International Journal of Statistical Distributions and Applications
2025, Vol. 10, No. 1, pp. 11-19
https://doi.org/10.11648/j.ijsda.20251001.12

Research Article

A New Exponentiated Siya Distribution and Its Biomedical Application

Shiny Chulliparambil Raj*, Mani Vijayakumar

Department of Statistics, Annamalai University, Annamalai Nagar, Tamil Nadu, India

Abstract

This article introduces a new exponentiated distribution called the Siya Distribution by incorporating a new parameter into the existing two-parameter Gamma distribution. It is a versatile three-parameter model designed to capture various data behaviors encountered in biological and environmental studies. Siya distribution accounts for data that exhibit variable degrees of skewness and kurtosis, making it suitable for complex datasets such as medical measurements, reliability analysis, and survival times. We derive fundamental properties including the probability density function (PDF), moments, the cumulative distribution function (CDF), and moment generating function (MGF), along with the hazard function to allow comprehensive analytical exploration of the distribution's behavior. Parameter estimation is conducted using Maximum Likelihood Estimation (MLE), providing robust estimators for real-world applications. Model performance is evaluated using two real datasets against three alternative existing parameter distributions using the Akaike Information Criterion (AIC), Corrected AIC (AICc), and Bayesian Information Criterion (BIC), demonstrating that the Siya Distribution consistently achieves a superior fit, especially with highly skewed data. Empirical applications to biological and medical data illustrate the model's adaptability and potential to improve data representation in fields requiring precise distribution modelling. The Exponentiated Siya distribution thus offers a significant tool for advanced statistical analyses in applied sciences, supporting more accurate and nuanced interpretation of complex data trends.

Keywords

Exponentiated, General Gamma Distribution, Incomplete Gamma, Poly Gamma, Di-Gamma Distributions, Maximum Likelihood Estimation

1. Introduction

It is crucial to understand lifetime data by applying a suitable probability model in order to comprehend the nature of the data obtained from various real-life sectors. The parameters in a probability model may not significantly enhance the fit of the data. However, models with more parameters are typically favoured to provide greater flexibility when applying a probability model to data. One way to add more parameters to pre-existing or classical models is the exponentiation technique. Investigators have demonstrated that exponentiated models are highly applicable in real-life situations and often fit real-life data better than other existing models. For instance, when analysing the lifespan of mechanical components, an exponentiated model can offer a more precise representation of data by accommodating variations that simpler models

*Corresponding author: shinycraj@gmail.com (Shiny Chulliparambil Raj)

Received: 12 February 2025; **Accepted:** 27 February 2025; **Published:** 13 March 2025

might miss. This enhanced flexibility allows for a more precise estimation of reliability and failure rates. Additionally, in medical research, exponentiated models can better capture the distribution of survival times among patients, leading to improved understanding and prediction of treatment outcomes.

A novel two-parameter Exponentiated distribution with its effectiveness in biometric applications [2, 6] and a New Exponential Gamma Distributions showing its superiority in real-world data modelling [3] was demonstrated. In addition to these, studies have revealed various distributions like an Exponentiated Power distribution and Exponentiated Lomax Geometric distribution focusing on its statistical properties and estimation methods [8, 10], Exponentiated Gamma distribution comparing different estimation methods [12], a three-parameter Lindely distribution enhancing the flexibility of the standard Lindely model [7], and exponentiated transmuted modified Weibul distribution integrating transmutation techniques for improved tail behavior [9]. The weighted Power Shanker distribution demonstrated its applicability managing in real lifetime datasets [11]. Weibull -Burr Bivariate model extending the Weibull framework for bivariate applications, and Rayleigh and Lindley model improved the dependency modelling [4, 5]. Failure time data application is explained in the Additive Dhillon -Chen distribution [1].

The above standard distributions have been widely used due to their interpretability. But they often exhibit limitations when modelling complex data structures such as skewed or heavy-tailed distributions. To address these limitations we introduce additional parameters to enhance flexibility and improve goodness of fit. In this investigation, we extend the existing 2-parameter Gamma distribution with an additional scale parameter to create a new distribution. In contrast to other distributions, we anticipate that our new distribution will yield better consequences and be more dependable and adaptable. The density along with hazard rate function shapes are among the several statistical characteristics of the suggested distributions that are produced.

2. Exponentiated SIYA Distribution

With 2 parameters, α as well as β, a random variable X has been claimed to have a Gamma distribution. The scale parameter is α, and the shape parameter is β. If its PDF is provided by

$$f(x;\alpha,\beta)=\frac{\beta^{\alpha}\ x^{\alpha-1}e^{-(\beta x)}}{\Gamma(\alpha)}\quad 0\le x\le\infty;\ \alpha,\beta\ge 0 \tag{1}$$

Its CDF is an incomplete Gamma function.

