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Research Article

Computational Analysis of Heat Transfer Enhancement in a Trapezoidal Cavity with Central Circular Obstacle

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Abstract

This study numerically explores natural convection in a trapezoidal enclosure with a wavy upper boundary and a centrally placed circular cavity of varying radius. The left vertical wall is maintained at a low temperature, while the right wall is heated. The remaining boundaries are adiabatic. Three obstacle sizes are considered with radius 0.05, 0.08 and 0.03 representing different levels of geometric blockage. Simulations are carried out in COMSOL Multiphysics for Rayleigh numbers between 10^3 and 10^6 under the Boussinesq approximation. Flow circulation temperature contours and heat transfer performance are analyzed for each configuration. The results indicate that enlarging the circular cavity alters the strength and structure of buoyancy-driven vortices, influencing thermal stratification and the effective heat transfer rate across the cavity. At low Rayleigh numbers conduction dominates and influence of cavity size limited whereas at higher Rayleigh numbers natural convection becomes significant and the obstacle radius strongly affects vortex dynamics and Nusselt number distribution. The findings provide insight into the coupled effect of cavity geometry and buoyancy intensity offering guidance for the design of thermal system with internal obstacles and irregular enclosures. The main objective of this paper is to find out the effect of natural convection of air within a wavy chamber using finite element methods and to investigate the influence of heated wall on free convection flow numerically.

Keywords

Heat Transfer, Natural Convection, Circular Cavity

1. Introduction

Nowadays in this world improving the heat transfer rate in different field such as electrical engineering, mechanical engineering, solar collectors, biomedical etc. is an interesting and valuable project. So a large number of researcher have tried to improve the heat transfer rate in different technique. From them Hammoodi [1] showed that synthesizes advancement in natural convection within trapezoidal cavities using mono and hybrid nanofluids emphasizing their geometric advantages for thermal management. Baytas□, A. C [2] investigated that laminar natural convection flow in trapezoidal enclosures to study the influence of the inclination angle on the flow and also the dependence of the average

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Nusselt number on the Rayleigh number. De Vahl Davis [3] investigated that natural convection in a square cavity using numerical methods to solve the Navier-Stokes and energy equations which provides benchmark solutions for validating computational fluid dynamics (CFD) models. Esfe, M. H et al [4] studied on Numerical Simulation of Thermal Radiation Influence on Natural Convection in Trapezoidal Enclosures and found that a theoretical and numerical study of natural convection in two-dimensional laminar incompressible flow in a trapezoidal enclosure in the presence of thermal radiation is conducted, motivated by energy systems applications. Mahapatra [5] studied the effect of the Rayleigh number on the Nusselt number at the top of a heat source and at the outer walls of the cavity. Hussein [6] described about numerical results for natural convection heat transfer in partially divided trapezoidal cavities investigating the effects of Rayleigh number, Prandtl number, baffle height and baffle location on the heat transfer. Teamah [7] provided a detailed understanding of the fluid flow and heat transfer characteristics inside the trapezoidal cavity under the influence of multidirectional magnetic impacts. Basak [8] examined magnetohydrodynamic natural convection heat transfer inside a trapezoidal enclosure for different Hartmann number, Rayleigh number and inclination angle of inclined walls. Lasfer [9] discussed about Numerical Study of Laminar Natural Convection in a Side-Heated Trapezoidal Cavity at Various Inclined Heated Sidewalls. This numerical study investigated laminar natural convection in a side-heated trapezoidal cavity at various inclined heated sidewalls. Hasan [10] analyzed heat flow during natural convection inside a trapezoidal porous cavity having a wavy top surface. Esfe [11] Simulated on Natural Convection Heat Transfer in a 2D Trapezoidal Enclosure where he found that numerical results are validated with previous results. The governing parameters namely Rayleigh number and Prandtl number on flow patterns isotherms as well as local Nusselt number are reported. Mahapatra [12] considered the natural convection in a 3D trapezoidal cavity filled with Al2O3-water nanofluid. The left and right vertical cavity walls were maintained at different hot and cold temperatures. Mirzaei, Amirmohammad, et al [13] investigated fluid flow and heat transfer in a circular cavity with a variable number of obstacles in each step. U Rashid et al [14] investigated the influence of nanoparticle shape on nanofluid flow within a lid-driven square cavity containing a fixed circular obstacle at its center. The top wall of the cavity is adiabatic and in motion, while the circular obstacle and the bottom wall are heated, and the remaining walls are maintained at a cold temperature. Hossain, M.S et al [15] M. S. Hossain investigated a steady, laminar, two-dimensional magnetohydrodynamic (MHD) natural convective flow within a porous medium-filled trapezoidal cavity containing a heated triangular obstacle of varying aspect ratios.

