

Research Article

Heat-generating Obstacle of Nanofluid Flow in a Rhombus-shaped Enclosure Filled with Porous Media

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Abstract

The main goal of the current research is to investigate the flow of an innovative nanofluid technology with a heat-generating obstruction in a rhombus-shaped the enclosure that the is filled with porous media. The current research addresses the implications of media with pores on the dimensionless Richardson and Darcy numbers for the heat-generating obstacle field. Numerical solutions to the problem have been found utilising the Galerkin weighted residual consider. The current study investigates the consequences of the Richardson and Darcy numbers on streamline equilibrium temperatures, devoid of dimension temperature, velocity characteristics, average fluid temperature, and Nusselt experiment numbers. The outcomes demonstrate that both components have significant implications on streamlines and equilibrium temperatures. Additionally, it is readily apparent that the Darcy number is a significant control parameter for heat transfer in fluid flow through the porous material that makes up an enclosure. A linear relationship for the average number obtained from Nusselt has been demonstrated according to various Darcy and Richardson principles. When there is an overwhelming concurrence amongst the results of the present investigation and previously published research, it has been validated.

Keywords

Nanofluid, Porous Medium, Heat Generating Obstacle, FEM, CFD

1. Introduction

In order to more fully comprehend system performance, computational fluid dynamics performs mathematical computations to provide estimates real-world systems. In these engineers to achieve results with exceptional precision and physical accuracy, CFD machine learning employing finite grids is needed. The practical significance of CFD goes transcend automated engineering procedures for design by encompassing understanding complex fluids through the

implementation of comprehensive Navier-Stokes equation solutions in place of experimental investigation. Nowadays, CFD has been used for modeling the movement of nanofluid in a rhombus-shaped enclosure with porous medium and a heat-generating impediment. Enclosures containing electrically conducting fluids are essential components for numerous architectural and meteorological systems, including solar collectors, astronomy, biology, ventilation, heating

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and cooling, elevators, escalators, and building heating and cooling exchangers. These applications have been thoroughly investigated and discussed in the literature due to Nield & Bejan [1] and Ingham & Pop [2], and their implications were additionally briefly explained due to Xuan and Li [3], Brinkman [4], Zienkiewicz [5], Maxwell-Garnett [6], and Wang [7]. Munshi and colleagues have carried out several types of studies with regard to mixed convection heat transfer, that include one in and these Munshi et al. [8] investigated the numerical simulation of mixed convection heat transfer of nanofluid in a lid-driven porous medium square enclosure. In an additional study, Munshi [9] investigated hydrodynamic convection and conduction in a square cavity driven by a lid and incorporating an elliptic heated block with a corner heater. Saha et al. [10] investigated the transmission of heat by convection and conduction in a hollow space driven by the lid with a wavy bottom surface. The authors of Munshi et al. [11] addressed the convection and condensation optimization in a lid-driven permeable square chamber with linear side walls and an inner elliptic isothermal block. Furthermore, Hussein [12] conducted a study on convection and conduction in a square lid-driven enclosure with an eccentric circular structure. Basak et al. looked into lid-driven convection and conduction throughout a porous square chamber with a continuously heated wall [13]. The results they obtained demonstrate that forced convection dominates at a Reynolds number of 100 for a continuously heated left wall and a chilled right wall. The combined convection of nanofluids in a square enclosure with a triangular heated block powered by a lid has been investigated by Boulahia et al. [14]. They determined that whenever the volume proportion of nanoparticles increases and the Richardson number decreases, the rate of heat transfer increases. Garoosi et al. [15] addressed mixed and natural convection of nanofluids in a square cavity in an entirely distinct investigation.

Through simulating nanofluid flow in a rhombus-shaped enclosure with a heat generating obstacle that contains porous media, the current study seeks to address a gap in the literature through addressing the issue of mixed convection heat transfer in a lid-driven enclosure using different characteristics.

This chapter's next portion has been structured as follows. Section 1 functions as an introduction, whereas Section 2 presents the physical configurations that are relevant to the ongoing research. The mathematical model that has the greatest potential for this investigation is subsequently investigated in section 3, including covers the governing equations as well as the boundary conditions. Sections 4 and 5 provided an explanation concerning the numerical scheme employed in the current investigation.

2. Physical Configuration

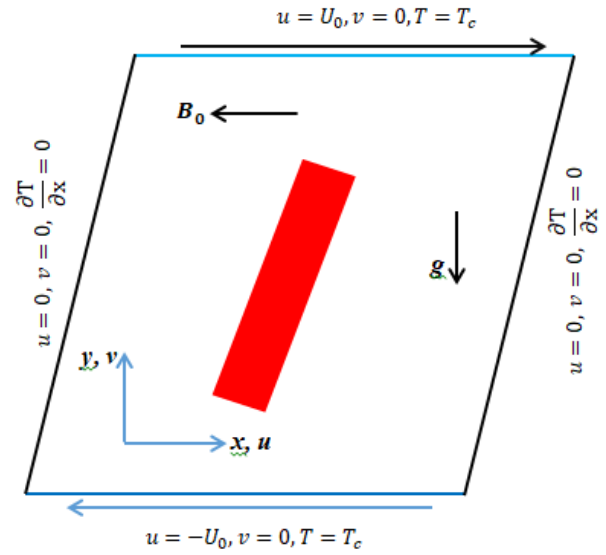


Figure 1. Schematics demonstrating exactly the finished model is now arranged.

The schematic diagram for simulating nanofluid flow inside a rhombus-shaped enclosure with a porous medium and a heat-generating obstruction is displayed in Figure 1. The enclosure's top and bottom walls have cold, and the lid moves from right to left and left to right, respectively. The thermal barrier is positioned beyond the enclosure's borders while therefore the partition walls are adiabatic.

