

Research Article

A Hybrid Adaptive Neuro-fuzzy Inference System and Physics-informed Neural Network (ANFIS-PINN) for Complex System Modeling

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Abstract

This work explores the integration of Adaptive Neuro-Fuzzy Inference Systems (ANFIS) and Physics-Informed Neural Networks (PINN) into a novel hybrid ANFIS-PINN framework. The proposed system aims to leverage the complementary strengths of both paradigms to address limitations inherent in individual approaches. ANFIS offers inherent interpretability, robust uncertainty handling, and adaptability to nonlinear relationships, applying the expert knowledge in the considered area, while PINN excels at incorporating physical laws, enhancing data efficiency, and improving generalization. The synergistic combination is envisioned to yield a more robust, interpretable, and physically consistent artificial intelligence (AI) solution, particularly for complex scientific and engineering problems characterized by nonlinearity, uncertainty, and sparse data, based on measurement data, a nonformal human expert's experience, and formal known physical laws. This paper details the foundational principles of ANFIS and PINN, outlines the compelling rationale for their integration, proposes several conceptual architectures and implementation strategies, and discusses the challenges and future directions for this promising hybrid AI paradigm.

Keywords

Adaptive Neuro-fuzzy Inference Systems (ANFIS), Artificial Intelligence (AI), Complex System, Physics-informed Neural Networks (PINN)

1. Introduction

The modelling of complex real-world systems, ranging from intricate fluid dynamics to biological processes, increasingly demands advanced artificial intelligence (AI) models that can effectively handle nonlinearity, inherent uncertainty, and sparse data. Traditional approaches, whether purely data-driven or solely model-driven, frequently encounter limitations in these challenging scenarios. There is a growing imperative for AI systems to not only achieve high predictive accuracy but also to demonstrate interpretability

and physical consistency, especially in critical engineering and scientific domains where decisions carry significant implications. This work focuses on two distinct yet powerful AI paradigms: the Adaptive Neuro-Fuzzy Inference System (ANFIS) and Physics-Informed Neural Networks (PINN). ANFIS, a hybrid intelligent system, combines the adaptive learning capabilities of neural networks with the human-like reasoning and interpretability of fuzzy logic, making it adept at nonlinear mapping and decision-making in uncertain en-

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vironments. Conversely, PINN integrates neural networks with known physical laws, typically expressed as differential equations, to enhance generalization and reduce reliance on extensive datasets. The core motivation for proposing a novel ANFIS-PINN hybrid system is to harness the complementary strengths of these two approaches. By doing so, the aim is to overcome their individual limitations and develop a more robust, interpretable, and physically consistent AI solution for complex systems. This paper is structured to guide the reader through the foundational concepts of ANFIS and PINN, articulate the rationale for their integration, present proposed architectural and implementation strategies, and conclude with a discussion of the challenges and future research directions for this innovative hybrid paradigm.

2. Foundations: Understanding ANFIS and PINN

2.1. Adaptive Neuro-fuzzy Inference Systems (ANFIS)

ANFIS represents a powerful hybrid intelligent system that merges the learning capabilities of Artificial Neural Networks (ANNs) with the knowledge representation and inference abilities of Fuzzy Inference Systems (FIS) [1]. This integration enables ANFIS to learn complex nonlinear relationships between inputs and outputs while retaining the inherent interpretability of fuzzy rules [1]. The architecture of ANFIS consists of five distinct layers, designed to mimic a Takagi-Sugeno-Kang fuzzy inferential system [5]. Each layer performs a specific function in mapping inputs to outputs based on fuzzy IF-THEN rules [1]:

1. Layer 1 (Input Layer): This layer receives crisp (non-fuzzy) input values and transmits them directly to the subsequent layer without any computation.
2. Layer 2 (Fuzzification Layer): In this layer, membership functions (e.g., triangular, trapezoidal, Gaussian, tangential, etc.) are applied to the input values, converting them into fuzzy sets and calculating their degrees of membership. These membership functions are parameterized and adjustable, forming what are known as "premise parameters". Each node in this layer typically represents a linguistic label, such as "low," "medium," or "high".
3. Layer 3 (Rule Layer): Each node within this layer corresponds to a specific fuzzy IF-THEN rule. The firing strength of each rule is computed by combining the membership values obtained from the fuzzification layer, usually through fuzzy operators like AND (e.g., minimum or product). This layer effectively represents the antecedent part of the fuzzy rules [8].
4. Layer 4 (Normalization Layer): This layer normalizes the firing strengths calculated in the rule layer. The firing strength of each rule is divided by the sum of all firing strengths, ensuring that the collective contribution of all rules sums to unity. These normalized strengths are crucial as they determine the relative importance of each rule in contributing to the final output.
5. Layer 5 (Defuzzification/Output Layer): The final crisp output is computed in this layer as a weighted average of the rule outputs. This layer incorporates "consequence parameters," which are typically linear combinations of the input variables. Standard ANFIS architectures are designed to produce a single output [6].

