y) = a x^2 + b y^2 + c,$$

$$z(x,y) = a x^2 - b y^2 + c,$$

These forms capture curvature sign changes between the eastern (steeper) and western (gentler) shelves; they are a convenient basis for translating vertical $\Delta\mu$ into lateral inun-

ation/emersions across Mazandaran's wide, shallow platform.

2.4. Sediment-Continuity (Morphodynamics) and Littoral Transport

For morphodynamic adjustments, we adopt the Exner equation for sediment mass conservation:

$$(1 - \alpha_p) \frac{\partial \alpha_s}{\partial t} + \nabla \cdot q_s = 0$$

2.5. Meta-analysis Scope and Screening

We synthesize peer-reviewed studies with (i) reported shoreline/level time series; (ii) explicit Caspian or Mazandaran focus; and/or (iii) transferable coastal-morphodynamics insight from the Yangtze/China coasts. We rely on DSAS documentation for metric definitions [14, 15]; remote-sensing indices and thresholding fundamentals [17-19]; and hydrologic diagnostics for the CS balance [1-6]. Case evidence for Mazandaran ports and wetlands comes from recent imaging/engineering and Ramsar-focused studies [7, 8]. All sources cited in text carry DOIs and appear in the reference list.

3. Results & Discussion

3.1. Vertical (Level) Change: Observations and Projections

Instrumental records and gravimetry indicate substantial twentieth-century CS oscillations followed by a post-1995 decline [1]. Multi-model projections show evaporation increases outpacing precipitation, yielding a persistently negative basin-integrated P - E and continued level fall through 2100 across emissions pathways [2]. Ecosystem risk assessments conclude that even 5–10 m of decline would drastically reduce marine protected area coverage, displace biota (e.g., Caspian seals), and strand infrastructure [3]. Thermohaline analyses also describe feedbacks between surface area, evaporation, and circulation that modulate budgets as the lake contracts [4, 5]. In a water-balance view (Section 2.1), these combine to maintain $\frac{d\mu}{dt} < 0$ unless Volga discharge compensates, which is unlikely given climatic and regulatory trajectories [6].

3.2. Horizontal Change Along Mazandaran

Because Mazandaran shelves have small, shoreline position is extremely sensitive to $\Delta\mu$ (Section 2.2). Empirical DSAS studies from the southern CS report multi-decadal land–water boundary migration on the order of tens to hundreds of meters, synchronized with level oscillations [14, 15]. Engineer-

ing-scale imaging of Amir-Abad Port documents shoaling and morphological instability that complicate berth accessibility—a characteristic response under falling water levels when nearshore bars migrate shoreward and channels intersect oblique littoral drift [7]. To the east, Gorgan Bay/Miankaleh exhibits circulation weakening, nutrient shifts, and exposure to dust-source formation during low stands; modeling suggests inlet deepening/realignment could partly restore exchange under ongoing decline [8]. These observations are consistent with Exner-governed adjustments where gradients in alongshore transport ($\nabla \cdot q_s$) reorganize bars/spits while base-level fall lowers accommodation space [16].

3.3. Areal Transformation and Basin Geometry

Areal contraction $A(\mu)$ accelerates as falls across gently sloping shelves, producing large areal loss per unit level in Mazandaran relative to steeper margins. The parabolic vs saddle idealizations (Section 2.3) help interpret this: regions approximated by an elliptic paraboloid (both principal curvatures positive) withdraw more uniformly, whereas saddle regions (opposing curvatures) can localize exposure along one axis while retaining submergence along the other. This geometry aligns with observed anisotropy between Iran's wide southwestern shallows and steeper eastern flanks [4, 5]. Practical implication: hazard footprints elongate along the gentle curvature direction, magnifying agricultural land exposure and expanding intertidal mudflats that hinder small-craft navigation and fishing.

3.4. Volga Regulation (Volgograd) and Catchment Forcing

The Volga River supplies most CS inflow; its cascade of reservoirs (including Volgograd) modifies discharge timing and sediment load [6]. While the Caspian's level is climatically constrained via evaporation, sustained reductions or seasonally altered hydrographs from the Volga further depress and suppress deltaic sediment delivery, which would otherwise partially offset shoreline retreat locally. Under the storage balance (Section 2.1), a plausible long-term trajectory is a continued negative $\frac{d\mu}{dt}$ with amplified seasonality, complicating port draught management along Mazandaran.

