

Research Article

Optimal Wealth Allocation to Interest-Bearing Central Bank Digital Currency in Investor Portfolios: A Merton Model Approach

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Abstract

Floating interest Central Bank Digital Currency (CBDC) is a moderately risky financial asset. Given the risks involved and the returns attached, a rational investor has to determine the optimal allocation of wealth to the CBDC in his or her portfolio. The paper sought to establish the optimal wealth allocation to a floating interest rate CBDC and a risk-free asset. The study adopted an analytical design to illustrate the theoretical optimal holding of floating interest rate CBDC by individual investors based on Merton's 1969 mathematical model. Using data collected from the Central Bank of Kenya and Kenya National Bureau of Statistics, the paper applied the Merton model to a hypothetical proxy for CBDC, and real-world data on inflation, interest, and 91-day T-bill, to allocate investors' wealth to floating interest CBDC and a risk-free asset. The results show that optimal wealth allocation to floating interest rate CBDC was a function of the risk premium, the degree of investor risk aversion and the volatility of the floating interest rate CBDC. The results further demonstrate that whenever CBDC offered a higher interest rate than a risk-free asset, investors would shift wealth to CBDC and vice versa. Further, whenever the volatility on CBDC returns increased, investors tended to hold fewer units of interest-bearing CBDCs and more of risk-free assets and vice versa. The optimal monthly consumption for the risk-averse investor was a function of subjective discounting rate, degree of investor risk aversion and previous wealth. A higher subjective discounting rate or a higher cumulative wealth, or a lower risk aversion was associated with increased optimal consumption, *ceteris paribus*, and vice versa is true. Our results therefore suggest that financial markets investment portfolios are sensitive to CBDC volatility, and this that can originate another strand of CBDCs literature. These findings provide useful insights to individual and institutional investors, and can guide policymakers and financial market regulators on the important link between CBDC and financial markets in the new digital-currency era. For example, policymakers and regulators can adjust fiscal and monetary policy by considering the possible impact on investor portfolios. This can guide investors to strategically adjust their portfolio positions.

Keywords

Central Bank Digital Currency, Optimal Wealth Allocation, Investor Portfolio, Risk-Free Asset, Cryptocurrency

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1. Introduction

Undoubtedly, inspired by recent advances in technology-driven payment systems such as mobile payment systems, cryptocurrencies, and blockchain technologies, central banks worldwide are currently exploring the possibility of issuing digital forms of fiat money called central bank digital currencies [CBDC] [1]. A thoughtfully designed CBDC can be a new means for central banks to satisfy numerous policy objectives (Choi *et al.*, 2021). A CBDC, therefore, is a digital currency issued by a central bank and intended to serve as legal tender [2]. CBDCs are ‘digital fiat currencies’ that do not have a physical form, which is a key distinction from conventional fiat money [3]. The IMF defined CBDC as a new form of money, issued digitally by the central bank and intended to serve as legal tender [4].

The CBDC may be interest-bearing or not, depending on what kinds of roles and functions the issuing central bank wants CBDCs to perform and objectives to be achieved [5]. If CBDCs are regarded as digitised payment instruments to be used instead of banknotes, CBDCs could be designed as non-interest-bearing as banknotes [6]. On the other hand, CBDCs should bear interest, since central bank deposits bear interest in many countries and CBDCs are similar to existing central bank deposits, in the sense that both of them are digitised central bank liabilities [7]. An interest-bearing CBDC is a form of digital currency issued by a central bank that accrues interest over time. Unlike cash or a non-interest-bearing CBDC, this type of CBDC functions more like a deposit at the central bank, providing monetary policy tools and influencing financial markets [8]. Interest-bearing CBDC carries some level of risk, such as interest rate fluctuations, market exposure, or credit risk [9]. Floating interest rates CBDC may be tied to policy rates, such as the central bank’s deposit rate or market benchmarks [10].

Floating interest CBDC have the features of moderately risky financial assets such as commercial banks’ time deposits, corporate bonds and money market funds. Unlike a fixed-rate CBDC, this model allows interest payments on CBDC holdings to change over time, making it a dynamic tool for monetary policy [8]. European Central Bank (ECB) is considering tiered interest rates, its Digital Euro version of CBDC, where small balances earn more, and large holdings receive lower or negative rates [11]. Further, China’s Digital Yuan (e-CNY) is currently non-interest-bearing, but future versions may allow for dynamic rate adjustments [7]. Additionally, Sweden’s e-Krona, issued by Riksbank, is exploring how a floating-rate CBDC could interact with commercial bank deposits [12].

