

Research Article

Effects of Friction Loss Models and Cycle Parameters on the Performance of the Irreversible Dual-Miller Cycle

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Abstract

Since the effects of various irreversible losses and cycle conditions on the internal combustion engine performance are significant, it is important to compare the effects of the parameters on the performance in a cycle study. Thermodynamic cycle studies should take into account the irreversible losses such as internal irreversibility, friction losses, heat transfer losses, and the variation of the specific heat of the working fluid. This study aims at deriving the analytic equation with respect to ecological function of the irreversible Dual-Miller cycle (DMC) by using the specific heat model of polynomial, and comparing results of performance calculation of different friction losses, and analyzing the effect of the cycle parameters for optimum performance. The calculation results show that the friction loss has a considerable influence on the engine performance, but there is no significant difference in the choice of the cycle parameters for optimum performance. It is also found that optimizing the ecological function is an effective way to simultaneously increase the power and efficiency, and the friction loss is not affected. The cut-off ratio, which gives the ecological function maximum, is between the cut-off ratio, which gives the power and efficiency maximum, and the Miller cycle ratio, respectively, which is closer to the efficiency maximum. The Miller cycle ratio that maximizes performance is slightly larger than the cut-off ratio. The presented models and results are expected to provide guidelines for the design and optimization of DMC engines, and for the performance improvement and to perform the cycle modeling close to the cycle of the actual engine.

Keywords

Dual-Miller Cycle, Friction Loss, Ecological Function, Irreversible Loss, Cycle Parameter

1. Introduction

Thermodynamic analysis of cycles is performed in order to evaluate theoretical boundaries of the performance and find out the ways of improving performance and compare the performance characteristics of different cycles. The studies on internal combustion engine (ICE) cycles can be divided into the following four aspects: the optimum performances of air standard ICE cycles, the optimum piston motion path of ICE cycles, the performance limits of ICE cycles with

non-uniform working fluid (WF), and performance simulation of ICE cycles [1]. Thermodynamic analysis of ICE cycles, models describing basic characteristics in the real process of engine can be established by using the FTT (Finite-time thermodynamics, FTT) [1-3].

In the recent years, numerous research works on the performances of air standard ICE cycles are focused on the reduction of exhaust emission and improvement of the engine

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performance [1, 4-9]. Numerous investigations have been conducted to improve engine performance and reduce NO_x emissions using the Miller cycle. Moreover, the ICE design through the Miller cycle has been optimized to reduce NO_x emissions and improve fuel economy [10-12]. Many researches show that Dual-Diesel cycle (DDC) has an advantage of high power and Otto-Miller cycle (OMC) has an advantage of low NO_x emissions. DMC has the advantages of high power and low NO_x emissions simultaneously, it is because the heat addition processes of DMC are similar to those of DDC, and the heat rejection processes of DMC are similar to those of OMC. However, in order to apply DMC for ICE design, there should be more complete theory and more mature technology.

So far, some progress has been made in the theoretical analysis and application of overexpansion cycles, including the Miller cycle, some of which have already been commercialized.

A review of the cycle studies of overexpansion ICE is presented, which optimizes the design and operating parameters with one or more performance parameters prioritized among performance parameters such as power, thermal efficiency, power density, exergy, and ecological functions as objective functions.

In our previous work, we have carried out a theoretical study of the cycle, such as the Atkinson cycle and the Miller cycle engine, based on finite-time thermodynamic theory. Here, the working fluid specific heat model can be broadly classified into constant specific heat model, variable specific heat model, and variable specific heat ratio model, using friction loss model with respect to piston mean velocity. Ge, Y [13] considered the power and compression ratio relationship of Atkinson cycle in the presence of heat transfer loss and friction loss, and found that heat transfer and friction loss have a serious effect on performance. Zhao et al. [14] optimized power output and efficiency of the irreversible OMC with heat transfer loss. They optimized power output and efficiency of the cycle with respect to the pressure ratio (maximum/minimum) of the working substance and analyzed optimum criteria of some important parameters such as the power output, efficiency. Mousapour et al. [15] analyzed the power and efficiency of the irreversible OMC and investigated the influences of the Miller cycle ratio and initial temperature, heat transfer on compression ratio. Gonca et al. [16] made a performance analysis based on the power output, thermal efficiency, maximum power output and maximum thermal efficiency criteria for an air standard irreversible DMC with late inlet valve closing (LIVC) version which covers internal irreversibility owing to the irreversible-adiabatic processes. They studied the influences of cycle temperature ratio and cycle pressure ratio of the DMC on the optimum performances as well. Ust et al. [17] optimized total exergy output and exergetic performance coefficient for an irreversible DMC. Their results show that thermal efficiency will get low when optimizing power, and on the con-

trary, power will turn low when optimizing η . Gonca et al. [18] investigated the comprehensive performance analyses and comparisons for irreversible thermodynamic cycle engines based on the power output, power density, thermal efficiency, maximum dimensionless power output (MP), maximum dimensionless power density (MPD) and maximum thermal efficiency (MEF) criteria. Gonca et al. [19] investigated the performance of the irreversible Dual-Miller cycle with LIVC, taking into consideration the influences of heat transfer loss and internal irreversibility. Mehmet CAKIR [20] analyzed the performance of an irreversible OMC in consideration of heat transfer effects and internal irreversibility. In the analyses, the influences of the Miller cycle ratio, combustion, and heat loss constants, and inlet temperature have been investigated. Gonca et al. [21] carried out an ecological performance analyses and optimization of irreversible gas cycle engines such as Atkinson cycle, Otto cycle, Diesel cycle, Miller cycle, DDC, DMC engines based on the ecological coefficient of performance criterion which covers internal irreversibility, heat leak and finite-rate of heat transfer. Gonca et al. [22] carried out the comparative performance analyses of the irreversible OMC engine, Diesel Miller cycle engine and DMC engine based on the maximum dimensionless power output, maximum dimensionless power density. Ebrahimi [23] showed that the work output the DMC has linear relationship with the intake temperature, friction coefficient, working fluid, chemical energy release by combustion and heat-transfer coefficient. Wu et al. [24] investigated power output, thermal efficiency and ecological function characteristics of an endo-reversible DMC with finite speed of the piston. Ust et al. [25] carried out comparative performance analysis and optimization based on the non-dimensional power output and thermal efficiency criteria for an irreversible DMC. The effects of the design parameters such as cycle compression ratio, cut-off ratio and Miller cycle ratio were investigated. You et al. [26] described the model of the DMC with two polytropic processes and heat transfer loss and analyzed influences of compression ratio, cut-off ratio, polytropic exponent on the performance. Shahriyar et al. [27] investigated the performance and the effect of critical parameters on the performance of the DMC with two polytropic processes. Multi-objective optimization was performed to obtain the best point of performance of the DMC. Researches mentioned above investigated an air standard Miller cycle with a constant specific heat.

In addition, studies of an air standard Miller cycle have been performed on the performance analysis and optimization of the cycle with the variable specific heat of WF. Al-Sarkhi et al. [28] investigated the influences of the linearly variable specific heat characteristic on the OMC. Al-Sarkhi et al. [29] analyzed the performance of a Miller engine under different specific heat models (i.e., constant, linear, and fourth order polynomial). Their results show that an accurate model such as fourth order polynomial is essential for accurate prediction of cycle performance. Lin and