ANFIS employs a hybrid learning algorithm that combines gradient descent with least squares estimation [7]. During a forward pass, input signals propagate through the network, and the consequent parameters are identified using the least squares method. In a subsequent backward pass, error signals propagate backward through the network, and the premise parameters (those associated with membership functions) are updated using gradient descent. This iterative process continues until a desired level of accuracy is achieved. A primary advantage of ANFIS is its inherent interpretability and transparency, allowing for direct translation into a fuzzy rulebase that is readily understandable to humans. It is often characterized as a "grey-box" model, offering a more transparent alternative to opaque "black-box" deep neural networks [2]. This interpretability is vital for fostering trust in AI systems, especially in sensitive applications [9]. ANFIS excels at uncertainty handling due to its foundation in fuzzy logic. Fuzzy logic inherently accommodates uncertainty and linguistic variables by allowing partial membership, making ANFIS robust in environments characterized by imprecise or vague information [3]. Furthermore, ANFIS demonstrates strong adaptability and nonlinear mapping capabilities. It can learn and adjust to complex nonlinear relationships from data, making it well-suited for dynamic scenarios and control systems [1]. Its ability to continuously optimize performance and adapt to changing conditions in real-time is a significant benefit [12]. The system also exhibits robustness to noise and efficacy with limited data. ANFIS has shown robust modeling performance, particularly when handling noisy data within nonlinear systems [4]. It also possesses a rapid learning capability even with limited or unevenly distributed datasets [4]. Despite its strengths, ANFIS faces certain limitations. A long-observed challenge is the trade-off between an algorithm's transparency and its predictive accuracy; interpretable systems often tend to be less accurate, while more accurate ones can be less transparent [5]. Traditional ANFIS models may also struggle with high-dimensional and evolving data, as they often lack explicit mechanisms for dynamic rule and attribute management [13]. This can limit transparency and interpretability, and adversely affect performance in domains where data characteristics and operational conditions change rapidly. The parameter optimization process in Neuro-Fuzzy Systems (NFS) models can be challenging, as conventional methods are prone to getting trapped in local optimum points

[2]. Additionally, standard ANFIS architecture is designed for a single output, which can be a constraint for multi-output systems [6]. Recent advancements have aimed to address some of these limitations. ADAR (Adaptive Dual-Weighting and Rule Management), for instance, integrates dual weighting mechanisms for attributes and rules with automated growth and pruning strategies. This streamlines complex fuzzy models, regulating rule complexity and feature importance to enhance scalability and transparency in high-dimensional and evolving data environments [13]. Dimensionality reduction techniques, such as deep auto-encoders, are employed to compress high-dimensional input data for NFS, significantly reducing computational complexity while maintaining predictive accuracy. The integration of optimization algorithms like Particle Swarm Optimization (PSO) and other metaheuristic algorithms can optimize the parameters of membership functions, leading to improved robustness, accuracy, and reduced model complexity. Hierarchical and modular NFS frameworks offer another approach by dividing high-dimensional input spaces into smaller, manageable subspaces where localized fuzzy inference is conducted independently before combining results. This strategy offers distinct advantages in interpretability and performance optimization [13]. Finally, Deep Convolutional Neuro-Fuzzy Inferential Systems (DCNFIS) represent a significant hybrid. This approach combines a Convolutional Neural Network's (CNN) convolutional base (acting as an automated feature extractor) with a modified ANFIS classifier. DCNFIS achieves state-of-the-art accuracy comparable to CNNs while retaining fuzzy rule interpretability and allowing for end-to-end training [5].

2.2. Physics-informed Neural Networks (PINN)

Physics-Informed Neural Networks (PINNs) represent a transformative paradigm in machine learning, integrating data-driven learning with the governing physical laws of a system [14]. Fundamentally, PINNs convert the challenge of approximating solutions to complex physical phenomena, often described by differential equations, into an unconstrained minimization problem [16]. The core principle of PINNs lies in modifying the neural network's loss function [15]. Unlike traditional neural networks that primarily minimize the discrepancy between predictions and observed data, PINNs incorporate an additional penalty term that quantifies the violation of the system's governing equations [17]. The total loss function (L) for a PINN is typically composed of a sum of different terms [17]:

1. L_{data} (Data Loss): This is the conventional loss term, often a mean squared error, which minimizes the difference between the neural network's predictions and the actual observed data points.
2. L_{equation} (Equation Loss): This is the defining "physics-informed" component. It represents the residual of the governing differential equation(s) (e.g., Partial

Differential Equations, PDEs). By minimizing this term, the neural network is compelled to satisfy the underlying physical laws.