Exponentiated Siya distribution's pdf is

$$f(x;\alpha,\theta,\beta)=\frac{\beta x^{\alpha-1}e^{-(x/\theta)^{B}}}{\theta^{\alpha}\Gamma\left(\alpha/\beta\right)}\quad ;0\le x\le\infty;\ \alpha,\beta,\theta\ \ \ge 0 \tag{2}$$

Its cdf is provided by

$$F(x;\alpha,\theta,\beta)=\int_0^x f(x;\ \alpha,\theta,\beta)\ dx=\int_0^x \frac{\beta x^{\alpha-1}e^{-(x/\theta)^{B}}}{\theta^{\alpha}\Gamma\left(\alpha/\beta\right)}dx$$

Put $u=(x/\theta)^{B}$ $x=\theta u^{1/\beta}$

$$F(x;\alpha,\theta,\beta)=\frac{1}{\Gamma\left(\alpha/\beta\right)}\int_0^{x/\theta} e^{-u}\,u^{\frac{\alpha}{\beta}-1}du$$

$$F(x;\alpha,\theta,\beta)=\frac{r\left[\frac{\alpha}{\beta},\left(\frac{x}{\theta}\right)^{B}\right]}{\Gamma\left(\alpha/\beta\right)} \tag{3}$$

Where $r\left[\frac{\alpha}{\beta},\left(\frac{x}{\theta}\right)^{\beta}\right]$ is incomplete gamma function.

The curve of the distribution are as Figures 1 and 2 respectively:-

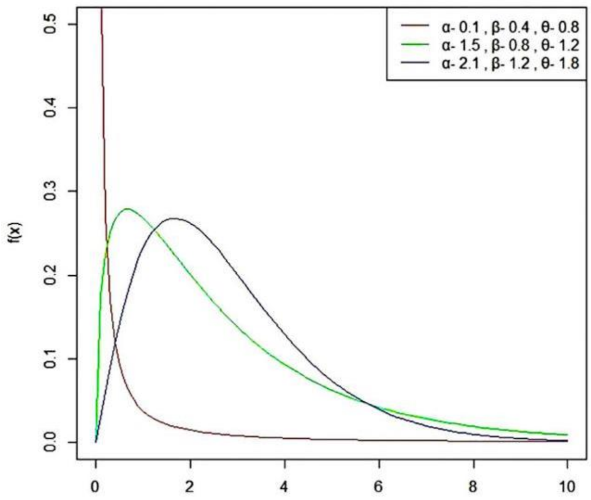

Figure 1. *pdf.*

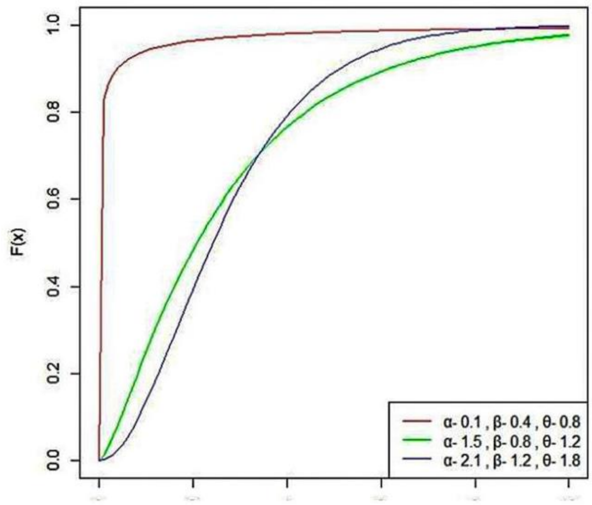

Figure 2. *cdf.*

3. Statistical Properties

The various structural characteristics of the 3 parameter Exponentiated Siya distribution are covered in this section.

3.1. Moments of the Distribution

Let a random variable be X subsequent to an Exponentiated Siya distribution with three parameters α, β & θ.

The mean and variance of the given distribution is

$$\text{Mean} = E(x) = \mu_1' = \int_0^\infty x \; f(x; \alpha,\theta,\beta)dx$$

$$E(x) = \int_0^\infty x \; \frac{\beta x^{\alpha-1} e^{-(x/\theta)^B}}{\theta^\alpha \Gamma(\alpha/\beta)} dx = \theta \; \frac{\Gamma[(\alpha+1)/\beta]}{\Gamma(\alpha/\beta)} = \frac{\theta(\alpha+1)}{\beta} \quad (4)$$

$$E(x^2) = \mu_2' = \int_0^\infty x^2 \; \frac{\beta x^{\alpha-1} e^{-(x/\theta)^B}}{\theta^\alpha \Gamma(\alpha/\beta)} dx$$

$$= \theta^2 \; \frac{\Gamma[(\alpha+2)/\beta]}{\Gamma(\alpha/\beta)} = \frac{\theta^2(\alpha+2)(\alpha+1)}{\beta^2}$$