Hou [30] investigated the effects of heat loss characterized by a percentage of fuel's energy, friction and variable specific heats of working fluid on the performance of an air-standard Miller cycle. Xing [31] analyzed the performance of the irreversible OMC with heat leak losses, the irreversibility, the specific heat of working fluid linearly varying with its temperature. Ebrahimi et al. [32] investigated the performance of the irreversible Miller cycle with the nonlinear specific heats of a working fluid, the frictional loss related to the mean velocity of the piston, and heat transfer loss through the cylinder wall. Lin et al. [33] investigated the performance of an irreversible air standard Miller cycle in a four-stroke free-piston engine with the variable specific heats, the heat transfer loss as a percentage of fuel's energy and the friction loss. Dobrucali [34] carried out the performance analysis for an irreversible OMC has been presented by taking into consideration heat transfer effects, frictions, variable specific heats. The results demonstrate that the engine design and running parameters have considerable effects on the cycle thermodynamic performance. Gonca [35] carried out the performance investigation of the power, power density, effective efficiency of a DMC engine with nonlinear specific heats. In this study, influences of the engine design and operating parameters on the performance characteristics and energy losses of a diesel engine have been investigated. G. Gonca and B. Sahin [36] numerically and empirically investigated the influences of the combination of the steam injection method, Miller cycle and turbo charging methods on the performance of a direct injection diesel engine. This report also showed a comprehensive comparison of the results of experiments. Ge et al. [37] investigated the ecological function performances of an irreversible Otto cycle considering internal irreversibility loss, friction loss and heat transfer loss, nonlinear specific heats. Their results show that optimization of the exergy-based ecological function not only represents a compromise between the power output and the rate of entropy generation but also represents a compromise between the power output and the thermal efficiency, the specific heat models have no qualitative effect and only have quantitative effect on the performance characteristics of ecological function versus power output and ecological function versus efficiency. Ebrahimi [38] analyzed the effect of the variable specific heats and engine speed on the power output and the efficiency of cycle. In order to make thermodynamic and economic performances of the heat engine cycle reach ideal values simultaneously, Angulo-Brown firstly proposed the ecological function, Yan [39] later revised as $E = P - T_0 \sigma$. Gonca et al. [40] optimized the ecological function of irreversible Miller cycle with heat transfer loss. Wu et al. [41] investigated an air-standard irreversible DMC with linearly variable SHR, and with heat transfer loss, friction loss and other internal irreversible losses. Their results indicated that the maximum power output, maximum efficiency and maximum ecological function of the DMC are superior to those of OC, DDC and OMC, and optimizing ecological function

is the best compromise between optimizing power output and optimizing efficiency. Wu et al. [42] optimized the ecological function of irreversible Miller cycle with internal irreversibility loss, friction loss and heat transfer loss, nonlinear specific heat ratio. Chen et al. [43] founded and analyzed an air standard universal reciprocating heat-engine cycle with heat transfer loss, friction loss and internal irreversibility loss. They analyzed performance characteristics of various special cycles (including Miller, Dual, Atkinson, Brayton, Diesel and Otto cycles) when the SHR of WF is constant and variable (including the SHR varied with linear function and nonlinear function of WF temperature). Gonca et al. [44] carried out the comprehensive performance examination of an engine running on Diesel-Miller cycle in terms of effective power, density of the effective power, and effective thermal efficiency using a novel thermodynamic simulation model. Their results show that variable specific heat values with respect to temperature variation for working fluid are used to get realistic results of engine performance.

Previous thermodynamic cycle studies based on finite-time thermodynamics have mostly dealt with heat transfer loss and friction loss in relatively simple terms. The heat transfer loss through the cylinder wall was assumed to be proportional to the average temperature difference between the working fluid and the cylinder wall. The heat transfer coefficient was considered constant, which did not change with the change of engine design parameters and cycle conditions. Also, the friction power loss term is considered only as viscous friction, which is related to the average piston speed, and the influence of important engine parameters such as cylinder pressure, stroke, etc. is not considered. In a real ICE, the heat transfer coefficient changes in response to changes in the charge pressure, temperature, average piston speed, etc. Moreover, in actual ICE, the frictional loss power is not only related to viscous friction associated with the average piston velocity, but also to the boundary friction affected by the maximum cylinder pressure. The influence of design and operating parameters on the heat transfer coefficient and boundary friction should be considered when establishing an FTT model of cycles. The results of the performance comparison analysis of several thermodynamic cycles in [13, 18] indicate that engine performance is significantly different in real engines if one does not consider heat transfer loss and friction loss or treats it as a simple relation. In [44, 45], Zhao considered the heat transfer and friction losses with the influence of the actual engine parameters. It was found that engine design parameters and cycle conditions have a significant effect on heat transfer and friction losses, and therefore heat loss and friction loss through the cylinder wall have a considerable effect on engine performance.

It is clear from this that if irreversible losses are not taken into account in detail, the reliability of the cycle performance comparison can be distorted, the advantages and disadvantages can be clearly revealed, the study of the influence

of engine design and operating parameters on performance cannot have practical significance, and it is difficult to provide more accurate guidance for the design and optimization of real engines. Although the above studies can provide strong reference to designers when designing engines, there is little literature comparing the effects of various friction and specific heat models on engine performance in detail.

The aim of this paper is to compare the performance calculation results of an irreversible DMC using different friction models to clarify the effect of friction models on performance and to provide a more detailed guidance in the performance optimization and cycle parameters selection of real engines. For this purpose, For the purpose of this study, first, an FTT model of the power, efficiency and ecological functions of the DMC considering the specific heat model of polynomial, heat transfer loss, internal irreversible loss and friction loss of the working fluid, and compare the effects of friction loss models on the performance. Then, cycle parameters that give power, efficiency and ecological function

maximum are analyzed and some important remarks are presented.

2. Cycle Model and Performance Analysis

P-v and T-s diagrams of the considered irreversible air-standard DMC (1-2-3-4-5-6-1) are shown in Figure 1.

In the diagrams, the process 1-2s is an isentropic compression, while the process 1-2 takes internal irreversibilities into account. The heat addition is done in two steps: processes 2-3 and 3-4. they are constant volume and constant pressure heat addition processes respectively. Process 3-4 also makes up the first part of the power stroke. Process 4-5s is an isentropic expansion while the process 4-5 takes into account internal irreversibilities. The heat rejection occurs in two steps: 5-6 and 6-1 are constant volume and constant pressure heat rejection processes respectively.

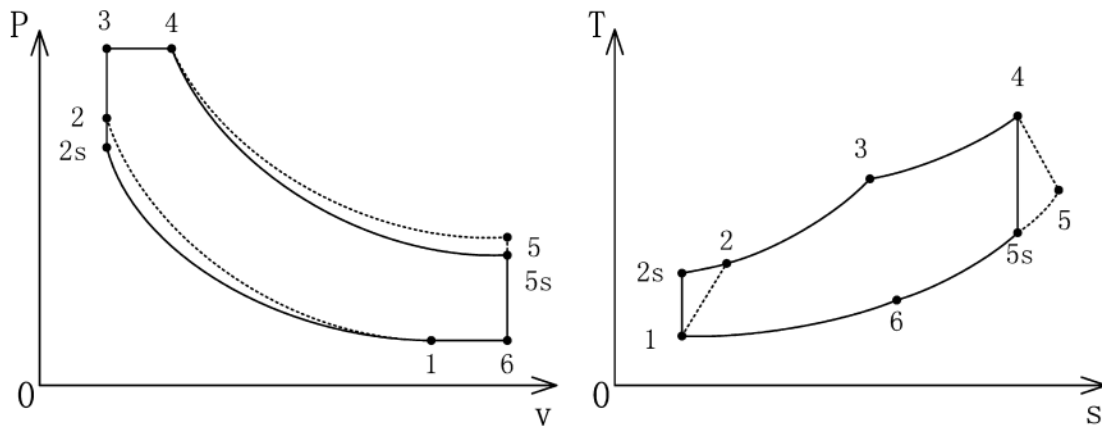


Figure 1. P-v and T-s diagrams for an air-standard irreversible DMC.