3. L_{IC} (Initial/Boundary Condition Loss): An optional but frequently crucial term that enforces initial and/or boundary conditions. This ensures that the model finds the specific solution relevant to the problem, as differential equations often have infinitely many solutions without such constraints. The architecture of a PINN itself is typically a standard neural network, such as a multi-layer perceptron. However, its training objective is fundamentally altered by the inclusion of these physics-based terms in the loss function. Some advanced PINN architectures may even include non-trainable layers designed to enforce hard constraints [18]. PINNs excel at incorporating physical laws and enhancing generalization. By embedding known physical constraints, PINNs produce predictions that are significantly more precise and generalize better to unseen data [14]. This ensures that the model adheres to fundamental scientific principles, which is critical for robustness in scientific and engineering applications. A notable advantage of PINNs is their data efficiency and reduced data dependency. They can learn effectively with limited or even no observational data, as their learning process is guided by the underlying equations. This makes them particularly suitable for scenarios where data collection is expensive, scarce, or impractical. PINNs offer a mesh-free formulation, providing an alternative to traditional numerical methods that require mesh generation. This characteristic allows for greater flexibility in handling complex geometries and can reduce manual effort in simulations. For differential equations where an analytical (closed-form) solution is unknown, PINNs can provide a robust alternative to conventional numerical methods, which can accumulate errors over many time steps and lead to a loss of accuracy [17]. Neural networks, and thus PINNs, are effective interpolators. This capability allows PINNs to provide prompt values for unseen data points that fall within the trained time interval, a task that traditional numerical methods might not perform as readily [16]. Furthermore, integrating physical knowledge directly into the training process helps to prevent overfitting, even when dealing with finite and noisy datasets [20]. The "physics-informed" nomenclature is also a generalization; the technique's broader applicability extends to any system with a well-defined mathematical model, not exclusively physical systems [17]. Despite their innovative features, PINNs have certain inherent restrictions. A significant drawback is that PINNs, in their prevalent form, utilize soft constraints and "cannot strictly satisfy" physical constraints in their predictions [18]. This means they typically strike a balance between approximating the ground truth and favoring first prin-

ciples, which can potentially lead to physically inconsistent results in critical applications [18]. The development of "hard constraints (hPINN)" is an active research area, but rigorous mathematical guarantees for these are often lacking [18]. PINNs, being a relatively recent numerical approach, possess a less rigorous theoretical foundation and a scarcity of effective error analysis tools compared to established numerical methods [16]. Training PINNs can be time-consuming, and convergence is not always guaranteed. Optimizing hyperparameters often requires substantial time and effort [16]. They are also prone to vanishing gradient problems, particularly in deep networks, and integrating multiple objectives into the loss function can bias gradients [16]. While offering a mesh-free approach, conventional PINNs can still incur a computational burden due to the large number of collocation points required for pointwise approximation [14]. Additionally, PINNs can face challenges in accurately learning high-frequency components of solutions [19].

Table 1 shows the comparative analysis between ANFIS and PINN structures.

3. Synergistic Integration: The ANFIS-PINN System

3.1. Rationale for Hybridization: Complementary Strengths

The conceptualization of an ANFIS-PINN system is driven by the desire to transcend the limitations of individual ANFIS and PINN models by leveraging their distinct yet complementary strengths. This hybridization aims to create a more powerful, robust, and trustworthy AI system for complex scientific and engineering problems. One compelling reason for this integration is bridging interpretability and physics adherence. PINNs excel at embedding physical laws, leading to high accuracy and generalization, but they often remain "black-box" models, making their internal workings opaque. ANFIS, conversely, provides inherent interpretability through its fuzzy rulebase, which can be directly understood by humans. While ANFIS can sometimes face accuracy trade-offs, an ANFIS-PINN system could offer "interpretable physics-informed AI" by allowing the fuzzy logic to explain the physical behaviors or decisions made by the network, thereby overcoming the opacity of deep neural networks while retaining physical consistency. This integration could lead to a deeper understanding, specifically providing interpretable physical regimes. PINNs learn to satisfy governing differential equations (PDEs) globally, but the internal representation of *how* these laws are satisfied across different physical regimes (e.g., laminar vs. turbulent flow, distinct phases of a chemical reaction) remains implicit within the neural net-