3. Mathematical Formulation

The following are the mathematical equations:

$$\nabla \cdot \underline{q} = 0$$

$$\rho_{nf} (\underline{q} \cdot \nabla \underline{u}) = -\frac{\partial p}{\partial x} + \mu_{nf} \nabla^2 u - \frac{(\rho\beta)_{nf}}{k} v u$$

$$\rho_{nf} (\underline{q} \cdot \nabla v) = -\frac{\partial p}{\partial y} + \mu_{nf} \nabla^2 v - \frac{(\rho\beta)_{nf}}{k} v v + (\rho\beta)_{nf} g \beta (T - T_c) - \sigma_{nf} B_0^2 v$$

$$\underline{q} \cdot \nabla T = \alpha_{nf} \nabla^2 T$$

$$\nabla^2 T_s + q = 0$$

The nanofluid possesses effective density and thermal diffusivity:

$$\rho_{nf} = (1 - \varphi) \rho_f + \varphi \rho_s$$

$$\alpha_{nf} = \frac{k_{nf}}{(\rho c_p)_{nf}}$$

The nanofluid has heat capacity and thermal expansion coefficients:

$$(\rho c_p)_{nf} = (1 - \phi)(\rho c_p)_f + \phi(\rho c_p)_s$$

$$(\rho \beta)_{nf} = (1 - \phi)(\rho \beta)_f + \phi(\rho \beta)_s$$

Brinkman [Brinkman (4)] as follows:

$$\mu_{nf} = \frac{\mu_f}{(1 - \phi)^{2.5}}$$

According to the Maxwell model [6], the nanofluid's effective temperature conductivity corresponds to the follows:

$$k_{nf} = \frac{k_s + 2k_f - 2\phi(k_f - k_s)}{k_s + 2k_f + \phi(k_f - k_s)} \times k_f$$

Properties of Nanofluid

Thermo-physical properties of nanofluid (H_2O, Cu) for heat capacitance (4179, 385) Density (997.1, 8933), Thermal conductivity (0.613, 401), Thermal expansion coefficient (2.1×10^{-4} , 1.6×10^{-5}) and dynamic viscosity (0.001003).

3.1. Dimensional Form of Boundary Conditions

No-slip boundary criterion regarding velocity ($u = 0, v = 0$) is applied on all the walls, except the top and bottom lids. On the top of the lid ($u = U, v = 0$) providing a cold temperature, while on the bottom lid ($u = -U, v = 0$) with cold temperature is applied. The right wall and left wall are adiabatic and also, heat generating heated obstacle inside the enclosure.

3.2. Dimensionless Analysis

To derive the dimensionless governing equations from the dimensional forms mentioned above, the following scaling parameters are applied,

$$X = \frac{x}{L}, Y = \frac{y}{L}, U = \frac{u}{U_0 \alpha_f}, V = \frac{v}{U_0 \alpha_f}, P = \frac{p}{\rho_{nf} U_0^2},$$

$$\theta = \frac{T - T_c}{T_h - T_c}$$

The dimensionless equations are given in the following form,

$$\nabla \cdot \underline{q} = 0 \text{ where } \underline{q} = U \underline{i} + V \underline{j}$$

$$\underline{q} \cdot \nabla U = -\frac{\partial P}{\partial X} + \frac{1}{Re} \frac{\mu_{nf}}{\rho_{nf} \nu_f} \frac{1}{(1 - \phi)^{2.5}} \nabla^2 U - \frac{(\rho \beta)_{nf}}{\rho_{nf} \beta_f} \frac{1}{Re Da} U$$

$$\underline{q} \cdot \nabla V = -\frac{\partial P}{\partial Y} + \frac{1}{Re} \frac{\mu_{nf}}{\rho_{nf} \nu_f} \frac{1}{(1 - \phi)^{2.5}} \nabla^2 V - \frac{(\rho \beta)_{nf}}{\rho_{nf} \beta_f} \frac{1}{Re Da} V + \frac{(\rho \beta)_{nf}}{\rho_{nf} \beta_f} Ri \theta - \frac{Ha^2}{Re} V$$

$$\underline{q} \cdot \nabla \theta = \frac{\alpha_{nf}}{\alpha_f} \frac{1}{Re Pr} \nabla^2 \theta$$

$$\nabla^2 \theta_s + Q = 0$$

The simulation undertaken in the current study involves the following dimensionless governing parameters:

$$Pr = \frac{\nu_f}{\alpha_f}, Da = \frac{K}{L^2}, Re = \frac{U_0 L}{\nu_f}, Gr = \frac{g \beta_f (T_h - T_c) L^3}{\nu_f^2},$$

$$Ri = \frac{Gr}{Re^2}, Ha = B_0 L \sqrt{\frac{\sigma_{nf}}{\rho_{nf} \nu_f}}$$

The dimensionless boundary conditions:

No-slip boundary criterion regarding velocity ($U = 0, V = 0$) is applied on all the walls, except the top and bottom lids. On the top of the lid ($U = 1, V = 0$) with cold temperature ($\theta = 0$) is applied, while on the bottom lid ($U' = -1, V' = 0$) with cold temperature ($\theta = 0$) is applied. The right wall and left wall are adiabatic. Also, fluid solid interface inside the enclosure.

Local and Nusselt numbers average: The local Nusselt number on the left wall can be defined as

$$Nu_{local} = \frac{k_{nf}}{K_f} \cdot \frac{\partial \theta}{\partial X} \Big|_{X=0}$$

To assess the overall rate of heat transmission, the average Nusselt number along the heated wall of the enclosure is taken into account. It is defined as $\int_0^1 Nu_{Local} dY$.

4. Program Validation

The average Nusselt number for each of the grid can be seen within Table 1. The grid with 6743 nodes and 12270 elements obviously provided an appropriate outcome for the numerical study at hand, and as may be observed from the table.

Table 1. Grid sensitivity check at $Pr = 0.71$, $Ri = 1$ and $Ha = 50$.

Nodes	831	1113	1724	6742	24135
Elements	1600	2145	3327	12271	48469
Nu_{av}	8.64277	8.66748	8.67728	8.70373	8.70842
Time (s)	9	12	13	19	34

Figure 2 presents an examination of comparisons between the graphical solutions of streamlines and isotherms. The figures indicate a strong correspondence between the two results.