The design parameters of compression ratio r_c , cut-off ratio ρ and Miller cycle ratio r_M for the DMC are defined as follows:

$$r_c = V_1/V_2 \quad (1)$$

$$\rho = V_4/V_3 = T_4/T_3 \quad (2)$$

$$r_M = V_6/V_1 = T_6/T_1 \quad (3)$$

$$\alpha = T_4/T_1 \quad (4)$$

According to Ref. [4, 34, 36], for the temperature range of 300 ~ 3500 K, the specific heats of air can be written as;

$$C_p = a_1 T^2 + a_2 T^{1.5} + a_3 T + a_4 T^{0.5} + a_5 + a_6 T^{-1.5} + a_7 T^{-2} + a_8 T^{-3} \quad (5)$$

$$C_v = a_1 T^2 + a_2 T^{1.5} + a_3 T + a_4 T^{0.5} + a'_5 + a_6 T^{-1.5} + a_7 T^{-2} + a_8 T^{-3} \quad (6)$$

$$R = C_p - C_v \quad (7)$$

where C_p - specific heat at constant pressure, C_v - specific heat at constant volume, R is gas constant and equals to 0.287 kJ/kgK, and T is the absolute temperature.

$$a_1 = 2.506 \times 10^{-11}, \quad a_2 = 1.454 \times 10^{-7}, \quad a_3 = -4.246 \times 10^{-7}, \quad a_4 = 3.162 \times 10^{-5}, \\ a_5 = 1.3303, \quad a'_5 = 1.0433, \quad a_6 = -1.512 \times 10^4, \quad a_7 = 3.063 \times 10^5, \quad a_8 = -2.212 \times 10^7$$

The heat added to the working fluid, during constant volume process 2→3 and constant pressure process 3-4, is expressed as

$$\dot{Q}_{in} = \dot{m} \left(\int_{T_2}^{T_3} C_v dT + \int_{T_3}^{T_4} C_p dT \right) \quad (8)$$

$$\dot{Q}_{in} = \dot{m} \left[\left(\frac{a_1}{3} T^3 + \frac{a_2}{2.5} T^{2.5} + \frac{a_3}{2} T^2 + \frac{a_4}{1.5} T^{1.5} + a_5 T - \frac{a_6}{0.5} T^{-0.5} - a_7 T^{-1} - \frac{a_8}{2} T^{-2} \right) \right]_{T_2}^{T_4} - (a_5 - a'_5) T \Big|_{T_2}^{T_3} \quad (9)$$

where m is flow rate of working fluid, kg/s or mol/s.

The heat rejected by the working fluid, during constant volume process 5-6 and constant pressure process 6-1, is as follows

$$\dot{Q}_{out} = \dot{m} \left(\int_{T_6}^{T_5} C_v dT + \int_{T_1}^{T_6} C_p dT \right) \quad (10)$$

$$\dot{Q}_{out} = \dot{m} \left[\left(\frac{a_1}{3} T^3 + \frac{a_2}{2.5} T^{2.5} + \frac{a_3}{2} T^2 + \frac{a_4}{1.5} T^{1.5} + a_5 T - \frac{a_6}{0.5} T^{-0.5} - a_7 T^{-1} - \frac{a_8}{2} T^{-2} \right) \right]_{T_1}^{T_5} - (a_5 - a'_5) T \Big|_{T_6}^{T_5} \quad (11)$$

Because Cp and Cv, the specific heat ratio k are dependent on temperature, the equation often used for a reversible adiabatic process with constant k cannot be used for a reversible

adiabatic process with a variable k.

According to Refs. [24, 36], the equation for the reversible adiabatic process with variable specific heat can be written as

$$\Delta S = S_j - S_i = \int_i^j C_v \frac{dT}{T} + R \ln \frac{v_j}{v_i} = 0 \quad (12)$$

During the isentropic processes 1 to 2s and 4 to 5s Eq. (10) can be used to get a relation between T₁ and T_{2s} and another one between T₄ and T_{5s}.

Therefore, after substituting the value for Cv from Eq. (6)

and performing the integration of Eq. (12), equations describing processes 1→2s and 4→5s can be respectively expressed as follows

$$\left(\frac{a_1}{2} T^2 + \frac{a_2}{1.5} T^{1.5} + a_3 T + 2a_4 T^{0.5} + a'_5 \ln T - \frac{a_6}{1.5} T^{-1.5} - \frac{a_7}{2} T^{-2} - \frac{a_8}{3} T^{-3} \right) \Big|_{T_1}^{T_{2s}} = R \ln \frac{V_1}{V_{2s}} = R \ln r_c \quad (13)$$

$$\left(\frac{a_1}{2} T^2 + \frac{a_2}{1.5} T^{1.5} + a_3 T + 2a_4 T^{0.5} + a'_5 \ln T - \frac{a_6}{1.5} T^{-1.5} - \frac{a_7}{2} T^{-2} - \frac{a_8}{3} T^{-3} \right) \Big|_{T_{5s}}^{T_4} = R \ln \frac{V_{5s}}{V_4} = R \ln r_e \quad (14)$$

The internal irreversibilities in irreversible adiabatic processes 1- 2s and 4- 5s can be defined as follows [41].

$$\eta_c = \frac{(T_{2s} - T_1)}{(T_2 - T_1)} \quad (15)$$

$$\eta_e = \frac{(T_4 - T_5)}{(T_4 - T_{5s})} \quad (16)$$

where η_c and η_e are irreversible compression and expansion efficiencies, respectively, which can reflect all irreversibilities including FL for the processes 1- 2s and 4- 5s.

The heat transfer loss through the cylinder wall to the cooling medium is very complex in the actual combustion process.

Assuming that the cylinder wall temperature is constant, the heat transfer loss per second through the cylinder wall can be estimated as;

$$\dot{Q}_{tr} = h_c A_{cc} (T_m - T_w) \quad (17)$$

where h_c is a heat-transfer coefficient whose unit is W/K; and T_0 is ambient temperature, K A_{cc} is the approximate surface area of the combustion chamber, and T_m and T_w are the average cylinder gas temperature and cylinder wall temperature, respectively.

The heat transfer coefficient h_c can be expressed as [46]:

$$h_c = CB^{-0.2} P^{0.8} w^{0.8} T_m^{-0.55} \quad (18)$$

Here, C is a constant, P is the instantaneous pressure of the cylinder, and the average filling velocity in the W-cylinder, which is expressed as

$$w = \left[C_1 \bar{S}_p + C_2 \frac{V_d T_r}{P_r V_r} (p - P_m) \right] \quad (19)$$

In Eq. (13), V_d is the displaced volume; p is the instantaneous cylinder pressure; and P_r , V_r , T_r are the working-fluid pressure, volume and temperature at a reference state. P_m is the motored cylinder pressure at the same crank angle as p . As shown in Figure 1, under ideal conditions P_m is equal to P_2 . Select the point 2 as a reference state; and use mean incylinder pressure to substitute for the instantaneous cylinder gas pressure. Thus, these values are given as:

$$P_m = P_2 \quad (20)$$

$$p = \frac{(P_3 + P_2)}{2} \quad (21)$$

$$P_r = P_2 \quad (22)$$

$$V_r = V_2 = \frac{1}{4} \pi B^2 \frac{L}{r_c - 1} \quad (23)$$

$$T_r = T_2 \quad (24)$$

From the equation of state of ideal gas, the following relation is satisfied:

$$PV = mRT \quad (25)$$

The combustion chamber surface area is approximately estimated as:

$$A_{cc} = \frac{1}{2} \pi B^2 + \pi BL \frac{1}{r_c - 1} \quad (26)$$

For the performance analysis, including the ecological function taking into account the friction loss, we use several representative friction loss models introduced in the early study.

In the friction loss of the engine, the friction loss of the piston is the main source, and the friction loss includes boundary friction and viscous friction. According to [46], the boundary friction is mainly affected by design parameters such as maximum cylinder pressure, stroke and cylinder diameter, and viscous friction is mainly affected by the average piston speed and design parameters.

$$(FMEP)_{boundary} = a \times \frac{L}{B^2} \times P_{max} \quad (27)$$

where the coefficient a is multiplied to consider the influence of peak cycle pressure P_{max} ; and can be calibrated to include the pressure-affected friction in camshaft and crankshaft system in a real engine. The viscous piston friction under hydrodynamic lubrication conditions is primarily related with mean piston speed. The viscous piston friction is correlated by:

$$(FMEP)_{hydrodyn} = b \times \frac{A_{p,eff}}{LB^2} \times \bar{S}_p \quad (28)$$

where $A_{p,eff}$ is the effective area of piston skirt in contact with the cylinder liner. The coefficient b is multiplied to include speed-dependent friction in the camshaft and crankshaft system in a real engine.