The main objective of this study is to numerically investigate the effect of varying the inner circular obstacle radius on natural convection heat transfer inside a trapezoidal cavity using COMSOL Multiphysics. The cavity is subjected to differential wall heating with the left wall maintained at high temperature and the right wall kept cold. Simulations are carried out for Rayleigh numbers ranging from 10^3 and 10^6 and obstacle radius of 0.05, 0.1 and 0.2 in order to analyze the resulting flow circulation, temperature distribution and heat transfer characteristics.

2. Physical Model

Here the Figure 1 shows the schematic problem domain of natural convection of fluid flow in Trapezoidal shape filled with air. The left wall of the figure is kept constant hot temperature (T_h) and the right wall is taken as an constant cold temperature (T_c) and the others are adiabatic. In this cavity heat source's geometry, location and size play an important role for improving the heat transfer rate. Here we thought 2D incompressible fluid flow where we also have taken a circle inside the cavity. The enclose length is considered L. The acceleration due to gravity acts in the negative y -direction. For all solid boundaries, no slip walls are considered.

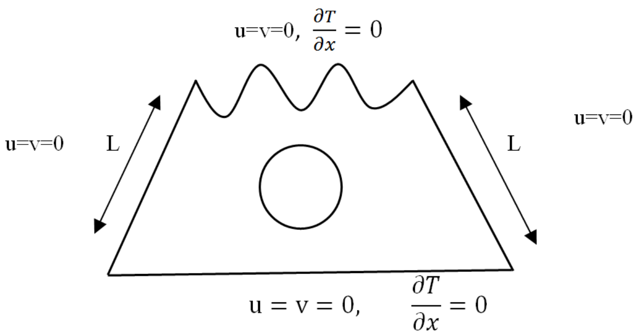

Figure 1. *The physical configuration of the cavity.*

2.1. Mathematical Formulation

The left and right wall are maintained at different constant temperatures to induce natural convection while the top and bottom walls are assumed adiabatic. The governing equations for laminar, incompressible, and steady-state natural convection based on the Navier–Stokes and energy equations with the Boussinesq approximation are solved in a dimensionless form. The Boussinesq approximation is employed to model buoyancy-driven natural convection within the trapezoidal cavity. Under this approximation density is considered constant throughout the fluid domain except in the buoyancy term of the momentum equations, where it is treated as a linear function of temperature.

$$\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial Y}\right) = 0 \quad (1)$$

$$\left(u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right) = -\frac{1}{\rho}\frac{\partial p}{\partial y} + \nu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) + g\beta(T - T_c) \quad (2)$$

$$\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=\alpha\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right) \quad (3)$$

Here some dimensionless variables are that are used in (1-3) to convert the equation into dimensionless

$$X=\frac{x}{L},\ Y=\frac{y}{L},\ U=\frac{uL}{\alpha},\ V=\frac{vL}{\alpha},\ \theta=\frac{T-T_c}{\Delta T} \quad (4)$$