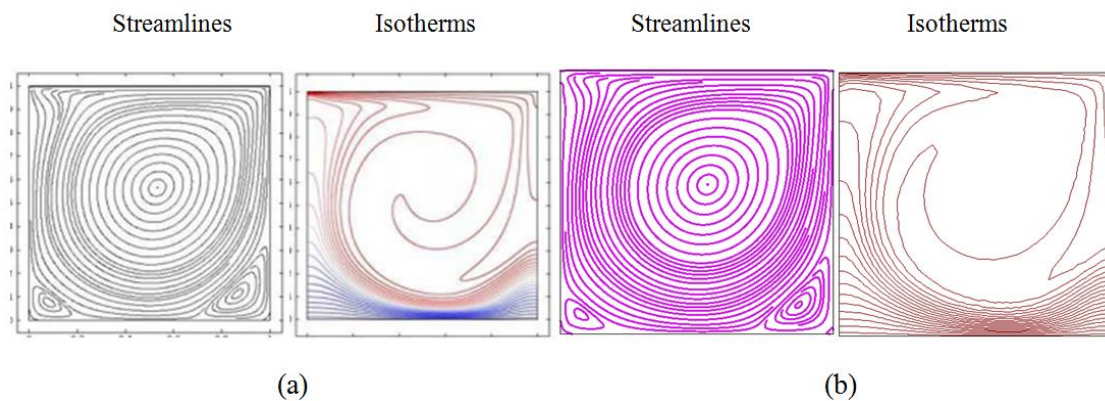


Figure 2. Comparison of streamlines as well as isotherms for the computational solution of (a) Shaha et al. [10] $Ra = 10^5$, $Re = 100$, $Ha = 50$ and (b) Present Study.

5. Results and Discussion

The mathematical and graphically outcomes obtained from simulating the flow of nanofluid in a rhombus-shaped enclosure with porous media and a heat-generating obstacle are demonstrated in this section. The simulations are divided into two parts. Firstly, the impact of Darcy numbers ($Da = 1e^{-5}, 1e^{-4}, 1e^{-3}, 1e^{-2}$) is examined. Secondly, the calculations are conducted for the Richardson numbers ($Ri = 0.1, 1, 5, 10$). The results of these simulations are discussed in the following two subsections.

5.1. Variation of Darcy Number

The changes in streamlines and isotherms inside the heat-generating obstacle according to various Darcy number values are displayed in Figures 3 as well as 4, while the fixed $Ri = 1$, $Ha = 50$, $Pr = 6.2$, $Q = 1$ and $\phi = 5\%$ respectively. The porous medium's permeability as well as the Darcy number have been completely correlated, and it has been set to a range $1e^{-5} - 1e^{-2}$. Generally, when the Darcy number is low ($Da = 1e^{-5}$), the streamlines flow near the top and

bottom of the enclosure. Also, Darcy number increases to $Da = 1e^{-2}$, the strength of the flow are increased. Fluids rise up and generate two vortices adjacent to the enclosure's top and bottom walls, as could have been predicted considering the moving walls. An additional heat transmission may come through mixed convection as a result of this stronger circulation, which concentrations the temperature contours around the heat-generating obstruction.

Figure 4 demonstrates the way Darcy affects the temperature distribution and flow patterns inside an enclosure that includes a heat-generating obstruction. Darcy raises the flow resistance in the presence of a porous medium, and when Darcy increases from $Da = 1e^{-5}$ - $1e^{-3}$, the porous medium's permeability increases as well, which results in a significant rise in upward flow. Additionally, increased circulation results from increasing Darcy values. The shear force that operates in the same direction as the buoyancy force at the moving walls causes the isotherm contours to shift towards the center of the enclosure as Darcy exceeds $1e^{-2}$. A highly permeable porous substance and a mixed convection mechanism are used to accomplish the high heat transfer the approach.

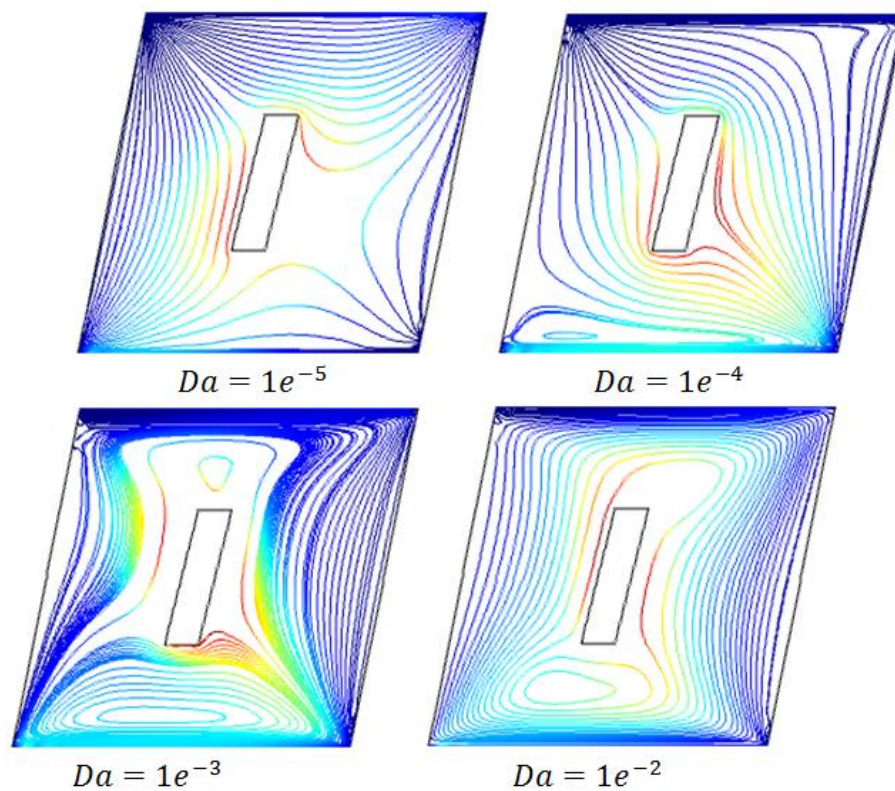


Figure 3. Streamlines to accommodate different Darcy values for numbers $Da = 1e^{-5} - 1e^{-2}$ when $Ri = 1, Ha = 50, Pr = 6.2, Q = 1$ and $\phi = 5\%$.