Therefore, the total friction loss due to boundary friction and viscous friction (denoted by *bound-visco loss* for simplicity later in the paper) output can be expressed as [45, 46]:

$$P_f (kW) = \frac{FMEP (kpa) \times V_d (dm^3) \times N (r/s)}{2 \times 10^3} \quad (29)$$

where the N is engine rotating speed and can be expressed as:

$$N = \bar{S}_p / 2L \quad (30)$$

The friction model of the engine was used by Chen-Flynn model [47, 48].

The frictional loss pressure by the Chen-Flynn model is calculated as follows:

$$\text{FMEP} = C + (\text{PF} \cdot P_{\text{Max}}) + (\text{MPSF} \cdot \text{Speed}_{\text{mp}}) + (\text{MPSSF} \cdot \text{Speed}_{\text{mp}}^2) \quad (31)$$

where FMEP is Friction Mean Effective Pressure, P_{max} is Maximum Cylinder pressure, Speed_{mp} is Mean Piston Speed, C is Constant part of FMEP, PF is Peak Cylinder Pressure Factor, MPSF is Mean Piston Speed Factor, MPSSF is Mean Piston Speed Squared Factor.

Another friction model is the one proposed by [37, 41], where the losses in the working process are determined by the sum of the friction losses in the intake and exhaust stroke. In this model (denoted by *in-exhaust loss* for simplicity later in the paper), the power loss due to friction loss is

$$P_{\mu} = 4\mu(4Ln)^2 = 64\mu(Ln)^2 \quad (32)$$

where μ is friction coefficient, kg/s, L is piston stroke, m; n is cycle index per second, s^{-1}

As a result, the power output can be computed as:

$$P_e = \dot{Q}_{in} - \dot{Q}_{out} - P_f \quad (33)$$

$$\eta = \frac{\dot{Q}_{in} - \dot{Q}_{out} - P_f}{\dot{Q}_{in} + \dot{Q}_{tr}} \quad (34)$$

The entropy generation of the DMC has four sources; HT

loss, heat loss due to FL in intake and exhaust strokes, irreversible compression and expansion in compression and power strokes, and heat loss due to WF with energy exhausting to environment. According to Ref. [37, 41], the EGR due to HT loss rate is

$$\sigma_q = \dot{Q}_{leak} \left[\frac{1}{T_0} - \frac{1}{(T_2 + T_4)/2} \right] = \frac{B(T_2 + T_4 - 2T_0)^2}{T_0(T_2 + T_4)} \quad (35)$$

Finally, the power loss due to FLs is transformed into heat. Such a heat loss rate is absorbed by environment totally, thus, it doesn't have the ability to make the useful power any longer. The EGR due to the above heat loss rate can be obtained by dividing the power loss into the ambient temperature as in below.

$$\sigma_{\mu} = P_{\mu} / T_0 \quad (36)$$

in-exhaust loss;

$$\sigma_{\mu} = P_{\mu} / T_0 = 64\mu(Ln)^2 / T_0 \quad (37)$$

The EGRs due to irreversible compression and expansion in compression and power strokes are

$$\sigma_{2s \rightarrow 2} = \dot{m} \int_{T_{2s}}^{T_2} C_v \frac{dT}{T} = \dot{m} \left(\frac{a_1}{2} T^2 + \frac{a_2}{1.5} T^{1.5} + a_3 T + 2a_4 T^{0.5} + a'_5 \ln T - \frac{a_6}{1.5} T^{-1.5} - \frac{a_7}{2} T^{-2} - \frac{a_8}{3} T^{-3} \right) \Bigg|_{T_{2s}}^{T_2} \quad (38)$$

$$\sigma_{5s \rightarrow 5} = \dot{m} \int_{T_{5s}}^{T_5} C_v \frac{dT}{T} = \dot{m} \left(\frac{a_1}{2} T^2 + \frac{a_2}{1.5} T^{1.5} + a_3 T + 2a_4 T^{0.5} + a'_5 \ln T - \frac{a_6}{1.5} T^{-1.5} - \frac{a_7}{2} T^{-2} - \frac{a_8}{3} T^{-3} \right) \Bigg|_{T_{5s}}^{T_5} \quad (39)$$

The EGR due to heat loss rate emitted into ambient medium from WF is given by

$$\begin{aligned} \sigma_{pq} &= \dot{m} \left[\int_{T_1}^{T_6} C_p dT \left(\frac{1}{T_0} - \frac{1}{T} \right) + \int_{T_6}^{T_5} C_v dT \left(\frac{1}{T_0} - \frac{1}{T} \right) \right] = \dot{m} \left[\frac{1}{T_0} \left(\int_{T_1}^{T_6} C_p dT + \int_{T_6}^{T_5} C_v dT \right) \right] - \dot{m} \left[\int_{T_1}^{T_6} \frac{C_p dT}{T} + \int_{T_6}^{T_5} \frac{C_v dT}{T} \right] \\ &= \dot{m} \left[\frac{b_{\lambda}}{T_0} \left(\frac{a_1}{3} T^3 + \frac{a_2}{2.5} T^{2.5} + \frac{a_3}{2} T^2 + \frac{a_4}{1.5} T^{1.5} + a_5 T - \frac{a_6}{0.5} T^{-0.5} - a_7 T^{-1} - \frac{a_8}{2} T^{-2} \right) \Bigg|_{T_1}^{T_5} - (a_5 - a'_5) T \Bigg|_{T_6}^{T_5} \right] \\ &\quad - \dot{m} \left[\left(\frac{a_1}{2} T^2 + \frac{a_2}{1.5} T^{1.5} + a_3 T + 2a_4 T^{0.5} + a_5 \ln T - \frac{a_6}{1.5} T^{-1.5} - \frac{a_7}{2} T^{-2} - \frac{a_8}{3} T^{-3} \right) \Bigg|_{T_1}^{T_5} - (a_5 - a'_5) \ln T \Bigg|_{T_6}^{T_5} \right] \end{aligned} \quad (40)$$

Thus, the total EGR of the DMC is

$$\sigma_{sum} = \sigma_q + \sigma_{\mu} + \sigma_{pq} + \sigma_{2s \rightarrow 2} + \sigma_{5s \rightarrow 5} \quad (41)$$

The ecological function of an irreversible DMC with linearly variable SHR is expressed as

$$E = P - T_0 \sigma_{sum} = P - T_0 (\sigma_q + \sigma_\mu + \sigma_{pq} + \sigma_{2s \rightarrow 2} + \sigma_{5s \rightarrow 5}) \quad (42)$$

3. Results and Discussion

An investigation has been performed on the effects of specific heat variation and heat transfer on the performance of the irreversible DMC based on the power output, thermal efficiency, and ecological function. Performance of the DMC has been compared with different specific heat models (i.e., constant, linear, and nonlinear polynomial). The effect of the design parameters such as compression ratio, pressure ratio, cut-off ratio and Miller cycle ratio on the performance of the irreversible DMC have also been investigated.

When the values of T_1 , r_C , r_M , α , ρ , η_c and η_e are given,

temperatures T_{2s} , T_2 , T_3 , T_{5s} , T_5 and T_6 can be calculated by using Eqs. (1)-(4) and (13)-(16). Then, substituting those temperatures into Eqs. (33), (34) and (42) yields P , η and E of the DMC. Assuming the ambient temperature T_0 is 300 K, the intake temperature T_1 should be higher than T_0 , so it can be assumed to be 340~400 K [25, 41], $\alpha=7$. According to Ref. [25, 41], the optimal r_M for MP is between 2 and 3, thus, ρ and r_M can be both taken as 1~4. To compare the present study with previous studies [41], in the calculations, the parameters are taken as; $\mu = 1.1 \text{ kg/s}$, $L = 0.06 \text{ m}$, $n = 30 \text{ s}^{-1}$, $\dot{m} = 1 \text{ mol/s}$, $\eta_c = 0.9$, $\eta_e = 0.9$.