2.2. Boundary Conditions of the Problem

On the left wall: $u=v=0, T=T_h$

On top(wavy) and bottom walls: $u=v=0, \frac{\partial T}{\partial x}=0$

On right wall: $u=v=0, T=T_c$

Putting the boundary condition and dimensionless variables into (1-3) we get the dimensionless equation:

$$\left(\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}\right)=0 \quad (5)$$

$$\left(U\frac{\partial U}{\partial X}+V\frac{\partial U}{\partial Y}\right)=-\frac{\partial P}{\partial X}+\Pr\left(\frac{\partial^2 U}{\partial X^2}+\frac{\partial^2 U}{\partial Y^2}\right) \quad (6)$$

$$\left(U\frac{\partial V}{\partial X}+V\frac{\partial V}{\partial Y}\right)=-\frac{\partial P}{\partial Y}+\Pr\left(\frac{\partial^2 V}{\partial X^2}+\frac{\partial^2 V}{\partial Y^2}\right)+Ra\Pr\theta \quad (7)$$

Now the dimensionless quantity is

$$\Pr=\frac{\nu}{\alpha},\ Ra=\frac{g\beta L^3\Delta T}{\alpha\nu}$$

2.3. Nusselt Number

The rate of heat transfer by convection in an encloser is calculated by the Nusselt number (Nu) and the local Nusselt number (Nu_L) is calculated at the fin walls by

$$Nu_L=\frac{hH}{k}=\left(-\frac{\partial\theta}{\partial n}\right)H$$

Where h is the heat transfer coefficient, n is the normal direction on the surface (a negative sign refers to the heat transfer from the hot wall to the field). The dimensionless normal temperature gradient can be written as

$$\frac{\partial\theta}{\partial n}=\frac{1}{H}\sqrt{\left(\frac{\partial\theta}{\partial X}\right)^2+\left(\frac{\partial\theta}{\partial Y}\right)^2}$$

Therefore, the average Nusselt number for the heated fin of the square cavity surface is defined as:

$$Nu_{av}=\frac{1}{S}\int_0^S Nu\,ds.$$

Where, S is the total chord length of the heated fin and s is the coordinate along the fin surface.

3. Methodology

The governing equations of incompressible laminar flow and heat transfer were solved using the finite element method under the Boussinesq approximation. A trapezoidal cavity with a centrally placed circular obstacle is considered where the left and right walls are maintained at hot and cold temperature respectively. And the upper wavy wall, based wall and obstacle are considered as adiabatic. Simulations are performed by COMSOL multiphysics for Rayleigh numbers between 10^3 and 10^6 and obstacle radius of 0.05, 0.1 and 0.2 times the cavity length. After discretization of the governing equations the system becomes nonlinear. To obtain accurate solutions the Newton-Raphson iteration method is employed. In this approach the solution is update iteratively until the residuals fall below the prescribed convergence tolerance. This method ensures numerical stability and accuracy for the wide range of Rayleigh numbers considered. The computational mesh is refined near solid boundaries to capture thermal and velocity gradients and grid independence tests are carried out. Key outputs included temperature contours, streamline patterns and local and average nusselt numbers to evaluate the influence of obstacle size on natural convection heat transfer.

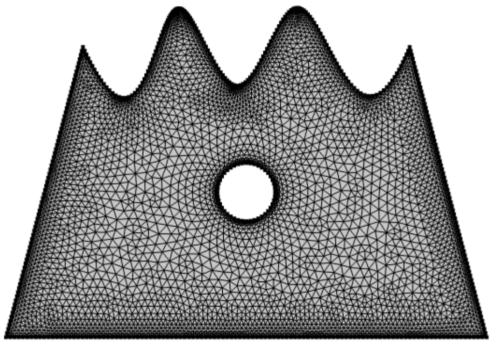

Figure 2. *The mesh of the shape.*

3.1. Validation and Grid Independence Test

For the Grid Independent Test we used $Ra=10^3$.Near the boundary layer we used Fine for the first case, Extra fine mesh for second cases and extremely fine for last case. The test results are given in the table blew:

Table 1. *The mesh validation.*

Name	Number of Mesh	Average Nusselt Number	Error
Fine	4567	0.90942	0.003
Finer	8540	0.90639	0.001

Name	Number of Mesh	Average Nusselt Number	Error
Extra Fine	22205	0.90517	-

Here we use finer mesh for the computational work.

To prove the reliability of the model we have to compare it with a published literature. The simulation result of this present model has been compared with De Vahl Davis [3] paper.

We compare the result for Ra= $10^3, 10^4, 10^5$ by streamlines, isotherms and Average Nusselt number. Here the result is described.