$$\text{Variance} = V(x) = \mu_2 = E(x^2) - [E(x)]^2$$

$$= \frac{\theta^{2(\alpha+2)(\alpha+1)}}{\beta^2} - \left(\frac{\theta(\alpha+1)}{\beta}\right)^2$$

$$V(x) = \frac{\theta^{2(\alpha+1)}}{\beta^2} \quad (5)$$

3.2. r^{th} Order Raw Moments

The r^{th} order raw moments of the given distribution $E(x^r)$ is

$$E(x^r) = \mu_r' = \int_0^\infty x^r f(x,\alpha,\beta,\theta) \; dx$$

$$= \int_0^\infty x^r \; \frac{\beta x^{\alpha-1} e^{-(x/\theta)^B}}{\theta^\alpha \Gamma(\alpha/\beta)}$$

$$= \frac{1}{\theta^\alpha \Gamma(\alpha/\beta)} \int_0^\infty \beta x^{r+\alpha+1} \; e - (x/\theta)^B dx$$

$$E(x^r) = \frac{\theta^r}{\Gamma(\alpha/\beta)} \; \Gamma\left[\frac{\alpha+r}{\beta}\right] \quad (6)$$

3.3. Moment Generating Function and Characteristic Function

Take into consideration, a random variable subsequent to Exponentiated Siya Distribution with three parameters α, β, θ is X.

MGF of new distribution is

$$Mx(t) = E[e^{tx}]$$

$$= \int_0^\infty e^{tx} \; f(x; \alpha, \theta, \beta)dx$$

$$= \int_0^\infty e^{tx} \; \frac{\beta x^{\alpha-1} e^{-(x/\theta)^B}}{\theta^\alpha \Gamma(\alpha/\beta)} dx$$

Using Taylor's series

$$Mx(t) = \int_0^\infty (1 + tx + \frac{(tx)^2}{2!} + \frac{(tx)^3}{3!} + \cdots \ldots \ldots \ldots \ldots) f(x; \; \alpha, \theta \; ,\beta) \quad dx$$

$$= \int_0^\infty \sum_{r=0}^\infty (tx)^r / r! \; f(x; \; \alpha, \theta \; ,\beta) \quad dx$$

$$= \sum_{j=0}^\infty \frac{t^r}{r!} \int_0^\infty x^r \; f(x; \; \alpha, \theta \; ,\beta) \quad dx$$

$$= \sum_{j=0}^\infty \frac{t^r}{r!} \; \mu_r'$$

$$Mx(t) = \sum_{j=0}^\infty \frac{t^r}{r!} \frac{\theta^r}{\Gamma(\alpha/\beta)} \; \Gamma\left[\frac{\alpha+r}{\beta}\right] \quad (7)$$

In a similar way, its characteristic function can be found as

$$\emptyset_x(t) = Mx(it)$$

$$\emptyset_x(t) = \sum_{j=0}^\infty \frac{(it)^r}{r!} \frac{\theta^r}{\Gamma(\alpha/\beta)} \; \Gamma\left[\frac{\alpha+r}{\beta}\right]$$

Special case:

If β=1 the Exponentiated Siya distribution reduces to general Gamma distribution and its pdf becomes

$$f(x; \alpha, \theta \; ,\beta) = \frac{x^{\alpha-1} e^{-(\frac{x}{\theta})}}{\theta^\alpha \Gamma(\alpha)}$$

Then MGF is provided by

$$Mx(t) = \int_0^\infty e^{tx} \frac{x^{\alpha-1} e^{-(\frac{x}{\theta})}}{\theta^\alpha \Gamma(\alpha)} dx = \frac{1}{\theta^\alpha \Gamma(\alpha)} \int_0^\infty x^{\alpha-1} e - (\frac{x}{\theta} - tx) dx$$

$$Mx(t) = (1 - \theta t)^{-\alpha}$$

4. Reliability Analysis

This section provides Exponentiated Siya Distribution's reliability function, hazard rate function, along with reverse hazard rate function.

4.1. Reliability Function

The reliability function frequently called as survivor or survival function, is that a system will last beyond a given period of time. As well as it is provided by

$$R(x) = 1 - F(x;\, \alpha,\theta,\beta) = 1 - \frac{\gamma\left[\frac{\alpha}{\beta}, \left(\frac{x}{\theta}\right)^{2}\right]}{\Gamma\left(\alpha/_{\beta}\right)}$$

$$R(x) = \frac{\Gamma\left(\alpha/_{\beta}\right) - \gamma\left[\frac{\alpha}{\beta}, \left(\frac{x}{\theta}\right)^{B}\right]}{\Gamma\left(\alpha/_{\beta}\right)} \quad (8)$$

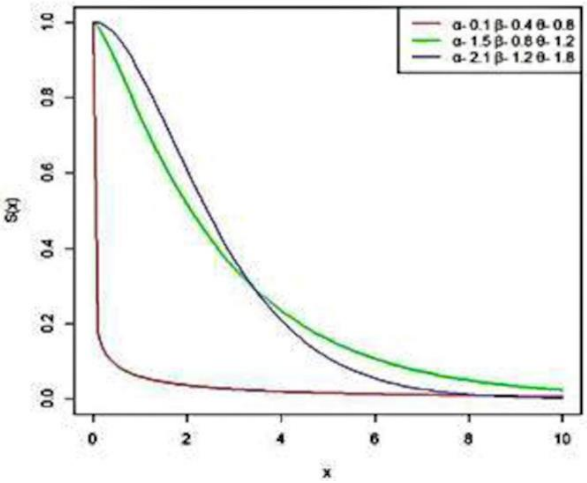

Figure 3. Plot of the Survival function.