3.1. Analysis of the Effect of Friction Models on Performance

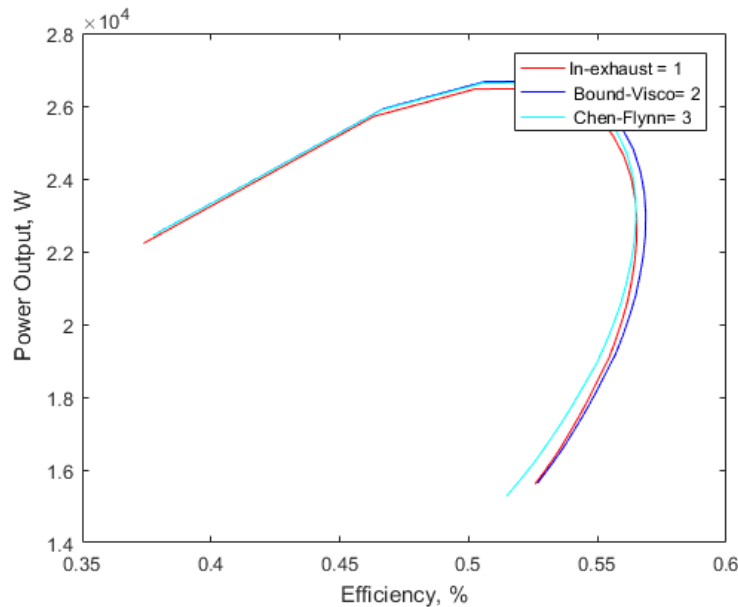


Figure 2. Results of power-efficiency calculations using different friction models.

The power-efficiency relationship, compression ratio-power, compression ratio-efficiency and compression ratio-ecological function relationship using different friction models are shown in Figures 2-4.

As shown in the figure, the loss values using the in-exhaust

loss model have the highest values, and in this case the optimal compression ratio values are further reduced.

It can also be seen that the performance calculation results using the boundary-viscous loss model and the Chen-Flynn model are close to 5%.

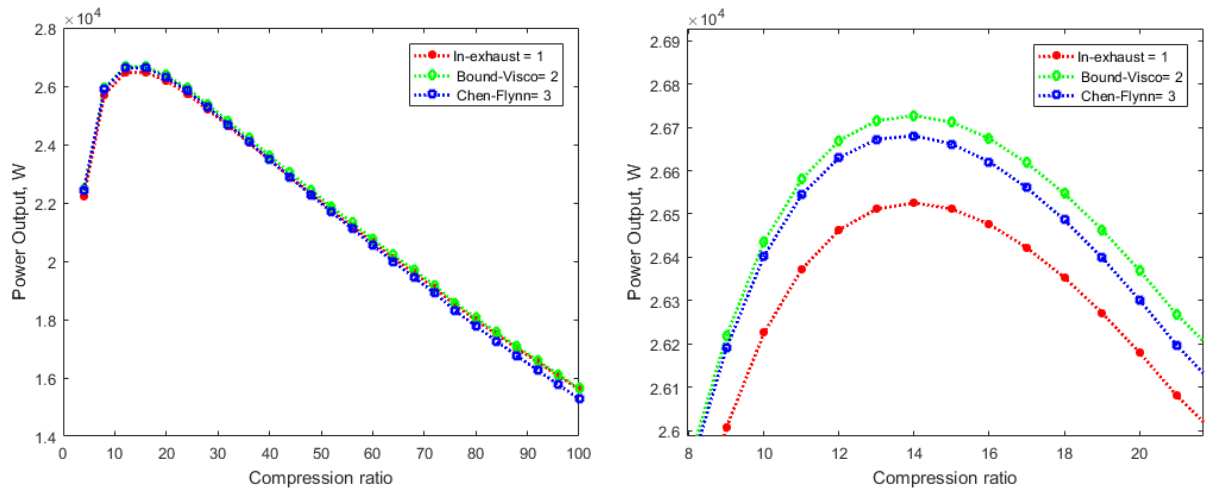


Figure 3. Results of power-compression ratio calculations using different friction models.

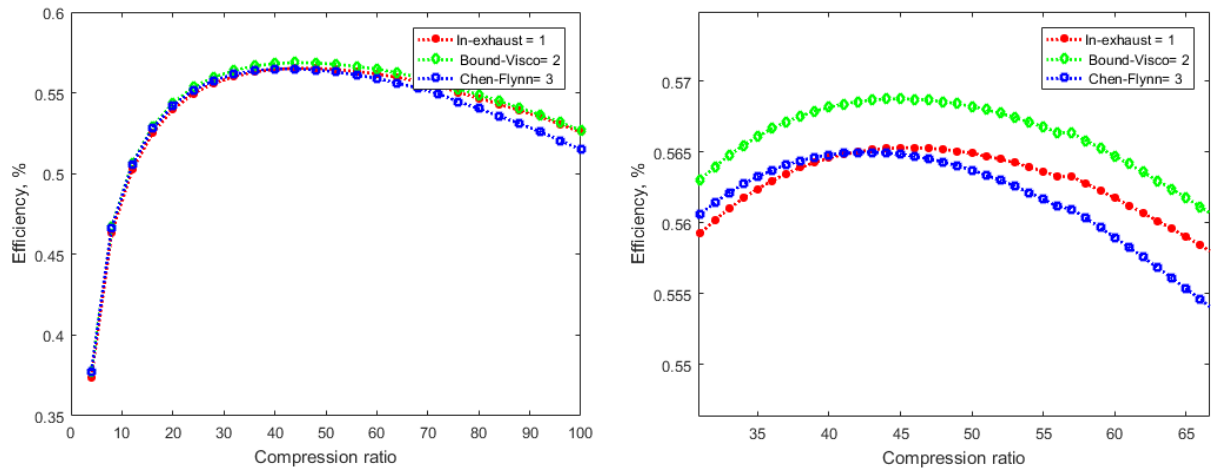


Figure 4. Results of thermal efficiency-compression ratio calculations using different friction models.

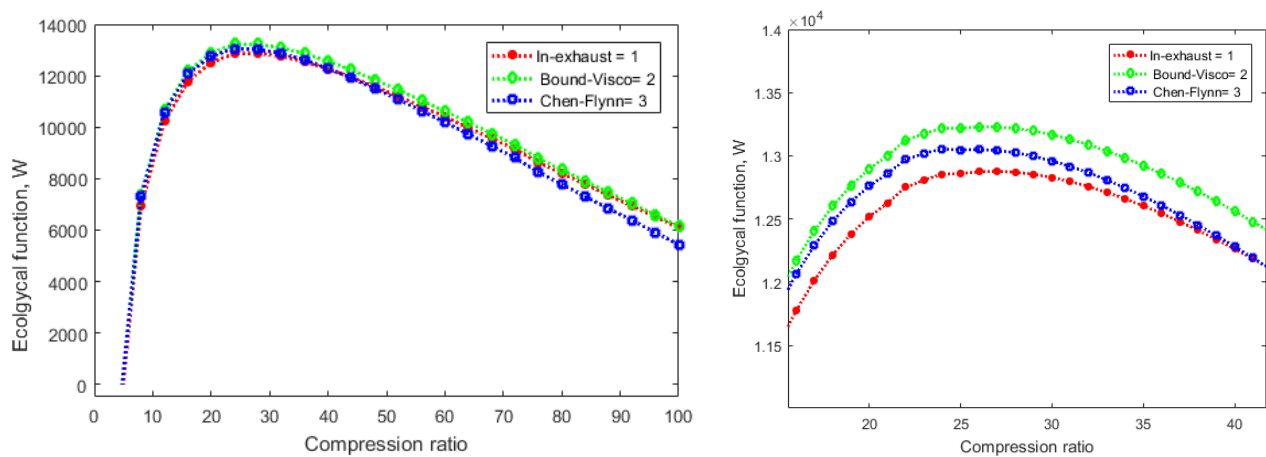


Figure 5. Results of ecological function-compression ratio calculations using different friction models.

In case of $p = 2$, $r_m = 1 \sim 4$, Miller cycle ratio r_m , which gives the maximum power and efficiency for different friction models, is shown in Table 1, respectively.

Table 1. Miller cycle ratio r_m , which gives maximum power and efficiency.