work's weights. ANFIS, with its fuzzy rules, naturally segments the input space into "fuzzy regions" where specific rules apply. If these fuzzy regions can be aligned with different physical regimes or operating conditions (e.g., "IF temperature is HIGH AND pressure is LOW THEN reaction is FAST"), the ANFIS component could provide human-understandable explanations for the PINN's behavior in those specific physical contexts. This moves beyond simple "saliency maps" for interpretability to provide rule-based explanations of physical phenomena, which represents a higher level of understanding for domain experts. This advancement could lead to a new class of "explainable scientific AI" where not just predictions, but the underlying physical reasoning, is made transparent, a critical step for scientific discovery, engineering design optimization, and regulatory compliance. Another significant benefit is enhancing robustness in uncertain environments. PINNs reduce data dependency by incorporating physics, while ANFIS is adept at handling noisy data and inherent uncertainty through fuzzy logic. The combination would create a system exceptionally robust to real-world data imperfections and inherent system uncertainties. This robustness extends to quantifying and propagating physical uncertainty. PINNs, in their prevalent form, use soft constraints and "cannot strictly satisfy" physical constraints, implying a degree of approximation in their physical adherence. Fuzzy logic, as utilized in ANFIS, is specifically designed to model and propagate uncertainty. The concept of "Fuzzy Physics-Informed Neural Networks (fPINN)" explicitly addresses "uncertain fields" and "fuzzy partial differential equations" [21]. The fPINN approach, which employs multiple interval PINNs (iPINNs) for different α -cut levels, demonstrates how fuzzy sets can characterize the possibilistic uncertainty in inputs and outputs of physical systems [22]. An ANFIS-PINN could leverage ANFIS's fuzzy reasoning to not only handle noisy data but also to quantify and propagate the uncertainty associated with physical parameters or boundary conditions through the PINN's predictions. This would provide not just a point estimate but a fuzzy range of possible outcomes, invaluable for risk assessment and decision-making in uncertain physical systems. This moves towards "uncertainty-aware physics-informed AI," where the system provides a confidence interval or a fuzzy set of solutions, reflecting the inherent uncertainties in the physical system or input data, which is particularly relevant for high-stakes applications. Finally, the integration allows for combining data-driven learning with expert knowledge. ANFIS excels at incorporating expert knowledge through fuzzy IF-THEN rules, while PINNs integrate fundamental physics laws. The hybrid system could bridge the gap between purely data-driven models and purely physics-driven models, allowing for the incorporation of quantitative data, precise physical laws, and qualitative expert heuristics or "soft" physical understanding. This leads to adaptive knowledge integration and discovery. PINNs are powerful when physics laws are precisely known. However, in many real-world

systems, the governing equations might be partially known, approximate, or even unknown in certain regimes. ANFIS can capture expert knowledge even when formal mathematical models are absent. The Perception-Informed Neural Networks (PrINNs) framework explicitly allows for "perception-based information" and "expert knowledge" to be integrated into neural networks, even for systems with "unknown physics laws". An ANFIS-PINN could use the ANFIS component to infer fuzzy rules from data or expert input, representing approximate physical relationships or behavioral heuristics. These fuzzy rules could then act as "soft" physical constraints or regularization terms in the PINN's loss function, even when a precise PDE is unavailable. Conversely, the PINN's learning could help refine the fuzzy rules or identify new, more precise relationships that were previously only qualitatively understood, leading to a synergistic discovery process where data, physics, and expert knowledge iteratively inform each other. This aligns with the idea of "discovering new physics" mentioned for PINNs [20]. This creates a "knowledge-evolving physics AI," where qualitative expert insights can be formalized, tested against data and approximate physics, and iteratively refined towards more precise physical models, marking a significant leap towards truly intelligent scientific modeling.

3.2. Conceptual Architecture of ANFIS-PINN

The integration of ANFIS and PINN can be conceptualized in several architectural paradigms, each leveraging their strengths at different points in the system. The overarching goal is to combine the adaptive, interpretable, and uncertainty-handling capabilities of ANFIS with the physics-adhering and data-efficient nature of PINN. Proposed Integration Points and Data Flow:

3.2.1. Sequential Integration (ANFIS as a Pre-processor/Feature Extractor for PINN)

In this configuration, ANFIS functions as a front-end, processing raw or high-dimensional input data. It would utilize its fuzzification and rule layers to extract meaningful fuzzy features or linguistic representations of the input state. This approach is analogous to the Deep Convolutional Neuro-Fuzzy Inferential Systems (DCNFIS) where a Convolutional Neural Network acts as a feature extractor for ANFIS. The defuzzified outputs or normalized firing strengths from ANFIS would then serve as inputs to the PINN. This process reduces the dimensionality of the input space for the PINN and potentially provides more robust, interpretable features that are less sensitive to noise, thereby benefiting the PINN's training.