3.5. Comparative Lens: Yangtze River Delta and China's Open Coasts

The Yangtze delta presents a cautionary analogue of human-altered sediment regimes: post-Three Gorges Dam, studies show subaqueous delta erosion, coarsening, and widespread riverbed incision extending hundreds of kilometers downstream [10, 11, 14, 15]. DSAS-type shoreline analyses and bathymetric differencing document net retreat/redistribution in the estuary under reduced sediment

supply and deepened navigation channels [12, 15]. China's open coasts, particularly around the South China Sea and adjacent shelves, have also experienced large-scale shoreline reconfiguration due to reclamation and harbor construction; national datasets quantify 1990–2019 coastline gains/losses with strong spatial heterogeneity [13]. Contrast with the CS: whereas Chinese coasts are open-ocean, wave-tide-dominated with mixed sediment sources and marine base level, the CS's endorheic geometry and uniform base-level fall transmit vertical change more coherently across shelves. Consequently, Mazandaran's response is dominated by base-level control (vertical) mapping into large horizontal retreat via small β , while Yangtze dynamics hinge on sediment-supply deficits and engineered channel deepening—a different, though instructive, pathway [9-13, 16].

3.6. Risk Pathways for Mazandaran

Agriculture. Regression increases the likelihood of saline intrusion in shallow aquifers, exposure of previously submerged saline soils, and dust emission from emergent flats, all of which threaten rice and orchard systems along the coastal plain [8]. Irrigation intakes may require relocation or conveyance upgrades as channels elongate across the retreating shoreline [2, 3, 8].

Shipping and ports. Falling levels shrink navigational windows and raise dredging demand to maintain design depths at entrance channels and basins. Amir-Abad's imaging and operational experience are consistent with this trajectory; flexible berth strategies and sediment-bypassing at channel mouths are increasingly necessary [7]. Under projected declines, design freeboards and quay elevations face obsolescence, and some assets risk stranding [3].

Fisheries and habitats. Nearshore nursery grounds contract or fragment; wetland exchanges weaken, reducing habitat quality. Sturgeon and other species tied to specific salinity/temperature envelopes and access corridors are at risk under even moderate declines [3, 8]. Management must anticipate changing seasonality of nearshore temperatures and ice regimes (north) that modulate habitat quality [4, 5].

4. Conclusions

The endorheic Caspian Sea is on a persistent downward level trajectory driven by a negative hydroclimatic balance, with the Volga's regulated inflow unable to compensate for increasing evaporation under warming. In Mazandaran, small vertical changes map into large horizontal shoreline retreat and areal loss due to gentle shelf slopes, reshaping the coast in patterns well described by parabolic/saddle geometries. Morphodynamically, Exner-governed littoral transport and bar/channel interplay, superimposed on base-level fall, explain recurrent shoaling at port inlets and estuarine closures. Comparative insights from China's coasts and the Yangtze delta underscore that, while marine-dominated systems re-

spond strongly to wave/tide and sediment-supply engineering, the Caspian's response is uniquely dominated by basin-wide base-level control. Without adaptation, Mazandaran faces rising costs for dredging and access, increased agricultural and wetland stress, and declining fisheries productivity. Evidence-based, flexible strategies can reduce impact envelopes but require action aligned with realistic level-decline scenarios [1-3, 6-9, 12-16, 19, 20].

5. Recommendations

- 1) Dynamic coastal zoning: Shift setback lines and land-use categories using DSAS-derived retreat rates (LRR, EPR) under scenario-dependent $\Delta\mu$ (e.g., -5 m, -10 m), prioritizing Mazandaran's low-slope sectors [2, 3, 14, 15].
- 2) Flexible navigation design: Adopt adaptive channel templates (movable alignment, sacrificial deposition basins) and sediment bypassing to limit shoal recurrence at Amir-Abad; integrate berth design for larger tidal/wave excursions as depth margins shrink [7, 16].
- 3) Wetland & inlet restoration: Dredge/realign Gorgan Bay inlets to sustain exchange and water quality under lower base levels; deploy nature-based breakwaters and oyster/artificial-reef units where appropriate [8].
- 4) Agricultural buffers & groundwater management: Establish salinity-resilient buffer zones, adjust irrigation intakes and drainage, and monitor coastal aquifers for salinity rise to protect rice cultivation [2, 3, 8].
- 5) Monitoring and decision support: Institutionalize shoreline mapping (NDWI/MNDWI + Otsu), DSAS transects, and bathymetric surveys; couple with routinely updated water-balance diagnostics to trigger pre-planned interventions [14, 17-19].
- 6) Transboundary coordination: Engage Volga-basin stakeholders on flow timing and ecological releases where feasible; align regional early-warning and evacuation/asset-relocation protocols with shared projections [2, 3, 6].

Abbreviations

CS	Caspian Sea
DSAS	Digital Shoreline Analysis System
NDWI	Normalized Difference Water Index
MNDWI	Modified Normalized Difference Water Index
EPR	End Point Rate
LRR	Linear Regression Rate

Author Contributions

Majid Ghorbani is the sole author. The author read and approved the final manuscript.

Conflicts of Interest

The author declares no conflicts of interest.

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