	MP	MEF	ME
Model 1(in-exhaust)	2.85	1.99	2.29
model 2(bound-visco)	2.85	1.96	2.28
model 3(Chen-Flynn)	2.85	2.06	2.33

In case of $\rho = 2$, $r_m = 1 \sim 4$, cut-off ratio ρ , which gives the maximum power and efficiency for different friction models, is shown in Table 2, respectively.

Table 2. Cut-off ratio ρ , which gives maximum power and efficiency.

	MP	MEF	ME
Model 1(in-exhaust)	2.45	1.55	1.89
model 2(bound-visco)	2.45	1.55	1.89
model 3(Chen-Flynn)	2.45	1.45	1.85

From the calculated results, it can be seen that friction loss has a significant effect on the performance loss of the cycle, but the choice of friction model has a relatively small effect on the power, efficiency and ecological function. It is also found that the increase of friction loss reduces the scope of power, efficiency, ecological function and optimum compression ratio.

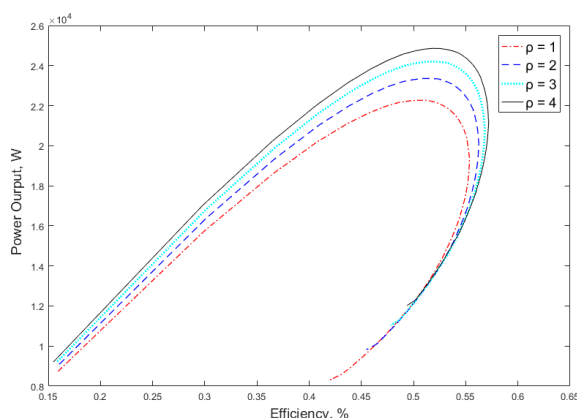
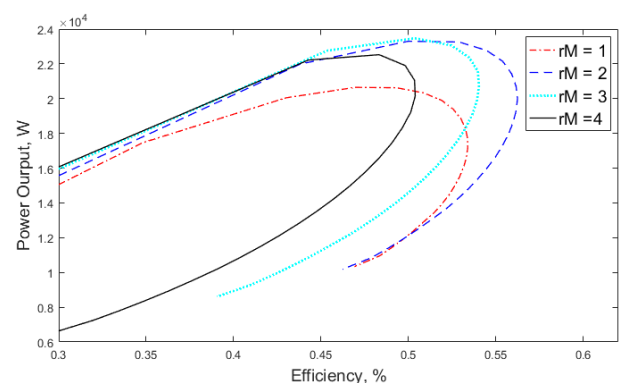
As a result, it can be seen that the Chen-Flynn model is more practical due to its simplicity in calculation, reflecting the maximum pressure of the cylinder and the engine rotational speed. Hence, in Section 3.2, we consider the effect of cycle parameters on the output, efficiency and ecological function maximum using this model.

3.2. Analysis and Performance Optimization of the Effects of Cycle Parameters and Design Parameters on Power and Efficiency

3.2.1. Analysis of the Effect of Cut-off Ratio and Miller Cycle Ratio on Power and Efficiency

Figure 6 reflects the effect of cut-off ratio on the power-efficiency variation at the Miller cycle ratio of 2.

The p - η curve has a ring shape, each curve has one point with maximum power (MP) and one point with maximum efficiency (MEF).

**Figure 6.** Effect of cut-off ratio on power-efficiency variation (Miller cycle ratio = 2).**Figure 7.** Effect of Miller cycle ratio on power-efficiency variation (cut-off ratio = 2).

There is a fundamental change compared to the double

Miller cycle without heat loss because there is a heat transfer loss between the working fluid and the coolant.

In the region $1 < \rho < 4$, there exist different ρ maximizing MP, MEF and ME, respectively.

Figure 7 reflects the effect of Miller cycle ratio on the power-efficiency variation when the cut-off ratio is 2. The p - η curve has a ring shape, with one point (MP) with maximum power and one point (MEF) with maximum efficiency. In the region $1 < r_M < 4$, there exist different Miller cycle ratios that maximize MP, MEF, and ME, respectively.

The power and efficiency variation diagram with

compression ratio for fixed cut-off ratio ($\rho=2$) and varying Miller cycle ratio ($r_M=1\sim 4$) are shown in Figure 4, respectively. With different values of Miller cycle ratio, the output power and efficiency produce a maximum peak, and as r_M increases, the compression ratio, which represents the maximum power (MP) and maximum efficiency (MEF), decreases. There also exists an r_M that maximizes the power and efficiency, respectively. Thus, the power and efficiency decrease when the Miller cycle ratio decreases or increases below a certain value.

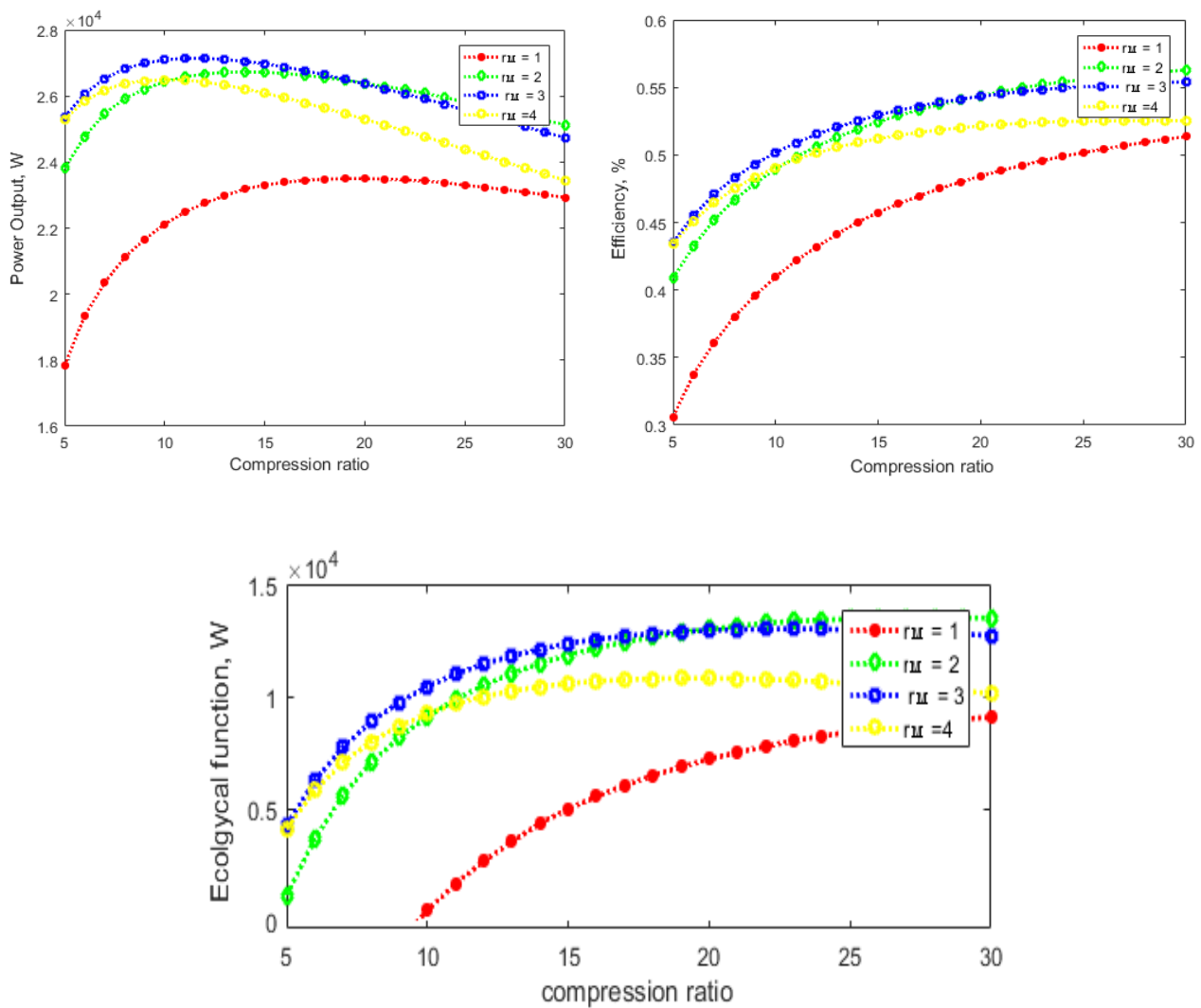


Figure 8. The effects of Miller cycle ratio on the (a) power output, (b) efficiency with compression ratio, $\rho=2$.