4.2. Hazard Function

Hazard function, frequently termed as hazard rate or failure rate or else force of morality, is provided by

$$h(x) = \frac{f\ (x;\ \alpha,\theta,\beta)}{R(x)} = \frac{1}{\theta^{\alpha}}\left[\frac{\beta x^{\alpha-1} e^{-(x/\theta)^{B}}}{\Gamma\left(\alpha/_{\beta}\right) - \gamma\left[\frac{\alpha}{\beta}, \left(\frac{x}{\theta}\right)^{B}\right]}\right] \quad (9)$$

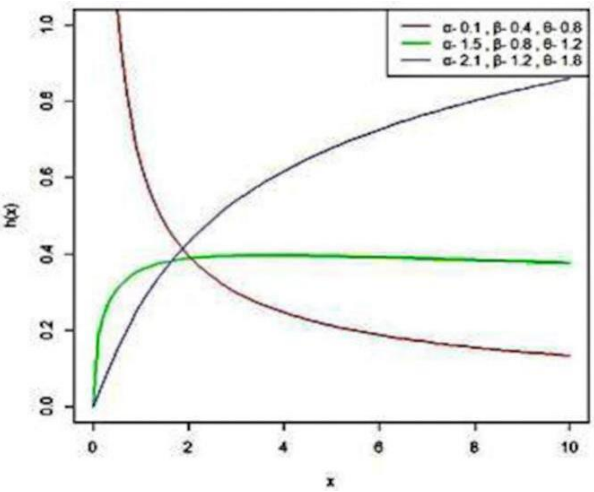

Figure 4. Hazard Function.

4.3. Reverse Hazard Function

The reverse hazard function of the Siya distribution has been provided by

$$h^{r}(x) = \frac{f\ (x;\ \alpha,\theta,\beta)}{F(x;\ \alpha,\theta,\beta)} = \frac{1}{\theta^{\alpha}}\left[\frac{\beta x^{\alpha-1} e^{-(x/\theta)^{B}}}{\gamma\left[\frac{\alpha}{\beta}, \left(\frac{x}{\theta}\right)^{B}\right]}\right]$$

5. Order Statistics

There are numerous applications for order statistics in the fields of life testing and reliability. Additionally, order statistics perform a significant role in many facets of statistical inference.

Consider, $X_{(1)}, X_{(2)}, \dots\dots\dots X_{(n)})$ be order statistics of a random sample $X_1, X_2, \dots\dots\dots. X_n$ selected from a continuous population with the probability density function $f_x(x)$ as well as cumulative density function with $F_x(x)$, pdf of r^{th} order statistic $X_{(r)}$ can be expressed as

$$f_{x(r)}(x) = \frac{n!}{(r-1)!(n-r)!} f_x(x)\ [F_x(x)]^{(r-1)}\ [1 - F_x(x)]^{(n-r)} \quad (10)$$

Utilizing equations (2), (3) and equation (8) in equation (10), probability density function of r^{th} statistics of Exponentiated Siya distribution are obtained,

$$f_{x(r)}(x) = \frac{n!}{(r-1)!(n-r)!} \frac{\beta x^{\alpha-1} e^{-(x/\theta)^{B}}}{\theta^{\alpha}\Gamma\left(\alpha/_{\beta}\right)} \times$$

$$\left[\frac{\gamma\left[\frac{\alpha}{\beta}, \left(\frac{x}{\theta}\right)^{B}\right]}{\Gamma\left(\alpha/_{\beta}\right)}\right]^{(r-1)} \left[\frac{\Gamma\left(\alpha/_{\beta}\right) - \gamma\left[\frac{\alpha}{\beta}, \left(\frac{x}{\theta}\right)^{B}\right]}{\Gamma\left(\alpha/_{\beta}\right)}\right]^{(n-r)} \quad (11)$$

Consequently, it is possible to obtain the probability density function of a higher-order statistics $X_{(n)}$, of the distribution. as

$$f_{x(n)}(x) = n\ \frac{\beta x^{\alpha-1} e^{-(x/\theta)^{B}}}{\theta^{\alpha}\Gamma\left(\alpha/_{\beta}\right)} \times \left[\frac{\gamma\left[\frac{\alpha}{\beta}, \left(\frac{x}{\theta}\right)^{B}\right]}{\Gamma\left(\alpha/_{\beta}\right)}\right]^{(n-1)}$$