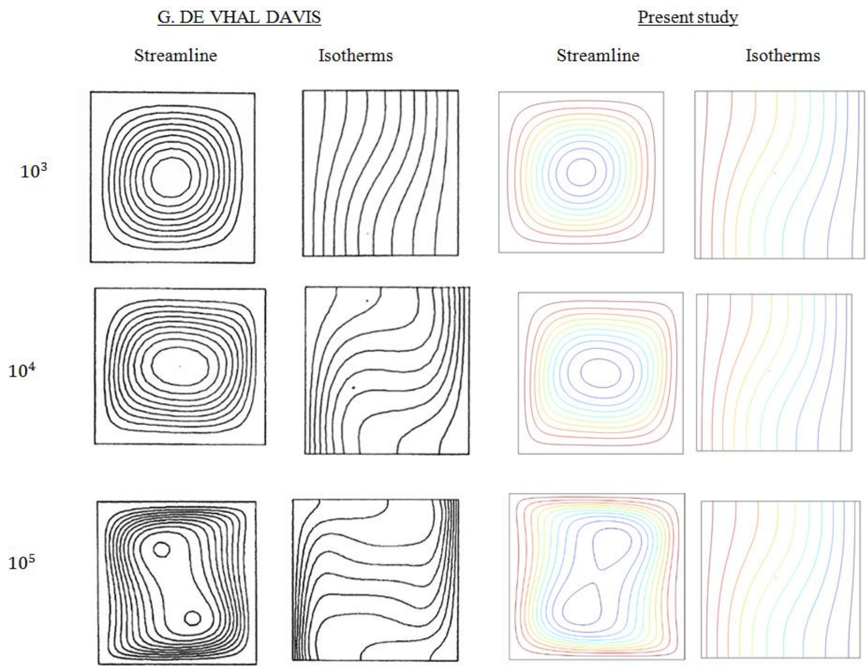

Figure 3. Comparison between Published work vs present studies.

3.2. Results and Discussion of Numerical Simulation

3.2.1. Velocity Distribution

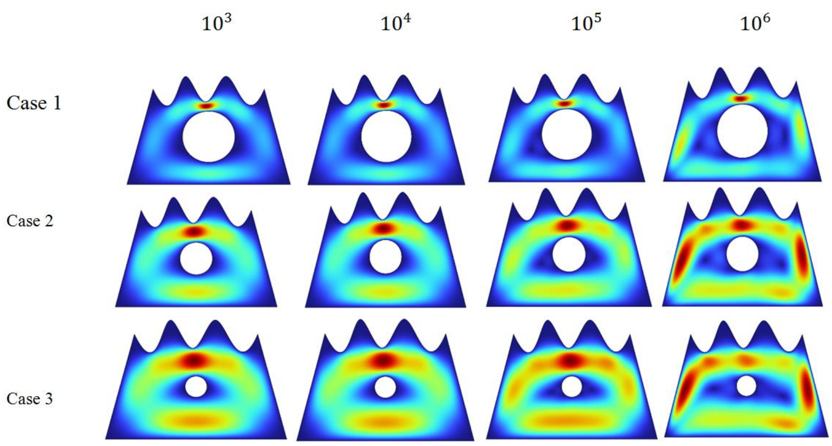

Figure 4. Velocity distribution for different Rayleigh number and inner circle radius.

1) *Low Ra*$(10^3 - 10^4)$: The velocity field shows weak fluid motion across all cases as thermal conduction is dominant. Only small recirculation form around the obstacle.

2) *Moderate Ra*(10^5): The velocity magnitude increases significantly. Stronger circulation cells appear especially in case 2 and case 3 where the obstacle forces fluid to accelerate around it.
3) *High Ra*(10^6): Velocity contours indicate strong upward.
4) and downward jets near the hot and cold walls. Case 3 exhibits the highest intensity as the longer obstacle constrains the flow and enhances circulation around it leading to stronger velocity gradients.