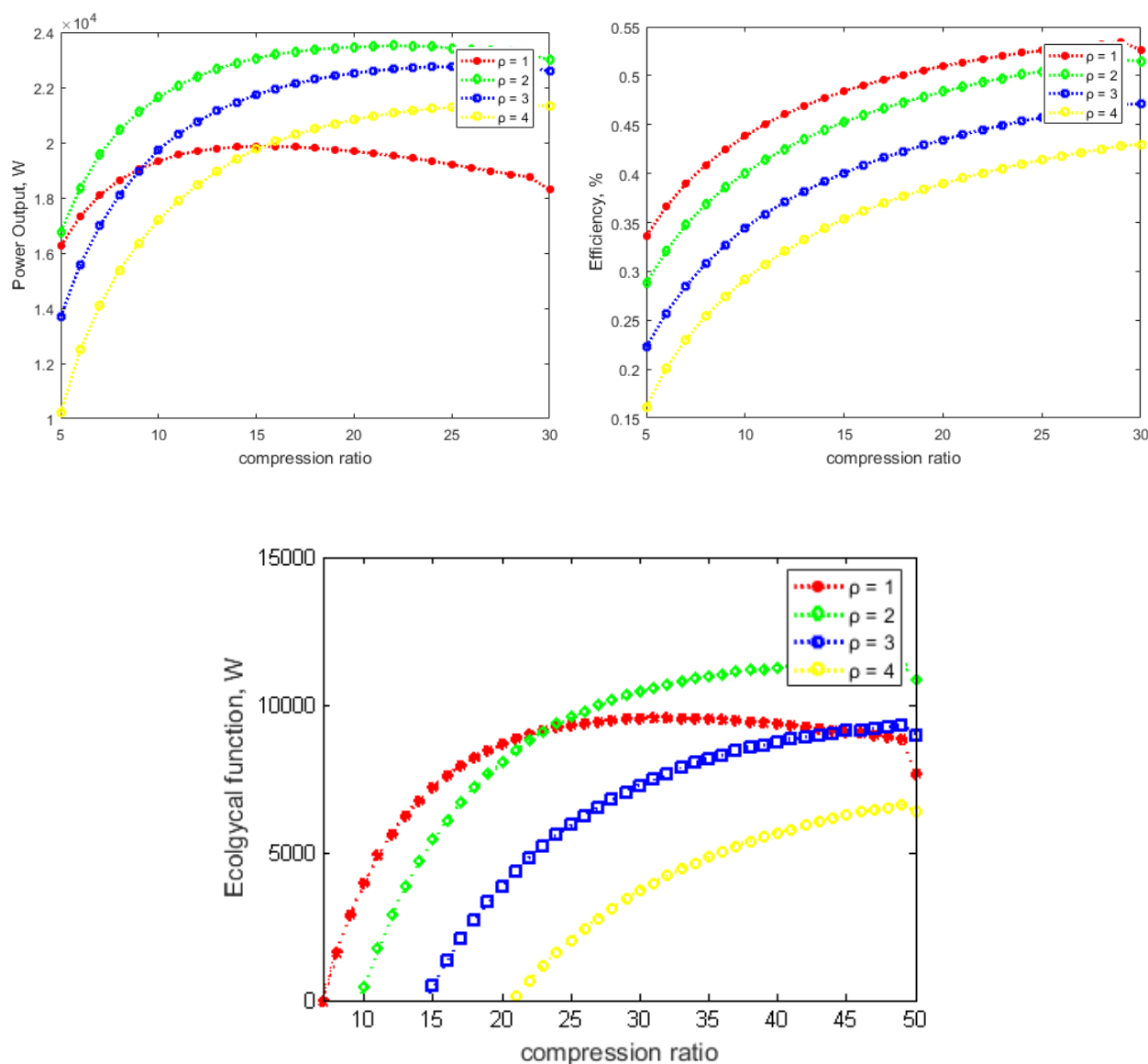


Figure 9. The effects of cut-off ratio on the (a) power output, (b) efficiency (3) ecological function with compression ratio; ($r_M=2$).

The power and efficiency variation diagrams with compression ratio are shown in Figure 9, respectively, for fixed Miller cycle ratio ($r_M=2$) and variable cut-off ratio ($p=1\sim 4$).

With different values of cut-off ratio, the power and efficiency increase and then decrease with increasing compression ratio. With increasing p , the compression ratio that gives maximum power (MP) and efficiency (MEF) increases. Also, there exists an optimal q that maximizes the

power and efficiency, respectively. Thus, the power and efficiency decrease when the cut-off ratio decreases or increases below a certain value.

The above calculation results are in agreement with the calculations of [41].

The power, efficiency and ecological function change relationship for constant compression ratio ($r_c = 12$) and variable cut-off ratio and Miller cycle ratio are shown in Figure 10.

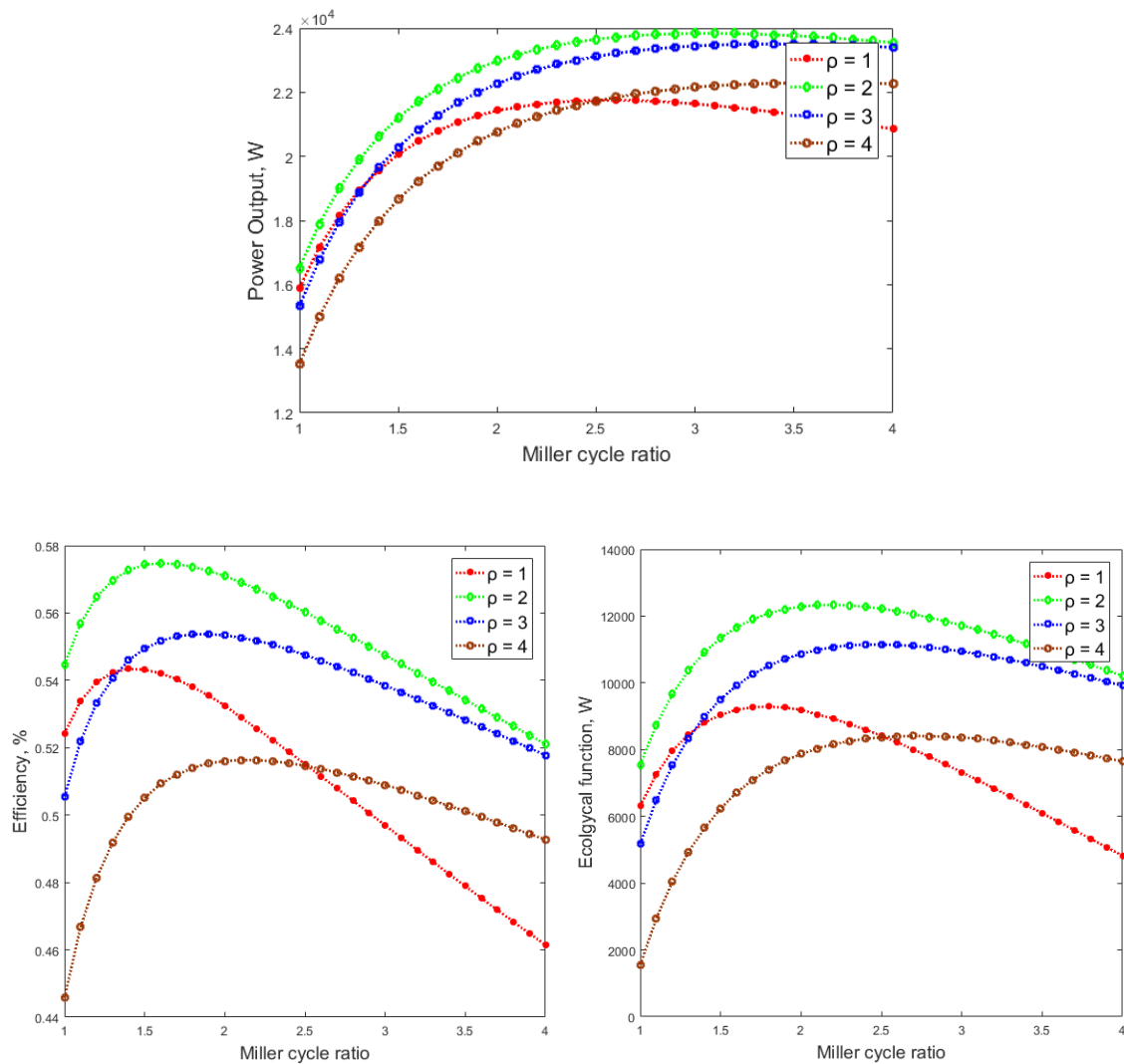
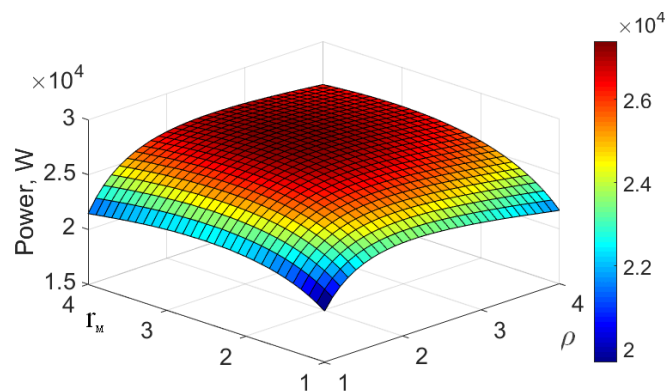