Similarly, pdf of the 1st order statistics can be found as

$$f_{x(1)}(x) = n\ \frac{\beta x^{\alpha-1} e^{-(x/\theta)^{B}}}{\theta^{\alpha}\Gamma\left(\alpha/_{\beta}\right)} \times \left[\frac{\Gamma\left(\alpha/_{\beta}\right) - \gamma\left[\frac{\alpha}{\beta}, \left(\frac{x}{\theta}\right)^{B}\right]}{\Gamma\left(\alpha/_{\beta}\right)}\right]$$

6. Maximum Likelihood Estimation and Fisher's Information Matrix

Evaluation of the Exponentiated Siya distribution's parameters utilizing the maximum likelihood estimation method will be covered in this section, along with the derivation of Fisher's information matrix. Consider, $X_1, X_2, \dots\dots\dots. X_n$ be a random sample of the size n from the Exponentiated Siya distribution, the likelihood function can therefore be expressed as

$$\mathrm{L(x)} = \prod_{i=1}^{n} f\ \ (x;\ \ \alpha, \theta, \beta)$$

$$\mathrm{L(x)} = \prod_{i\ =\ 1}^{n} \frac{\beta x_i^{(\alpha-1)} e^{-(\frac{x_i}{\theta})^\beta}}{\theta^\alpha\ \Gamma\left({}^{\alpha}/_{\beta}\right)}$$

The log likelihood function is given by

$$logL = \sum_{i=0}^{n}[log\beta + (\alpha-1)logx_i - (\tfrac{x_i}{\theta})^\beta - \alpha log\theta - log\gamma(\tfrac{\alpha}{\beta})]$$

$$logL = n\mathrm{Log}[\beta] - n\alpha\mathrm{Log}[\theta] - n\mathrm{Log}[\Gamma\left(\frac{\alpha}{\beta}\right)] + (\alpha-1)\ \sum_{i=1}^{n}\mathrm{Log}[x_i] - \sum_{i=1}^{n}\left(\frac{x_i}{\theta}\right)^\beta$$

Taking Derivative of $logL$ concerning the parameters and equating it to 0, subsequent normal equations are obtained:

$$\frac{dlogL}{d\alpha} = 0, implies\ \ \frac{n}{\beta} + \frac{n\alpha\mathrm{PolyGamma}[0,\frac{\alpha}{\beta}]}{\beta^2} - \sum_{i=1}^{n}\theta^{-\beta}(-\mathrm{Log}[\theta] + \mathrm{Log}[x_i])x_i^\beta = 0,$$

$$\frac{dlogL}{d\beta} = 0, implies\ \ \frac{n}{\beta} + \frac{n\alpha\mathrm{PolyGamma}[0,\frac{\alpha}{\beta}]}{\beta^2} - \sum_{i=1}^{n}(-\mathrm{Log}[\theta] + \mathrm{Log}[x_i])\ \left(\frac{x_i}{\theta}\right)^\beta = 0,$$

$$\frac{dlogL}{d\theta} = 0, implies\ \ -\frac{n\alpha}{\theta} - \sum_{i=1}^{n} -\beta\theta^{-(\beta+1)}x_i^\beta = 0.$$

It is to be noted that $PolyGamma[z]$ is the logarithmic derivative of gamma function, given by $\Psi(z) = \frac{\Gamma'(z)}{\Gamma(z)}$. $PolyGamma[n,z]$ is given for a positive integer n by $\Psi^n(z) = d^n\frac{\Psi(z)}{dz^n}$. Here $\Psi(z)$ is the Di Gamma function.

It is crucial to remember that the analytical solution to the previous system of nonlinear equations is too complicated to solve algebraically. Consequently, we utilize R along with Wolfram Mathematica to forecast parameters of the suggested distribution.

Let $\widehat{\Theta} = (\hat{\alpha}, \hat{\beta}, \hat{\theta})$ represents the MLE of the parameters α, β, θ. The asymptotic normality findings of the maximum likelihood estimators of α, β, θ are stated below as per asymptotic characteristics of ML estimators under regularity constraints and multivariate central limit theorem. [4, 5].

$$\sqrt{n}\left[(\hat{\alpha}-\alpha), \left(\hat{\beta}-\beta\right), \left(\hat{\theta}-\theta\right)\right]\ \ \rightarrow N_3[0, I^{-1}(\Theta)]$$