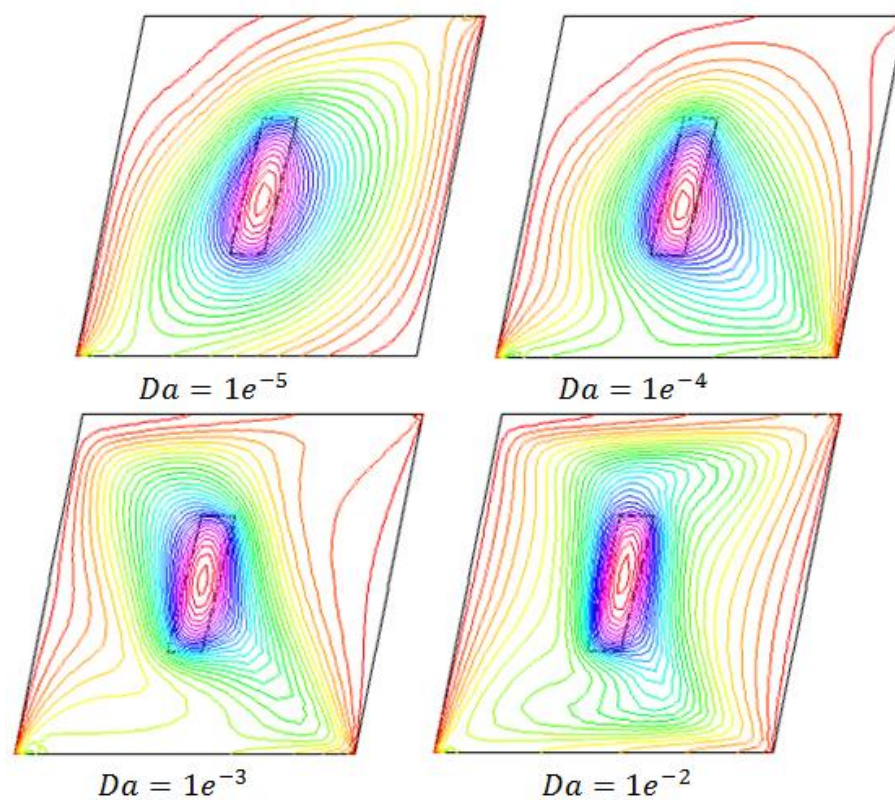


Figure 4. Isotherms to accommodate different Darcy values for numbers $Da = 1e^{-5} - 1e^{-2}$ when $Ri = 1, Ha = 50, Pr = 6.2, Q = 1$ and $\phi = 5\%$.

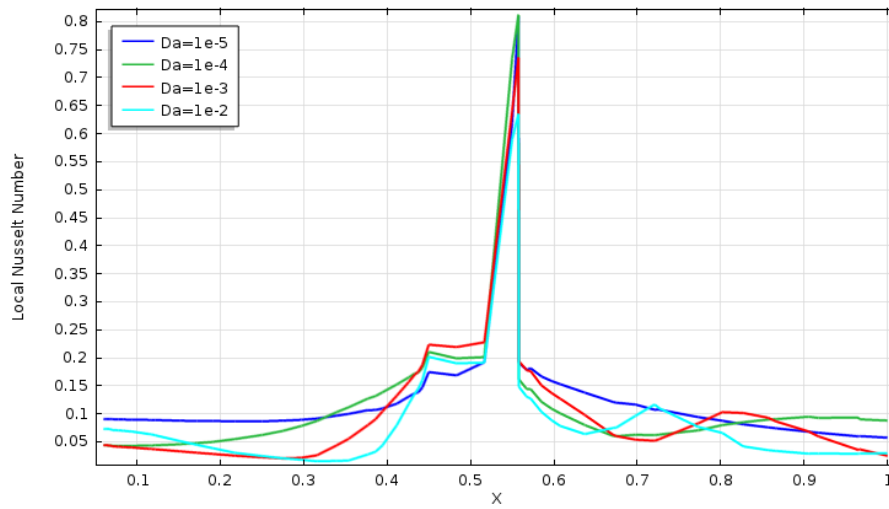


Figure 5. Local number Nusselt modification based on different Darcy number values along the enclosure's X -axis $Da = 1e^{-5} - 1e^{-2}$ when $Ri = 1, Ha = 50, Pr = 6.2, Q = 1$ and $\phi = 5\%$.

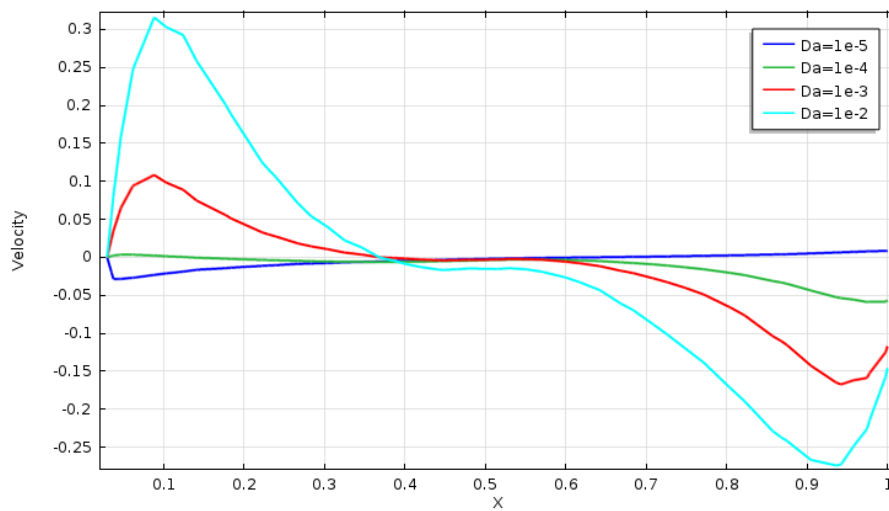


Figure 6. Modification of profiles of velocity according to different Darcy numbers along the enclosure's X -axis $Da = 1e^{-5} - 1e^{-2}$ when $Ri = 1, Ha = 50, Pr = 6.2, Q = 1$ and $\phi = 5\%$.