Data Flow: Raw Data → ANFIS (Fuzzification, Rule, Normalization, Defuzzification) → Interpretable Fuzzy Features → PINN (Neural Network + Physics Loss) → Physically Consistent Output.

3.2.2. Sequential Integration (PINN as a Pre-processor/Physics-informed Feature Extractor for ANFIS)

Conversely, a PINN could serve as the initial layer, learning a low-dimensional, physically consistent representation of the system dynamics. The PINN would be trained with both data and physics loss terms, ensuring that its internal states or intermediate outputs adhere to the governing equations. These physically-informed latent features or predicted states from the PINN could then be fed into an ANFIS. The ANFIS would subsequently use these refined, physics-consistent features to generate interpretable fuzzy rules and make final predictions or control decisions, providing explainability to the PINN's output. This approach could be particularly useful for control systems where interpretable decision-making based on physical states is critical.

Data Flow: Raw Data → PINN (Neural Network + Physics Loss) → Physically Consistent Latent Features/Predictions → ANFIS (Fuzzification, Rule, Normalization, Defuzzification) → Interpretable Output.

3.2.3. Parallel/Hybrid Integration (ANFIS and PINN in a Unified Loss Function)

This represents the most direct form of integration, where the ANFIS component directly influences the PINN's training through a modified loss function. This aligns with the concept of Fuzzy-Informed Neural Networks (FINNs) within the broader Perception-Informed Neural Networks (PrINNs) framework [22]. The loss function would include terms for data adherence, physical law adherence (PINN's core objective), and crucially, fuzzy rule adherence. The fuzzy rule adherence term would penalize deviations from desired fuzzy relationships or expert heuristics. Instead of strictly defined differential equations, some physical constraints could be expressed as fuzzy rules. For example, a rule might be "IF velocity is HIGH THEN air resistance is VERY HIGH." These fuzzy rules, derived from expert knowledge or learned from data, could be incorporated into the loss function as "fuzzy residuals." The concept of fPINN [21] and FcINNs [22] directly demonstrates this by solving fuzzy partial differential equations or incorporating fuzzy differential equations into the loss function. This allows for the modeling of systems with inherent uncertainty in their physical parameters or governing laws. The ANFIS component could define the membership functions and fuzzy rules that represent these "fuzzy physical laws," and their violation would contribute to the overall loss, guiding the neural network towards physically plausible and interpretable solutions. This could potentially lead to "harder" fuzzy constraints than typical PINN soft constraints [18].

Data Flow: Raw Data → Hybrid ANFIS-PINN Network (single or modular neural network architecture) → Output. Training involves a composite loss:

$$L = L_{\text{data}} + \lambda_1 L_{\text{physics}} + \lambda_2 L_{\text{fuzzyrules}} \quad (1)$$

3.3. Proposed Implementation Strategies

Implementing an ANFIS-PINN system requires careful consideration of how to merge their distinct mechanisms. Several strategies can be pursued:

3.3.1. Loss Function Integration

Fuzzy Regularization of Physical Laws. The most promising avenue for deep integration involves modifying the PINN's loss function to incorporate terms derived from ANFIS's fuzzy logic. This mechanism would involve defining a fuzzy loss component (L_{fuzzy}) that quantifies the degree to which the network's predictions violate a set of fuzzy physical rules or expert-defined constraints. For instance, if a physical quantity X is expected to be "Low" when Y is "High," L_{fuzzy} would increase if the network predicts X to be "Medium" or "High" under "High" Y . This approach aligns with the concept of "fuzzy residuals" in differential equations, as explored in fPINNs [21], where fuzzy numbers or functions are directly used within the equations. The overall loss function would then aim to minimize the "fuzziness" or uncertainty of these residuals. Furthermore, the "dual weighting mechanism" from ADAR [13] could be adapted for this hybrid system. An ANFIS-like component could dynamically adjust the weights (λ_1, λ_2) of the data, physics, and fuzzy loss terms during training. This adaptive weighting would allow the system to prioritize adherence to physical laws or fuzzy constraints when data is sparse, or to relax them when data is abundant but potentially noisy. This strategy could effectively address the issue of "biased gradients" that can arise in PINNs when integrating multiple objectives into the loss function [16].