Here, $I^{-1}(\Theta)$ indicates matrix of Fisher information. Elements of the Fisher information matrix can be computed using the following information given by:

$$E\left[\frac{\partial^2 logL}{\partial\alpha^2}\right] = -\frac{n\mathrm{PolyGamma}\left[1,\frac{\alpha}{\beta}\right]}{\beta^2},$$

$$E\left[\frac{\partial^2 logL}{\partial\alpha\partial\beta}\right] = \frac{n\left(\beta\mathrm{PolyGamma}[0,\frac{\alpha}{\beta}] + \alpha\mathrm{PolyGamma}[1,\frac{\alpha}{\beta}]\right)}{\beta^3},$$

$$E\left[\frac{\partial^2 logL}{\partial\alpha\partial\theta}\right] = -\frac{n}{\theta},$$

$$E\left[\frac{\partial^2 logL}{\partial\beta^2}\right] = -\frac{n\left(\beta^2 + 2\alpha\beta\mathrm{PolyGamma}[0,\frac{\alpha}{\beta}] + \alpha^2\mathrm{PolyGamma}[1,\frac{\alpha}{\beta}]\right)}{\beta^4} - \sum_{i=1}^{n}\mathrm{Log}\left[\frac{x_i}{\theta}\right]^2\left(\frac{x_i}{\theta}\right)^\beta,$$

$$E\left[\frac{\partial^2 logL}{\partial\beta\partial\theta}\right] - \sum_{i=1}^{n}\left(-\frac{\beta\mathrm{Log}\left[\frac{x_i}{\theta}\right]x_i\left(\frac{x_i}{\theta}\right)^{\beta-1}}{\theta^2} - \frac{\left(\frac{x_i}{\theta}\right)^\beta}{\theta}\right),$$

$$E\left[\frac{\partial^2 logL}{\partial\theta^2}\right] = \frac{n\alpha}{\theta^2} - \sum_{i=1}^{n}\left(\frac{2\beta x_i\left(\frac{x_i}{\theta}\right)^{\beta-1}}{\theta^3} - \frac{\beta x_i\left(\frac{x_i\left(\frac{x_i}{\theta}\right)^{\beta-2}}{\theta^2} - \frac{\beta x_i\left(\frac{x_i}{\theta}\right)^{\beta-2}}{\theta^2}\right)}{\theta^2}\right)$$

Whenever Θ is unknown, Fisher information matrix $I^{-1}(\Theta)$ is estimated or predicted by $I^{-1}(\widehat{\Theta})$.

7. Likelihood Ratio Test

Consider $X_{(1)}, X_{(2)}$…………X(n) be the random sample of size n drawn from a 3-parameter exponential Siya distribution. To consider its value, a hypothesis has been considered for our testing

H_0: f(x) = f (x; α, β) against H_1: f(x) = $f\ (x;\ \alpha,\theta,\beta)$

The test statistics illustrated below are employed to ascertain if random samples of size n are from Siya distribution

$$\Delta = \frac{L_1}{L_0} = \prod_{i=1}^{n} \frac{f\ (x;\ \alpha,\theta,\beta)}{f\ (x;\ \alpha,\beta)}$$

$$\Delta = \frac{L_1}{L_0} = \prod_{i=1}^{n} \frac{\dfrac{\beta xi^{\alpha-1} \mathrm{e}{-}(xi/\theta)^{B}}{\theta^{\alpha}\Gamma\left({\alpha}/{\beta}\right)}}{\dfrac{\beta^{\alpha}\ xi^{\alpha-1}\mathrm{e}{-}(\beta xi)}{\Gamma(\alpha)}}$$

$$\Delta = \frac{\Gamma(\alpha)}{\beta^{\alpha-1}\theta^{\alpha}\Gamma(\alpha/\beta)} \prod_{i=1}^{n} e^{-(\frac{x_i}{\theta})^{\beta})+\beta x_i}$$

8. Application

This section compares fit over three parameters using two real-life data sets in the Exponentiated SIYA distribution for ascertaining its goodness of fit. Weibul, Exponentiated Gamma Exponential, Power Gamma, Three Parameter Generalized Lindley, and Exponentiated Lomax distributions.

The R software approach has been utilized for the evolution of unknown parameters and to establish values of the model comparison criteria. We utilize criterion values BIC (Bayesian Information Criterion), AIC (Akaike Information Criterion), and AICc (Akaike Information Criterion Corrected), along with -2logL. To compare the performance of the Exponentiated SIYA distribution over the 3 Parameter Weibul, Power Gamma, 3 Parameter Generalized Lindley, Exponentiated Gamma Exponential, and Exponentiated Lomax distributions [13-15]. The distribution having lower criterion values for AIC, AICc, and BIC, along with -2logL is better. The following formulas are used to determine the criterion, including AICc, BIC, AIC, as well as -2logL.

$$\text{AIC} = 2k - 2\log L,\ \text{BIC} = k \log n - 2\log L \text{ and AICc} = \text{AIC} + \frac{2k(k+1)}{n-k-1}$$

Where sample size is n, number of parameters in the statistical model is k as well as -2log$L$ is maximised value of log-likelihood function under the suggested model.