3.2.2. Streamline Distribution

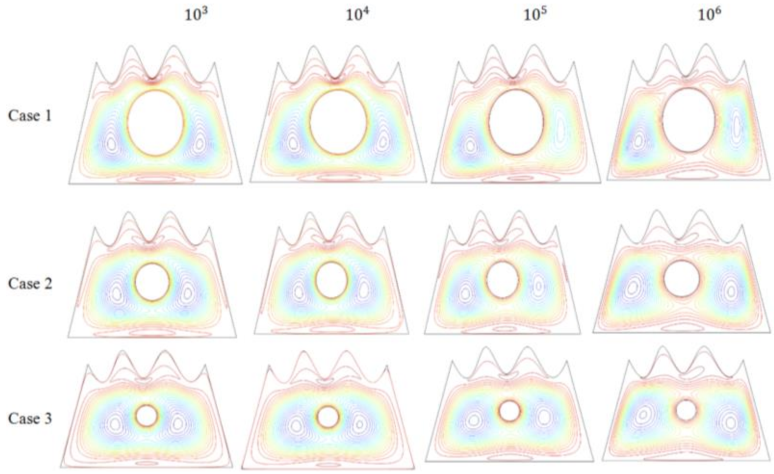

Figure 5. *Streamline distribution for different Rayleigh number and inner circle radius.*

Case 1(large obstacle): The larger radius significantly modifies the streamline pattern. At higher Ra strong recirculation zones are formed around the obstacle promoting more vigorous convection. This case demonstrates the most complex flow field.

Case 2(medium obstacle): The streamlines are more distorted compared to case 3. Secondary circulation cells appear around the obstacle especially at Ra=10^5 and 10^6 including enhanced convective mixing.

Case 3(small obstacle): Streamline show a single dominant circulation cell at lower Ra which becomes more complex at Ra=10^6. The flow bypasses the small obstacle easily causing moderate disturbance.

3.2.3. Isotherms Distribution

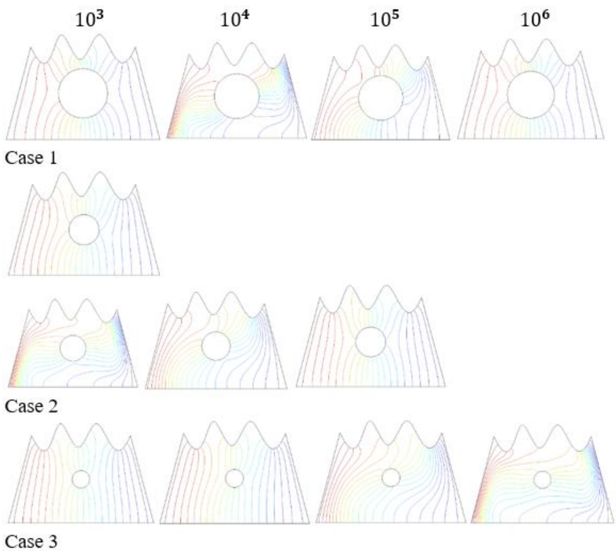

Figure 6. *Isotherms distribution for different Rayleigh number and inner circle radius.*

Low Ra$(10^3 - 10^4)$:Isotherms are nearly vertical and evenly spaced indicating conduction dominated heat transfer. The presence of the obstacle only slightly disturbs the thermal field.

Moderate Ra(10^5):Isotherms begin to bend and cluster near the hot and cold walls showing the increasing influence of convection. Around the obstacle distortion of isotherms becomes evident particularly in cases 2 and 3.

High Ra(10^6): Strong convection leads to significant isotherm clustering near the vertical walls indicating intense heat transfer. In case 3 the distortion is maximum as the large obstacle enhances circulation and thermal mixing.