3.3.2. Hybrid Network Architectures

Modular and Interconnected Designs. Various architectural designs can facilitate the integration:

i. Modular ANFIS-PINN

- 1) ANFIS as a Rule-Based Controller/Corrector for PINN: A pre-trained or adaptively trained ANFIS could continuously monitor the PINN's output for physical inconsistencies or deviations from fuzzy expert rules. If such inconsistencies are detected, the ANFIS could provide corrective feedback or adjust certain parameters of the PINN. This functions as an automated "human-in-the-loop" system [9-11], but with the fuzzy logic component providing the automated corrective intelligence.
- 2) PINN for High-Fidelity Simulation, ANFIS for Decision/Control: In this setup, the PINN would handle the underlying complex physical simulation, leveraging its strengths in solving differential equations. The outputs from the PINN would then be fed into a separate ANFIS, which would use its fuzzy logic to make interpretable decisions or control actions, similar to its established applications in control systems.

ii. Deep Neuro-Fuzzy Integration (Inspired by DCN-

FIS/FINNs)

- 1) Embedding Fuzzy Logic within Deep PINN Layers: Rather than having a completely separate ANFIS module, fuzzy logic operations (e.g., fuzzification, rule inference) could be integrated as custom layers or activation functions directly within the deep neural network architecture that forms the PINN. This approach would make the internal representations of the PINN more interpretable and enable end-to-end training of the entire hybrid system.
- 2) FINNs as a Precedent: The Fuzzy-Informed Neural Networks (FINNs) concept [22] provides a strong precedent for this. FINNs embed fuzzy logic constraints directly into a deep learning architecture's loss function without requiring defuzzification or fixed ANFIS-like structures. This allows for the development of arbitrarily deep or wide networks that are still "fuzzy-informed," combining the flexibility of deep learning with the interpretability of fuzzy logic.

3.3.3. Adaptive Parameter Optimization via Fuzzy Logic

Leveraging ANFIS's adaptive learning capabilities could enable the optimization of PINN-specific parameters, such as the weighting coefficients (λ) in the loss function, or even the network's hyperparameters (e.g., learning rate, number of layers/neurons). An outer ANFIS controller could observe the training progress (e.g., convergence of data loss versus physics loss, physical consistency metrics) and dynamically adjust the PINN's training parameters using fuzzy rules (e.g., "IF physics loss is HIGH AND data loss is LOW THEN INCREASE physics loss weight SLIGHTLY"). This adaptive approach could help address the limitation that "optimizing hyper-parameters requires a lot of time and effort" in PINNs, and potentially mitigate issues like getting trapped in local optima.

3.3.4. Uncertainty Quantification and Explainability Enhancement

The ANFIS component can naturally provide fuzzy outputs, representing degrees of certainty or possibility, rather than crisp point predictions. When combined with PINN, this would allow the system to quantify the uncertainty in its physically-informed predictions, providing a more complete picture for decision-makers, especially in scenarios involving imprecise data or inherent system variability. Furthermore, the fuzzy rules learned by the ANFIS component can serve as direct, human-interpretable explanations for the hybrid system's behavior, particularly concerning how it adheres to or deviates from physical principles. This directly addresses the "black-box" nature of traditional PINNs. Techniques like saliency maps derived from fuzzy rules, as demonstrated in DCNFIS [5], could also be employed to visually explain the system's focus. Table 2 shows the proposed ANFIS-PINN integration paradigms.