Table 1. *Diastolic blood pressure of randomly selected 184 males working in Siachen Glacier (high altitude regions).*

90	78	72	80	84	70	88	84	78	80	78	72	84	84
70	78	84	98	76	84	84	80	80	78	76	84	82	88
80	74	70	80	80	76	80	82	74	60	80	70	80	
64	82	82	74	76	80	82	78	82	80	86	78	72	
80	78	74	76	86	84	78	78	78	72	72	78	74	
98	80	82	84	90	86	82	72	76	76	80	80	80	
82	84	78	80	84	88	84	80	78	82	86	84	68	
70	84	80	78	88	86	76	82	84	84	70	82	88	
84	80	78	74	70	74	80	60	84	96	86	112	86	
72	74	82	80	98	86	84	84	78	86	80	80	74	
88	84	84	80	84	80	80	72	84	88	80	86	76	
80	80	86	88	76	84	80	92	84	84	84	72	82	
80	78	84	86	80	80	88	98	86	78	78	80	84	
82	80	78	86	74	78	78	80	84	84	82	78	86	

Table 2. *Comparison and performance of fitted distributions.*

Models	MLE	S. E	-2Log L	AIC	BIC	AICc
Exponentiated Siya	α = 73.6779, β = 1.9427 θ = 12.5121	α = 17.7875, β =0.4701 θ =8.6905	1335.7495	1341.7495	1351.6445	1341.872
3-parameter Weibull	α = 26.2782, β =3.5454 θ =57.2204	α = 1.6785, β =0.2804 θ =1.5055	1351.6555	1357.6555	1367.5505	1357.7780
Power gamma	α = 25.6202, β =1.7798 θ =0.0102	α = 2.2949, β =0.026 θ =0.001	1361.4773	1367.4773	1377.3722	1367.5997
3-parameter generalised Lindley	α = 78.4076, β =32.6255 θ =1.1478	α = 8.0315, β =679.6864 θ =0.1153	1385.7188	1391.7188	1401.6138	1391.8413
Exponentiated gamma exponential	α = 81.979, β =3.6012 θ =3.6999	α = 8.2385, β =42.0219 θ =43.1322	1436.6794	1442.6794	1452.5743	1442.8018
Exponentiated Lomax distribution	α = 62.8752, β =0.0064 θ =10.536	α = 9.3394, β =46.04 θ =0.5507	1674.3437	1680.3437	1690.2387	1680.4662

Table 3. *The body fat percentage of randomly selected 150 women between 35 to 50 of age in Thrissur.*

38.3	37.7	33.5	39.1	33.5	31.9	38.3	32.8	32.5	41.8	31.5
27.2	38.8	44.8	36.6	37	37	25	34.9	29	29.5	35.1
39	33.2	38.1	34.2	41.6	44.5	27.5	38.5	32.2	35	25.9
39.5	36.6	40.3	38.4	35.4	38.9	29.5	43.6	34.5	39.6	40.6
32.4	33.4	43.3	30.9	35.8	28.2	24.2	34.5	44.6	26.2	39.4
33	44.8	34.2	49.6	36.4	26.9	30.1	37.3	37.6	39.7	39.4
37.4	39.4	31.6	37.6	41.2	26.4	38.8	37.6	36.4	37.2	44
37.4	35.4	36.2	38.2	38.7	39.7	33	34.2	31.7	36.9	37.1
34.9	35.7	38.4	42.3	35.2	38.3	34.6	31.5	33.8	30.1	41.5
33.5	33.7	30.1	35.7	35.3	37.7	33.3	27.5	34.3	40.8	40.3
33	37.3	34.4	35.5	37.8	34	28.4	36.8	35.2	43.2	
42.8	35	29.7	35.6	36.1	31.9	40.8	33.06	35	37.1	
36.3	35.2	36	38	42.2	36	35.4	27.2	34	35.2	
34.9	31.9	42.9	37.3	47.6	33.4	35.4	34	37.8	41.7	

Table 4. Comparison and Performance of fitted distribution.

Models	MLE	S. E	-2Log L	AIC	BIC	AICc
Exponentiated Siya	α = 19.1648, β =3.3176 θ =21.5493	α = 8.3137, β =01.5549 θ =11.3877	883.4203	889.4203	898.4522	889.5847
3Parameter Weibull	α = 17.0474, β =3.6972 θ =20.4877	α = 2.5043, β =0.6265 θ =2.364	883.9458	889.9458	898.9777	890.1102
Power Gamma	α = 29.4141, β =1.4276 θ =0.1763	α = 11.5323, β =0.2783 θ =0.2434	885.0009	891.0009	900.0328	891.1653
Exponentiated Gamma Exponential	α = 59.1957, β =0.837 θ =1.3804	α = 6.8248, β =3.4213 θ =5.6531	885.8165	891.8165	900.8484	891.9809
Exponentiated Lomax	α = 62.2974, β =0.0075 θ =18.8011	α = 10.9294, β =0.00005 θ =1.3278	1007.4598	1013.4598	1022.4917	1013.6242

The Exponentiated SIYA distribution has lesser *AIC*, *BIC*, *AICc*, as well as *-2logL* values than 3 Parameter Weibul, Power Gamma, 3 Parameter Generalized Lindley, Exponentiated Gamma Exponential, and Exponentiated Lomax distributions, as can be seen from the above table. Therefore, one can conclude that the Exponentiated SIYA distribution offers a better fit over 3 Parameter Weibul, Power Gamma, 3 Parameter Generalised Lindley, Exponentiated Gamma Exponential, and Exponentiated Lomax distributions.