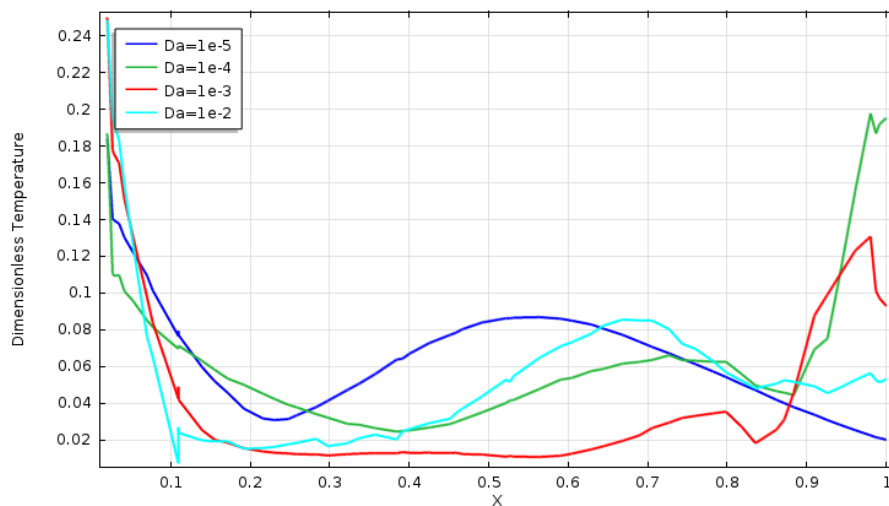


Figure 7. Changes in the enclosure's dimensionless temperature along the X -axis according to different Darcy numbers $Da = 1e^{-5} - 1e^{-2}$ when $Ri = 1, Ha = 50, Pr = 6.2, Q = 1$ and $\phi = 5\%$.

The Figure shows how the local number of Nusselt changes with according different Darcy numbers, and it indicates that the heat transfer increases as the buoyancy ratio and Darcy number increase. Notably, the graph reveals that the heat transfer reaches its peak at $X = 0.55$, regardless of the specific parameters. This is because $X = 0.55$ corresponds to the stagnation point, which is where the velocity of the fluid becomes zero, leading to a maximum heat transfer rate at that location. In summary, the graph demonstrates that the local Nusselt number is influenced by various factors, but the maximum heat transfer occurs at the stagnation point, which is at $X = 0.55$ in this case. Figure 6 illustrates how the velocity components vary along a vertical line for different values of the Darcy number. The graph shows that as the Darcy number increases, both the maximum and minimum values of the velocity also increase. In other words, higher Darcy numbers lead to higher maximum and minimum velocities along the vertical line. This finding indicates that the Darcy number has a significant impact on the velocity distribution of the fluid.

The dimensionless variations in temperature along the enclosure's X-axis for different Darcy numbers can be observed within Figure 7. The graph reveals that as the number of Darcy increases, the absolute values of both the maximum and minimum temperatures also increase. However, it is worth noting that the rate of change in the temperature slows down with increasing Darcy numbers. In other words, higher Darcy numbers result in slower changes in temperature along

the X-axis of the enclosure. This finding highlights the influence of Darcy numbers on the temperature distribution within the fluid, and it suggests that the higher Darcy numbers lead to a more gradual change in temperature along the X-axis.

5.2. Heat Transfer Rates

The numerical simulation of nanofluid flow in a rhombus-shaped enclosure with a heat-generating impediment and a porous medium is demonstrated in this section. For varying Richardson numbers and heat transfer rates, Figure 8 demonstrates the average number Nusselt as a function of Darcy number. The graph reveals that the heat transfer rate decreases with increasing Richardson number, indicating that higher buoyancy ratios lead to a reduction in the heat transfer rate. Figure 9 shows the variation of the average fluid temperature with Darcy number for different Richardson numbers. The graph indicates that the average fluid temperature increases linearly with the buoyancy ratio, which is represented by the Richardson number. Specifically, as the Richardson number increases, the average fluid temperature inside the enclosure also increases. These findings demonstrate the influence of the buoyancy ratio and Darcy number on the fluid flow and heat transfer characteristics, and they provide insight into how changes in these parameters affect the systems overall behavior.

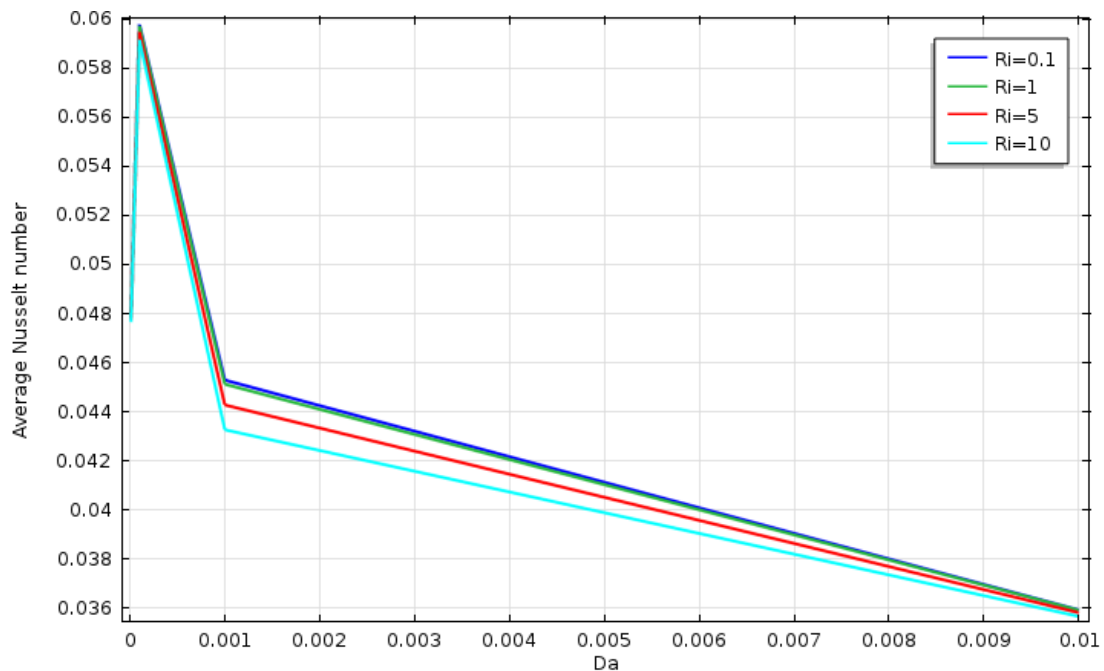


Figure 8. Difference between the averages Darcy number as well as Nusselt number $Da = 1e^{-5} - 1e^{-2}$ using different Richardson values for numbers when $\phi = 0.5$, $Ha = 50$, $Pr = 6.2$ and $Q = 1$.