Figure 10. Effect of Miller cycle ratio and cut-off ratio on power, efficiency and ecological function change (fixed compression ratio $r_c = 12$).

3.2.2. Analysis of the Effect of Cut-off Ratio and Miller Cycle Ratio on Performance Optimization

Three-dimensional diagrams of MP and MEF versus ρ and

r_M are shown in Figure 11. Comparing Miller cycle ratio and cut-off ratio which provide maximum output power (or maximum efficiency), it can be seen from Table 3 that Miller cycle ratio is greater than cut-off ratio ρ .



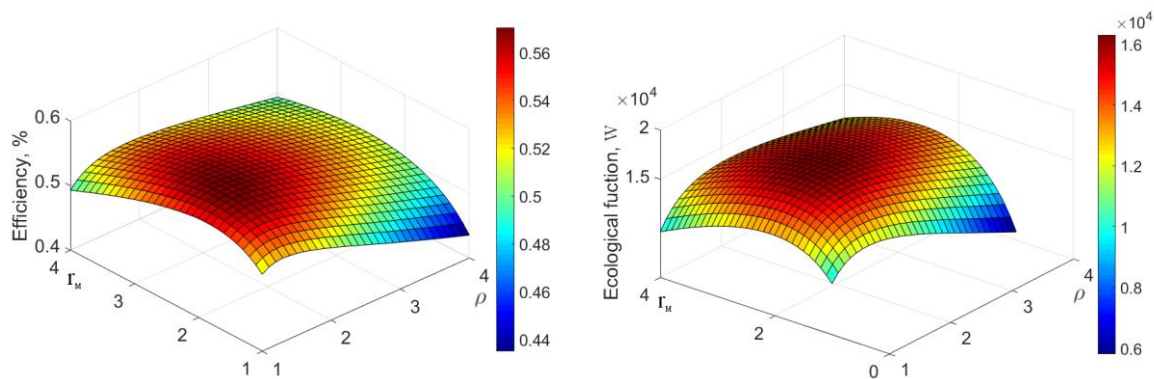


Figure 11. Effects of cut-off ratio and Miller cycle ratio on (a) MP, (b) MEF.

Table 3. Cut-off ratio and Miller cycle ratio on MP, MEF.

	MP	MEF
cut-off ratio	2.5	2
Miller cycle ratio	2.85	2.85

As shown in Table 3, it can be seen that the Miller cycle ratio that maximizes the power (efficiency) is larger than the cut-off ratio. It can also be seen that the cut-off ratio giving the ecological function maximum is between the cut-off ratio giving the power maximum and the cut-off ratio giving the efficiency maximum, and more close to the efficiency maximum.

Similarly, it can be seen that the Miller cycle ratio giving the ecological function maximum is between the Miller cycle ratio giving the power maximum and the Miller cycle ratio giving the efficiency maximum, and more closely approaches the efficiency maximum. It is well known that friction loss has a considerable effect on engine performance, but optimizing the ecological function does not affect the power output and efficiency simultaneously.

4. Conclusions

The FTT relationship for the power, efficiency and ecological functions of the dual Miller cycle considering the heat transfer change, heat transfer loss, internal irreversible loss and friction loss of the working fluid was derived and the influence of friction loss models on the performance and the relationship of the circulation parameters on the performance maximum were analyzed.

Based on the constructed FTT model, the following important conclusions were obtained:

- 1) It can be seen that friction loss has a significant effect on the performance loss of the cycle, but the choice of friction model has a relatively small effect on the power,

efficiency and ecological function. Also, with the increase of friction loss, the scope of power, efficiency, ecological function, and optimum compression ratio is reduced.

- 2) There exist values of the optimum compression ratio, Miller cycle ratio and pre-expansion ratio that best improve power, power density and energy efficiency, and Miller cycle ratio that maximizes performance is greater than the pre-expansion ratio.
- 3) It can also be seen that the pre-expansion ratio giving the ecological function maximum is between the pre-expansion ratio giving the power maximum and the pre-expansion ratio giving the efficiency maximum. Similarly, it can be seen that the Miller cycle ratio giving the ecological function maximum is between the Miller cycle ratio giving the power maximum and the Miller cycle ratio giving the efficiency maximum, and more closely approaches the efficiency maximum.

It is expected that the models and computational results presented in this paper will be used to model the Miller cycle close to the real engine cycle and to study the cycle parameters selection and optimal design of Miller engine.

Abbreviations

B	Constant of Heat Transfer
C	Specific Heat (kJ/kg K)
DDC	Dual-Diesel Cycle
DMC	Dual-Miller Cycle
E	Ecological Function (W)
EGR	Entropy Generation Rate
FL	Friction Loss
FTT	Finite Time THERMODYNAMICS
HTL	Heat Transfer Loss
ICE	Internal Combustion Engine
K	Specific Heat Ratio
L	Stroke Length (m)
LIVC	Late Inlet Valve Closing
MEF	Maximum Thermal Efficiency
MP	Maximum Power Output

MPD	Maximum Dimensionless Power Density
m	Flow rate of Working Fluid (mol/s or kg/s)
n	Cycle Index Per Second (s^{-1})
OC	Otto Cycle
OMC	Otto Miller Cycle
P	Power Output (w)
p	Pressure (pa)
Q	Quantity of Heat Transfer (j)
R	Gas Constant
rc	Compression Ratio
r_M	Miller Cycle Ratio
SH	Specific Heat
SHR	Specific Heat Ratio
T	Temperature (k)
T_0	Ambient Temperature
V	Volume (m^3)
W	Work (j)
WF	Working Fluid
η	Efficiency
η_c	Isentropic Compression Efficiency
η_e	Isentropic Expansion Efficiency
μ	Friction Coefficient (kg/s)
ρ	Cut-off Ratio
σ	Entropy Generation Rate (W/K)
α	Cycle Temperature Ratio
in	Input
leak	Leakage
max	Maximum Value
min	Minimum Value
opt	Optimal Value
out	Output
pq	Effect of Working Fluid Exhausting to Environment
q	Effect of Heat Transfer
s	Reversible
μ	Effect of Friction
1; 2; 3; 4; 5; 6	Cycle State Points

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Conflicts of Interest

The authors declare no conflicts of interest.

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