9. Conclusion

The Exponentiated Siya distribution is a novel three-parameter distribution examined in this paper. Estimates have been made for its mathematical characteristics, including moments, survival functions, order statistics, and parameters. Ultimately, a real-world data set was fitted, and AIC, BIC, as well as AICc criteria, were examined and contrasted. The use of exponentiated models offers a potent instrument for lifetime data analysis. Their versatility and usefulness have increased by their capacity to include other parameters which results in better fits and more precise forecasts in practical situations. Exponentiated models are therefore increasingly becoming a crucial part of contemporary statistical analysis.

Abbreviations

AIC	Akaike Information Criterion
AICc	Corrected Akaike Information Criterion
BIC	Bayesian Information Criterion
CDF	Cumulative Distribution Function
PDF	Probability Density Function
MGF	Moment Generating Function
MLE	Maximum Likelihood Estimation
SE	Standard Error

Author Contributions

Shiny Chulliparambil Raj: Conceptualization, Data curation, Formal Analysis, Funding acquisition, Investigation, Methodology, Project administration, Resources, Software, Visualization, Writing – original draft, Writing – review & editing

Mani Vijayakumar: Data curation, Formal Analysis, Investigation, Methodology, Project administration, Resources, Software, Supervision, Validation, Writing – review & editing

Conflicts of Interest

The authors declare no conflicts of interest.

References

[1] Faisal Muhammad Amiru et.al (2025) The Additive Dhillon-Chen Distribution: Properties and Application to failure time data International Journal of Statistical Distribution and Applications, https://doi.org/10.11648/j.ijsda.20251101.11

[2] Rajagopalan, V. P. Soumya & Rashid a Gunaie (2021) A new Exponentiated Two parameter distribution with Biometric Application. October 2021 International Journal of Engineering and Management Research 06(10): 56-65.

[3] Muhammed Adamu Umar and Waheed Babatunde (2021) A New Exponential Gamma Distribution with Application. Journal of Modern Applied Statistical Methods.

[4] Thomas, P. Yageen and Jose, Jitto (2021). Weibull-Burr impounded bivariate distribution. Japanese Journal of Statistics and Data Science, 4(1): 73-105.

[5] Thomas, P. Yageen and Jose, Jitto (2020). A new bivariate distribution with Rayleigh and Lindley distributions as marginals Journal of Statistical Theory and Practice. 14 Article ID 28.

[6] C Subramanian & Aafaq A Rather (2019). Exponentiated Power Distribution: Properties and Estimation. Science, Technology and Development, Volume VIII issue 2019 September.

[7] Rama Shanker, et al A three-parameter Lindley distribution (2017). American Journal of Mathematics and Statistics. 7(1): 15-26. https://doi.org/10.5923/j.ajms.20170701.03

[8] Amal Soliman Hassan and Marwa Abdallah (2017) Exponentiated Lomax Geometric distribution properties and Application. Pakistan Journal of Statistics and Operational Research. https://doi.org/10.18187/pjsor.v1313.1437

[9] Manish Pal and Montip Jiensuwan (2015) Exponentiated Transmuted modified Weibul Distribution. European journal of Pure and Applied Mathematics. Volume 8 No.1 (2015).

[10] Rama Shanker (2015) Akash Distribution and Its Application. International Journal of Probability and Statistics. https://doi.org/10.5923/j-ijps.20150403.01

[11] Rashid A Ganaie, V Rajagopalan and Saeed Aldulaimi. The Weighted Power Shanker Distribution with Characterisations and Applications of Real Lifetime Data. Journal of Statistical Application & Probability. https://doi.org/10.18570/jsap/100122

[12] I. Shawky and R. A. Bakoban. Exponentiated Gamma distribution: Different methods of Estimations". Journal of Applied Mathematics. Volume2012. https://doi.org/10.1155/2012/284296

[13] Amal Soliman Hassan (2017) Exponentiated Lomax Geometric Distribution-Its properties and application. Pakistan Journal of Statistics and Operation Research.

[14] Surya Jabeen & Bilal Para (2018) Exponentiated Gamma Exponential Distribution. Sohag Journal of Mathematics. https://doi.org/10.18576/sjm/050301

[15] Nosakhare Ekhosuchi (2018) A Three parameter Generalised Lindly Distribution - Properties and Application. STATISTICA, anno LXXVIII n 3, 2018.