4. Challenges and Future Directions

The development and deployment of a hybrid ANFIS-PINN system present several significant challenges that require dedicated research and innovation. One major hurdle is computational complexity and scalability. Combining two sophisticated neural-network-based systems inherently increases computational demands. Training deep PINNs is already known to be time-consuming and resource-intensive [16], and integrating ANFIS's parameter optimization and dynamic rule management could exacerbate these requirements. While techniques such as dimensionality reduction and modularity exist for ANFIS, and Physics-Informed Neural Operators (PINOs) aim to reduce the computational burden of PINNs [14], their synergistic integration demands careful consideration. Future research must focus on developing efficient parallel computing implementations, optimizing hybrid training algorithms, and leveraging advanced hardware accelerators (e.g., GPUs, TPUs). Exploring sparse fuzzy rule bases and adaptive network pruning for the ANFIS component within the hybrid context could also help mitigate complexity. Another critical challenge lies in defining and integrating fuzzy physical laws. While the concept of "fuzzy physical constraints" is powerful, as evidenced by work on Fuzzy-Informed Neural Networks (FINNs) and fuzzy partial differential equations [22], precisely defining these fuzzy rules and their corresponding membership functions for complex physical systems can be arduous. This often necessitates deep domain expertise and meticulous calibration [3]. Furthermore, the mathematical rigor for fuzzy differential equations and their seamless integration into neural network loss functions is still evolving [22], particularly for complex, real-world systems. Future directions include developing automated or semi-automated methods for generating fuzzy physical rules from data or expert input. More theoretical work on fuzzy calculus and its application to physics-informed learning is also essential to provide stronger mathematical guarantees and ensure robust integration. Exploring methods to adaptively learn fuzzy membership functions from data while simultaneously adhering to physical principles is another promising avenue. Theoretical guarantees and convergence analysis represent another significant area for advancement. PINNs themselves are acknowledged to have a "less rigorous theoretical foundation" and a "dearth of effective error analysis tools" [16]. Combining them with ANFIS, which also presents its own set of challenges in interpretability and parameter optimization, further complicates the establishment of theoretical guarantees for convergence, stability, and generalization error. Ensuring that the hybrid system converges to a physically consistent and accurate solution, especially when confronted with potentially conflicting data, precise physical laws, and fuzzy constraints, will be a substantial research hurdle. Future efforts should include rigorous mathematical analysis of the hybrid loss function's landscape and the combined learning algorithm.

Developing new error bounds and convergence proofs specifically tailored for ANFIS-PINN systems is crucial. Investigations into the conditions under which fuzzy constraints can act as "harder" constraints and their impact on solution uniqueness and stability are also warranted. Finally, real-world applications and validation are paramount. While both ANFIS and PINNs have demonstrated success in various domains individually, the real-world validation of a novel ANFIS-PINN system across diverse applications will be crucial. Demonstrating tangible improvements in interpretability, robustness to uncertainty, and adherence to physical principles compared to standalone models will be key to widespread adoption. Future work should focus on applying ANFIS-PINN to a variety of complex engineering and scientific problems, such as advanced control systems, climate modeling, biomedical simulations, plasma physics and material science, to thoroughly evaluate its performance and identify its optimal use cases.

5. Conclusion

The exploration of a hybrid ANFIS-PINN system reveals a compelling pathway toward developing more sophisticated and reliable AI models for complex scientific and engineering challenges. By synergistically combining the strengths of Adaptive Neuro-Fuzzy Inference Systems—namely, their inherent interpretability, robust uncertainty handling, and adaptability to nonlinear relationships—with those of Physics-Informed Neural Networks—their capacity to embed physical laws, enhance data efficiency, and improve generalization—the proposed ANFIS-PINN framework offers a multifaceted solution. The rationale for this integration is rooted in the complementary nature of the two paradigms. The hybrid system promises to bridge the gap between black-box accuracy and human-understandable explanations, enabling the development of interpretable physics-informed AI. This capability extends to providing transparent explanations of physical behaviors across different operating regimes, moving beyond mere predictions to offer rule-based insights into complex phenomena. Furthermore, by integrating ANFIS's ability to handle imprecision with PINN's data efficiency, the hybrid system can enhance robustness in uncertain environments, allowing for the quantification and propagation of physical uncertainty in predictions. This leads to a more comprehensive understanding of potential outcomes, crucial for risk-sensitive applications. The combination also facilitates a powerful adaptive knowledge integration process, where qualitative expert heuristics can inform and refine precise physical models, fostering a continuous cycle of scientific discovery. Conceptual architectures, ranging from sequential processing to unified loss function integration, demonstrate the versatility of this hybridization. Implementation strategies, including fuzzy regularization of physical laws within the loss function, modular and deep neuro-fuzzy network designs, and adaptive parameter optimization, provide concrete avenues for devel-

opment. While significant challenges remain, particularly in computational complexity, the precise definition of fuzzy physical laws, and establishing rigorous theoretical guarantees, the potential benefits of ANFIS-PINN are substantial. Future research efforts should focus on addressing these hurdles through advancements in optimized algorithms, automated rule generation, and comprehensive theoretical analyses. Successful validation in real-world applications will be critical to realizing the full promise of this innovative hybrid AI paradigm, paving the way for a new generation of intelligent systems that are not only accurate and efficient but also transparent, robust, and deeply informed by the underlying principles governing our physical world.

Abbreviations

AI	Artificial Intelligence
ANFIS	Adaptive Neuro-fuzzy Inference Systems
PINN	Physics-informed Neural Networks

Appendix

Author Contributions

Oleg Agamalov is the sole author. The author read and approved the final manuscript.