Rational investors, whether risk averse, risk neutral or risk lovers, often seek to maximise the expected utility of their wealth. If the investors have a choice to hold floating interest-bearing CBDC against other alternative financial assets, such as treasury bonds and bills, they need to make an optimal decision. Utility theory suggests that economic agents such as investors make decisions to maximise expected utility, rather than just expected returns. Utility theory helps determine the

optimal amount of interest-bearing CBDC an individual or institutional investor should hold, considering factors such as liquidity, return, risk, and alternative investment opportunities. Asset allocation was based on the Merton Model. Given that there is scanty information on how investors can design their investment portfolios through an optimal wealth allocation to floating interest CBDC, this paper sought to establish the monthly optimal wealth allocation to floating interest CBDC and risk-free security, the total monthly wealth evolution and total monthly consumption for risk risk-averse investor.

1.1. Research Question

What is the optimal proportion of wealth that a risk-averse investor should allocate to floating interest-bearing CBDCs versus risk-free assets?

1.2. Research Objectives

- 1) To establish the monthly optimal wealth allocation to floating interest CBDC and risk-free security.
- 2) To examine the monthly optimal consumption for risk risk-averse investor out of returns from investment.
- 3) To establish the total monthly wealth evolution for risk risk-averse investor.

2. Literature Review

2.1. Theoretical Foundation of Portfolio Management

This paper is based on utility theory, modern portfolio theory and the Merton model. Utility theory plays a pivotal role in investment decision-making by helping investors evaluate their preferences and align their choices with their risk tolerance and expected returns. This theory originates from the broader framework of expected utility theory, which was first articulated by von Neumann and Morgenstern through the formalisation of the expected utility maximisation principle [13]. The theory posits that individuals make decisions based on the expected utility of outcomes, where utility is a measure of the relative satisfaction or happiness derived from a particular choice. The theory suggests that individuals assess risky choices by considering the expected utility of outcomes rather than their expected monetary values [14]. A fundamental application of utility theory in investment is Modern Portfolio Theory (MPT), introduced by Harry Markowitz. Modern Portfolio Theory suggests that rational investors will construct a portfolio that maximises expected utility by balancing risk and return, using diversification to minimise risk [15]. The expected utility function can be expressed mathematically as:

$$EU = \sum p_i U(W_i)$$

Where: EU is the expected utility, p_i represents the probability of a given outcome, and (W_i) is the utility derived from wealth level W_i [13].

The MPT provides a framework for asset allocation, utilising optimisation techniques like mean-variance optimisation (MVO) to construct efficient portfolios that aim to maximise expected return for a given level of risk. While MVO offers a mathematically rigorous approach, its effectiveness is highly dependent on the accuracy of input assumptions regarding expected returns and risks, and it can sometimes lead to highly concentrated portfolios.

This paper mainly adopted the Merton model as a foundational framework to describe optimal investment portfolio allocation and consumption decisions under uncertainty. The Merton model of portfolio optimisation extends the classical Markowitz portfolio theory by incorporating continuous-time dynamics and intertemporal decision-making under uncertainty. It is a foundational model in modern portfolio theory and asset pricing, offering insights into optimal consumption and investment decisions over time [16]. The model is based on continuous-time stochastic control and solves for an investor's optimal asset allocation over time, given their risk preferences and market conditions. Unlike Markowitz's static one-period model, Merton's model considers a continuous-time framework where investors make decisions dynamically, adjusting their portfolio as market conditions change [16]. Merton assumes that asset prices follow a geometric Brownian motion. Merton derives an optimal investment strategy where an investor allocates wealth between a risky asset (such as stocks) and a risk-free asset (such as bonds or cash). The optimal proportion of wealth allocated to the risky asset (π^*) is:

$$\pi^* = \frac{(\mu - R_f)}{\gamma \sigma^2}$$

Where $\mu - R_f$ = excess return of the risky asset, σ^2 = variance of the risky asset's returns, γ = risk aversion parameter and π^* = optimal portfolio allocation to risky assets.

Further, Merton also incorporates optimal consumption decisions, determining how much of their wealth an investor should consume over time versus reinvesting. The Merton Model of Portfolio Optimisation is a cornerstone of intertemporal portfolio theory, allowing investors to make optimal decisions in a dynamic, uncertain environment. Its integration of stochastic processes, continuous-time decision-making, and consumption-investment trade-offs makes it a fundamental tool in both academic finance research and practical investment management.