Now the average Nusselt number distribution of this three cases between the Rayleigh number (10^3-10^6):

Table 2. *The table of all Average Nusselt number varying Rayleigh number and obstacle size.*

Rayleigh number(Ra)	Nu_{avg} for 10^3	Nu_{avg} for10^4	Nu_{avg} for10^5	Nu_{avg} for10^6
Case 1	0.63801	0.64288	0.95459	3.4212
Case 2	0.82578	0.84208	1.4535	4.0619
Case 3	0.90639	0.92582	1.5867	4.1153

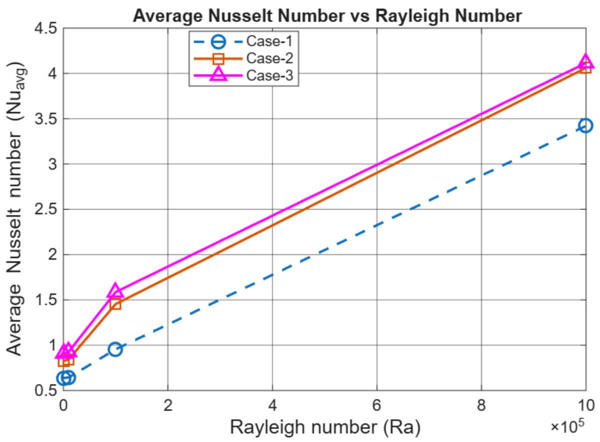

Figure 7. *Average Nusselt number for the different cases.*

4. Conclusion

This numerical study investigated the influence of a circular obstacle's radius on natural convection heat transfer within a trapezoidal cavity across a range of Rayleigh numbers(Ra) from $10^3 - 10^6$. Three distinct obstacle radius were analyzed: Case 1(R=0.08), Case 2(R=0.05), Case 3(R=0.03). The findings are summarized as follows:

1) *Enhanced Heat Transfer with Smaller Obstacles:* The results demonstrate a clear correlation between the obstacle size and average Nusselt number (Nu_avg), the key metric for heat transfer performance. For all Rayleigh number studied Case 3 consistently yielded the highest Nu_avg followed by Case 2 and then Case 1. This indicates that a smaller obstacle presents less resistance to the convective flow allowing for more efficient thermal plumes as visually confirmed by the isotherm distributions in Figure 6.
2) *Rayleigh Number Dominance at High Intensities:* The impact of the obstacle size is most pronounced in the conduction dominated (Ra=$10^3 - 10^4$) and early convection regimes (Ra=10^5) where the relative differences between cases are significant. However at the highest Rayleigh number (10^6) the Nu_avg values for all three cases converge to a much closer range. This suggests that while the obstacle size is critical design parameter its relative influence is mitigated by the overwhelming strength of buoyancy driven flows at very high Ra where the flow dynamics become increasingly turbulent.

In conclusion this work establishes that minimizing the size of an internal obstacle in an effective strategy for enhancing natural convection heat transfer. Here we can see that for small circle the velocity is increasing because there have much place to spread the temparature as case 3. On the Other hand there have low velocity when the circle radius will increasing as case 1. The optimal design for a system operating across a wide range of thermal intensities would incorporate the smallest feasible obstacle size to minimize flow obstruction thereby leveraging the full potential of buoyancy driven convection.

Abbreviations

CFD	Computational Fluid Dynamics
g	Acceleration of Gravitation
k	Thermal Conductivity
L	Cavity Length
Nu_{av}	Average Nusselt Number
p	Dimensional Pressure

P	Non-dimensional Pressure
Pr	Prandtl Number
Ra	Rayleigh Number
T	Fluid Temperature
u, v	Dimensional Velocity Component
U, V	Non-dimensional Velocity Component
x, y	Dimensional Coordinates
X, Y	Non-dimensional Coordinates
α	Thermal Diffusivity
β	Thermal Expansion Coefficient
ψ	Stream Function
θ	Non-dimensional Temperature
ρ	Density
σ	Electric Conductivity

Author Contributions

Abdullah Ahmed Foisal: Formal Analysis, Methodology, Supervision, Validation, Writing – original draft, Writing – review & editing

Akimul Islam Saikat: Conceptualization, Formal Analysis, Investigation, Methodology, Software, Validation, Visualization, Writing – original draft, Writing – review & editing

Azam Khan Muzahid: Formal Analysis, Validation, Writing – original draft, Writing – review & editing

Mainul Islam: Formal Analysis, Validation, Writing – original draft, Writing – review & editing

Conflicts of Interest

The authors declare no conflicts of interest.

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