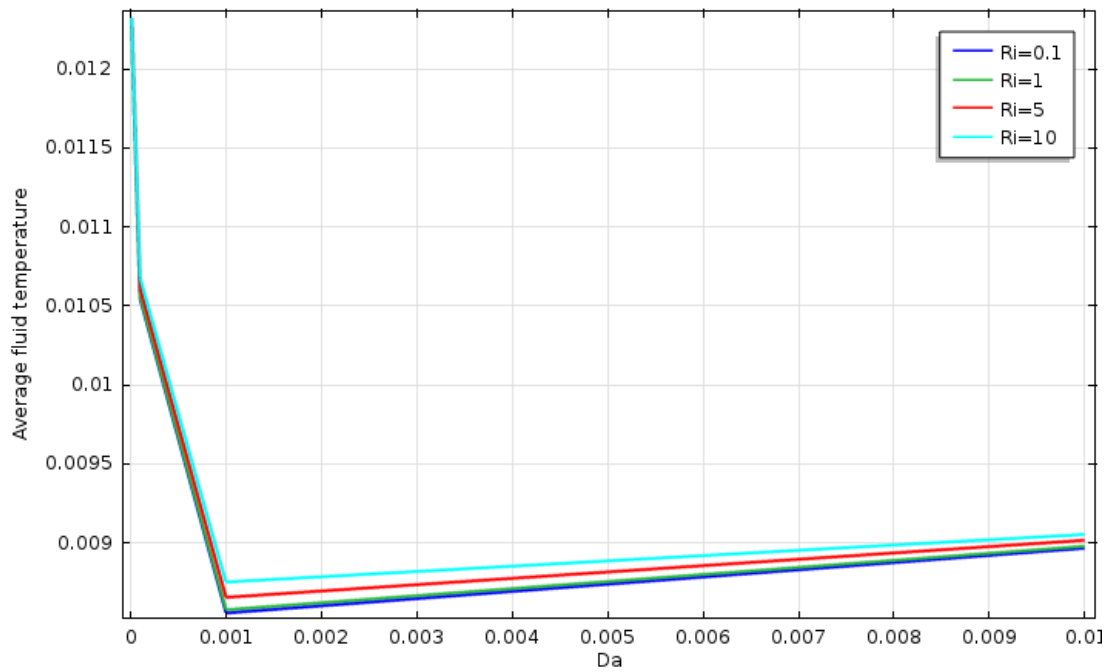


Figure 9. Average fluid temperature variation within connection with the Darcy number $Da = 1e^{-5} - 1e^{-2}$ using different Richardson values for numbers when $\phi = 0.5, Ha = 50, Pr = 6.2$ and $Q = 1$.

5.3. Variation of Richardson Number

To investigate the behavior of nanofluid flow in a rhombus-shaped enclosure filled with porous media and a heat-generating obstacle, the Richardson number is considered as a crucial parameter that represents the relative significance of natural convection caused by buoyancy versus forced convection due to the lids motion. In this section, simulations were conducted for different Richardson numbers to examine their impact on the flow structure and heat transfer inside the enclosure. It is worth noting that the parameters such as porosity, Hartmann number, Prandtl number and heat generating rate are kept constant in this section at values of 0.5, 50, 6.2 and 1 respectively. By varying the Richardson number, the researchers were able to analyze its effects on the nanofluid flow behavior and heat transfer rate within the enclosure.

Figures 10 and 11 display the variation of streamlines and isotherms inside a heated obstacle for different Richardson numbers ranging from 0.1 to 10. The results show that the position of the obstacle significantly affects the center of the

vortex and the formation of a clockwise rotating vortex inside the enclosure due to shear forces. When the obstacle is located in the middle of the enclosure, the clockwise rotating vortex near the top and bottom walls is stronger due to the influence of the moving lids compared to the buoyancy force. Furthermore, the fluid flow underneath the heat generating obstacle is much higher than that of the upper region, which is influenced by the lids motion. As the Richardson number increases, the strength of buoyancy also increases, making the influence of the moving lid even stronger.

Figure 11 demonstrates that condition-dominated heat transfer is observed from the isotherms. The isotherms appear parallel to the side walls for the lowest value of Richardson number, indicating low heat transfer through convection. However, as the Richardson number increases, the isotherms bend more near the heat generating obstacle, indicating increased heat transfer through convection. These findings shed light on the complex interaction between the flow and heat transfer characteristics inside the enclosure, highlighting the important role of the Richardson number in controlling the heat transfer rate.