Data Availability Statement

The data supporting the outcome of this research work has been reported in this manuscript and Appendix.

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Conflicts of Interest

The author declares no conflicts of interest.

Table 1. Comparative Analysis of ANFIS and PINN.

Feature	Adaptive Neuro-Fuzzy Inference System (ANFIS)	Physics-Informed Neural Network (PINN)
Core Paradigm	Hybrid Neuro-Fuzzy System	Physics-Informed Neural Network
Architecture	Five-layer (Takagi-Sugeno-Kang fuzzy inference system embedded in a neural network structure)	Deep Neural Network (e.g., Multi-Layer Perceptron) with a modified loss function
Knowledge Representation	Fuzzy IF-THEN rules, Membership Functions	Differential Equations, Physical Laws
Learning Mechanism	Hybrid (Gradient Descent for premise parameters, Least Squares for consequent parameters)	Gradient-based (minimizing combined data, physics, and initial/boundary condition loss)
Data Dependency	Moderate (can learn effectively with limited or noisy data)	Low (physics-driven, can learn with sparse or no observational data)
Uncertainty Handling	Explicit (inherent through fuzzy logic, allowing partial membership and linguistic variables)	Implicit (via data/equation error, but not designed for linguistic uncertainty)
Interpretability	High (grey-box model, direct translation to fuzzy rulebase, human-understandable explanations) ¹	Low (black-box model, internal representations are opaque) ²
Physics Integration	Indirect (learns relationships from data, can incorporate expert heuristics as rules)	Explicit (physical laws directly encoded as penalty terms in the loss function) ¹⁵
Typical Applications	Control systems, prediction, pattern recognition, decision support systems ⁴	Solving Partial Differential Equations (PDEs), system identification, fluid dynamics, material science ¹⁴
Key Strengths	Adaptability to nonlinearity, transparency, robustness to noise, rapid learning with limited data ¹	Enhanced precision, better generalization, data efficiency, mesh-free formulation, prevention of overfitting ¹⁴
Key Limitations	Accuracy-interpretability trade-off, challenges with high-dimensional/evolving data, potential for local optima, single output constraint ²	Soft constraint enforcement (cannot strictly satisfy physics), less rigorous theoretical foundation, training convergence issues, vanishing gradients, computational burden ¹⁴

Table 2. Proposed ANFIS-PINN Integration Paradigms.

Integration Paradigm	Mechanism	Expected Benefits	Potential Challenges
Sequential (ANFIS as Pre-processor)	ANFIS processes raw/high-dimensional input into interpretable fuzzy features; PINN then uses these features.	Reduced PINN input dimensionality; More robust, noise-resilient features for PINN; Interpretability of input space via fuzzy sets.	Potential information loss during fuzzification; Designing optimal fuzzy feature extraction for PINN.
Sequential (PINN as Pre-processor)	PINN learns physically-informed latent features or predictions; ANFIS then interprets these refined features for decision/control.	Interpretable output from physically consistent features; Explainable control or decision-making based on physical states.	Complexity of PINN feature extraction; Ensuring ANFIS captures subtle physical nuances from PINN output.
Parallel (Unified Loss Function)	A single network trained with a composite loss function: $L = L_{data} + \lambda_1 L_{physics} + \lambda_2 L_{fuzzyrules}$.	Direct enforcement of fuzzy physical constraints; Enhanced robustness to uncertainty; Adaptive weighting of loss terms.	Defining appropriate fuzzy loss terms; Balancing gradients from disparate loss components; Increased computational complexity.
Adaptive Optimization Loop	An outer ANFIS controller observes PINN training metrics (e.g., loss convergence, physical consistency) and dynamically adjusts PINN training parameters (e.g., learning rate, loss weights).	Improved convergence and stability of PINN training; Adaptive hyperparameter tuning; Potential to escape local optima.	Designing effective fuzzy rules for optimization logic; Potential for instability if control rules are poorly defined.

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Biography



Oleg Agamalov is the Head of the Electrical Department of TPSP. He completed his Doctor of Engineering Science in Electrical Engineering from the Institute of Electrodynamics (Kyiv, Ukraine) in 2017, and his PhD in Electrical Engineering from Kyiv Polytechnical Institute in 2005. Recognized for his long-standing collaboration with the work of the Association, Dr. Agamalov has been honored with the title of Distinguished Member of CIGRE in 2016. Now he is working as an independent researcher in the fields of clean and efficient energy sources, as well as domains of AI applications.

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