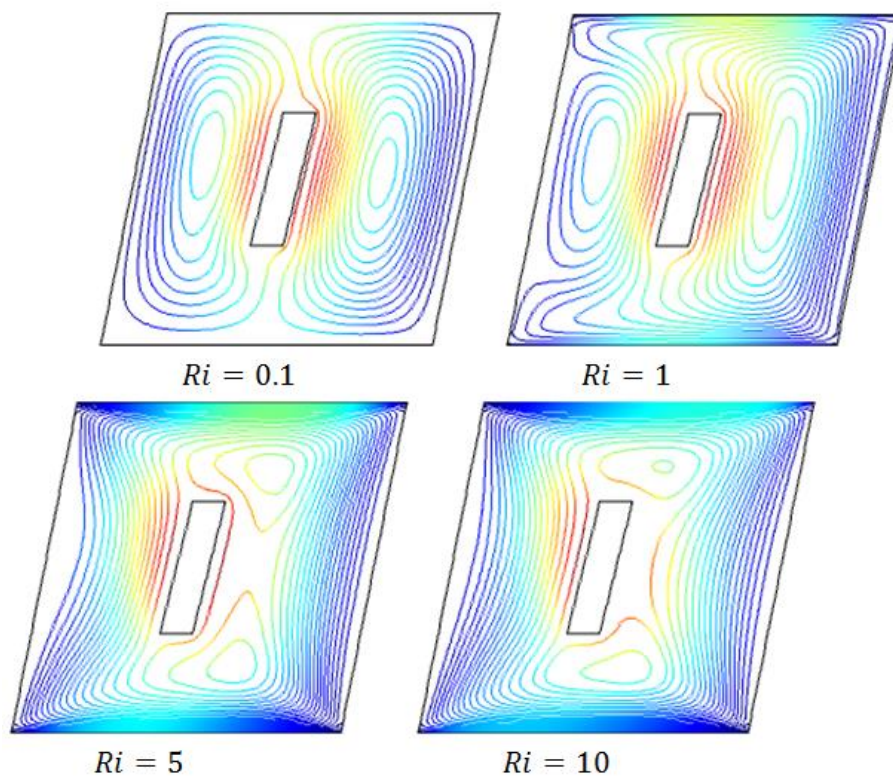


Figure 10. Streamlines at different values of Richardson numbers $Ri = 0.1, 1, 5, 10$ when $Da = 1e^{-5}$, $Pr = 6.2$, $Ha = 50$, $Q = 1$ and $\varphi = 5\%$.

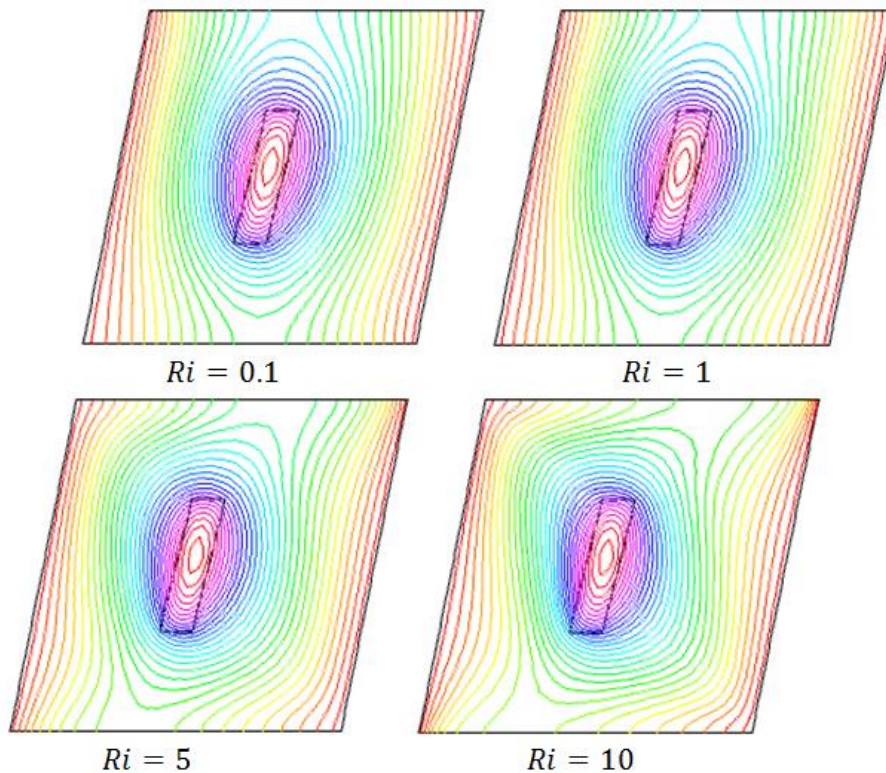


Figure 11. Employing various Richardson values for values in numbers $Ri = 0.1, 1, 5, 10$ when $Da = 1e^{-5}$, $Ha = 50$, $Pr = 6.2$, $Q = 1$ and $\varphi = 5\%$.

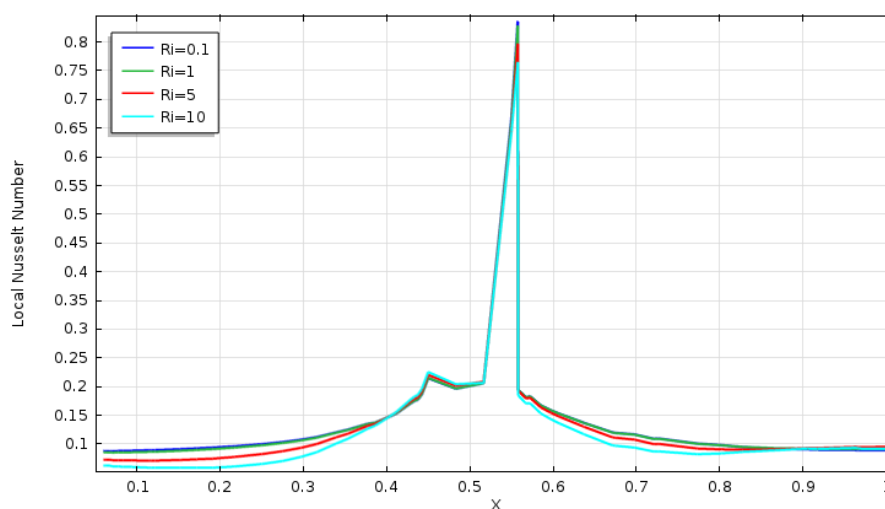


Figure 12. Local Nusselt value variation along the enclosure's X-axis according to different Richardson values for numbers $Ri = 0.1, 1, 5, 10$ when $Da = 1e^{-5}$, $Ha = 50$, $Pr = 6.2$, $Q = 1$ and $\varphi = 5\%$.

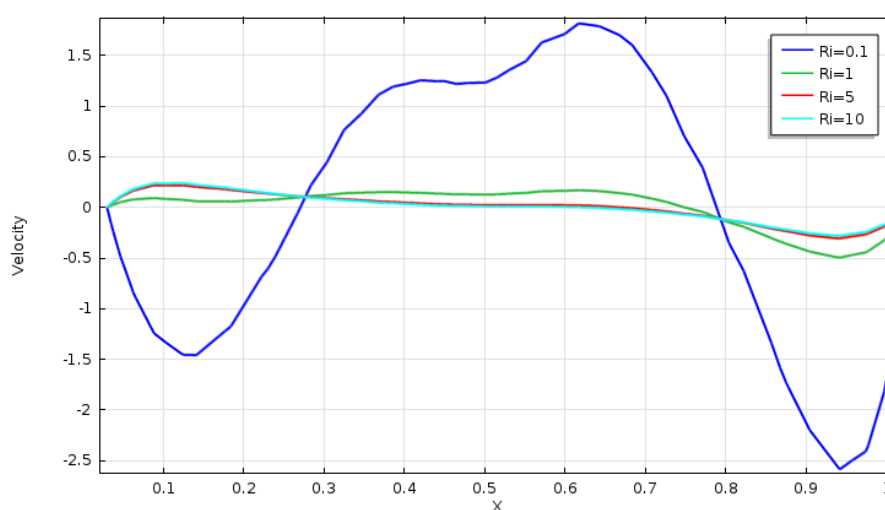


Figure 13. Modification of profile velocity according to different Richardson numbers along the enclosure's X-axis $Ri = 0.1, 1, 5, 10$ when $Da = 1e^{-5}$, $Ha = 50$, $Pr = 6.2$, $Q = 1$ and $\varphi = 5\%$.

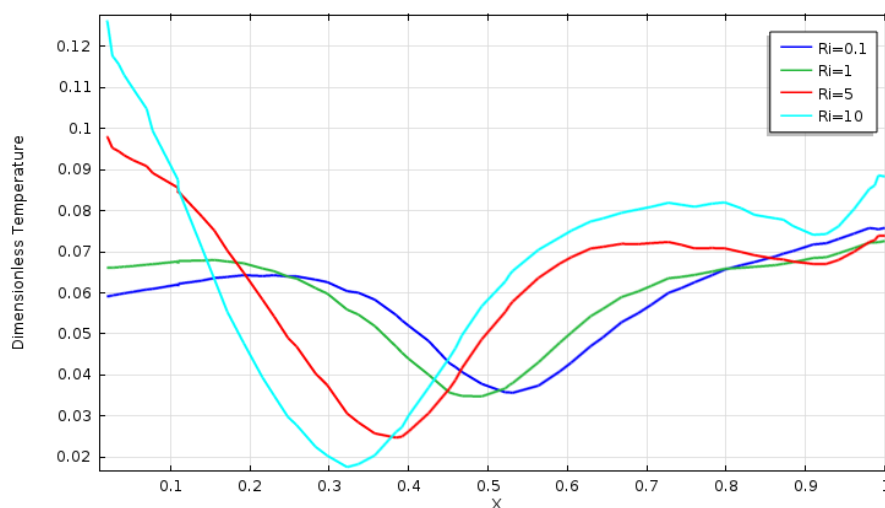


Figure 14. Variation of dimensionless temperature along X – axis of the enclosure with different values of Richardson numbers $Ri = 0.1, 1, 5, 10$ when $Da = 1e^{-5}$, $Ha = 50$, $Pr = 6.2$, $Q = 1$ and $\varphi = 5\%$.

The variation of local Nusselt numbers along X-axis of the enclosure, is depicted in Figure 12, at $Re = 100$, $Gr = 10^4$, for different Richardson numbers. The results show that the local Nusselt number increases with increasing Richardson number. Figure 13 illustrate the velocity component along X-axis for different Richardson numbers, revealing that the heat generating obstacle locations experience the same conditions. Figure 14 shows the dimensionless temperature profiles along the X-axis for different Richardson numbers. The result indicate that the temperature decreases as the Richardson number increases. Although the temperature value does not change significantly for lower Richardson numbers, it changes considerably for higher Richardson numbers.

6. Conclusion

The numerical modeling of nanofluid flow in a rhombus-shaped enclosure with porous media has been demonstrated in this article, which includes accounting for a heat-generating obstruction. In order to determine the results, a variety of relevant dimensionless groups were taken into consideration, including Richardson number and Darcy number. To improve heat transport, the heat-generating obstruction is essential. The findings indicate that employing copper-water nanoparticles as the heat-generating barrier yields the highest values. The heat-generating obstacle has a significant impact on the enclosure's fluid flow and heat transfer properties. The flow and thermal fields, as well as the average Nusselt number results, have been significantly impacted by the heat-generating obstruction. Using a heat-generating obstruction within the enclosure results in the fastest methods of heat transfer. Furthermore, the heat transfer rate increases when the heat generating enclosure's Darcy number increases.

Abbreviations

B_0	Constant Magnetic Field
ΔT	Dimensional Temperature Difference
g	Acceleration Due to Gravity
k	Thermal Conductivity of Fluid
K	Thermal Conductivity Ratio Fluid
x, y	Coordinantes
X, Y	Dimensionless Coordinates
Gr	Grashof Number
Ha	Hartmann Number
Pr	Prandtl Number
Re	Reynolds Number
Da	Darcy Number
Ri	Richardson Number
Nu	Nusselt Number
T	Dimensional Fluid Temperature
T_c	Temperature of Cold Top and Bottom Wall

u, v	Dimensional Velocity Component
U, V	Dimensionless Velocity Component
p	Pressure
Nu_{av}	Average Nusselt Number
Nu_{local}	Local Nusselt Number
U_0	Velocity of the Moving Wall
ϕ	Nanoparticle Volume Fraction
θ	Dimensionless Temperature
α	Thermal Diffusivity
β	Coefficient of Thermal Expansion
ρ	Density of the Fluid
c	Cold
f	Fluid
s	Nanoparticle
nf	Nanofluid
σ	Fluid Electrical Conductivity

Conflicts of Interest

The author has no competing interests.

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