2.2. Empirical Review

2.2.1. Interest-Bearing CBDC and Interest Pegging

Central Bank Digital Currency (CBDC) is a digital form of

central bank money that serves as a liability of the central bank, similar to physical cash and reserves. While most discussions on CBDCs focus on non-interest-bearing digital cash, the concept of an interest-bearing CBDC introduces new possibilities for monetary policy, financial stability, and banking system dynamics [8]. An interest-bearing CBDC functions similarly to central bank reserves but is accessible to the public. The Central Bank can set a positive or negative interest rate on CBDC holdings, which affects consumption, savings, and overall liquidity in the financial system [17].

The central bank can modify the CBDC's interest rate to implement monetary policy more effectively [18]. Unlike traditional cash, an interest-bearing CBDC competes with commercial bank deposits, affecting bank lending [19]. Unlike physical cash, which has a zero lower bound, interest-bearing CBDC allows for negative interest rates, incentivising spending during recessions [20]. With interest-bearing CBDC, central banks directly influence public savings and investment decisions by adjusting the digital currency's interest rate. This improves monetary policy transmission, especially in environments where traditional interest rate tools are less effective [8]. If depositors withdraw funds from banks to hold CBDC, commercial banks may face funding shortages, reducing their ability to lend [21].

A Floating Interest CBDC is a digital currency issued by a central bank where the interest rate is variable and adjusts based on macroeconomic conditions, market rates, or policy objectives. Unlike fixed-rate CBDCs, floating-rate CBDCs offer greater flexibility in monetary policy transmission, enhancing financial stability and market efficiency [8]. Compared to a fixed-rate CBDC, which maintains a predetermined return, a floating-rate CBDC adjusts based on: monetary policy rate, inflation rate, money market rates and bond yield.

Monetary Policy Rate: The central bank may link the CBDC rate to its policy rate (such as the Repurchase rate, federal funds rate, European Central Bank's Refinancing Rate). For instance, the Repurchase Rate (Repo Rate) is the rate at which commercial banks borrow from the central bank [19]. In using repo rate, the floating rate on CBDC can be set such that: $R_{CBDC} = \text{Repo rate} + \alpha$ where α is a spread controlled by the central bank. **Inflation Rate-** The rate may be adjusted in response to inflation expectations, liquidity conditions, and real interest rates [18]. For instance, to maintain the real value of digital holdings, a CBDC can be pegged to inflation indicators such as the Consumer Price Index (CPI). In using CPI, the floating rate on CBDC can be set such that: $R_{CBDC} = \text{CPI} + \beta$, where α is where β is an adjustment factor for market stability. This approach ensures that CBDC users receive inflation-adjusted returns, maintaining purchasing power [18].

Money Market rates: A floating interest rate, CBDC can be linked to money market rates, reflecting short-term liquidity conditions and borrowing costs. Major money market rates include Secured Overnight Financing Rate (SOFR) in USD Markets, Euro Overnight Index Average (EONIA) in Euro

pean Markets and Shanghai Interbank Offered Rate (SHIBOR) in China. In using the money market rate, the floating rate on CBDC can be set such that:

$$R_{\text{CBDC}} = \text{Money Market Rate} + \beta$$

where β is an adjustment factor for a risk premium. This approach aligns CBDC interest rates with real-time financial market conditions [17]. Government Bond Yield- A floating CBDC rate could be pegged to government bond yield, reflecting long-term and short-term interest rate expectations. Linking the CBDC rate to bond yields, the central bank can influence investment decisions and manage market liquidity [19].

Even though CBDCs are often considered risk-free assets due to central bank backing, a floating-rate CBDC introduces moderate risk stemming from interest rate volatility, liquidity concerns, monetary policy shifts, and macroeconomic conditions [8]. If the CBDC's interest rate fluctuates frequently, holders may face uncertainty regarding future returns, making it less attractive as a store of value [18]. For instance, if inflation spikes, central banks may rapidly adjust CBDC interest rates, leading to higher volatility in returns. Further, while CBDCs are generally liquid, limits on convertibility, withdrawal caps, or restrictions on large-scale transfers could arise in response to financial instability [19]. For instance, if a central bank imposes holding limits or delays in redemption, users may face temporary liquidity constraints. Therefore, unlike fixed-rate government bonds or stable CBDCs, floating-rate CBDCs are subject to monetary policy changes and market conditions, making them less stable and riskier compared to traditional risk-free assets. However, they are less risky when compared to corporate bonds and equities as they are backed by central banks and do not carry default risk [18].

Therefore, a floating-rate CBDC can serve as a cash-equivalent asset with higher returns than savings accounts but lower risk than corporate bonds and equities. Floating-rate CBDC competes with commercial bank deposits but offers central bank guarantees and policy-driven adjustments, potentially making it a safer savings instrument [19]. If CBDC interest rates are too attractive, depositors may shift funds away from commercial banks, leading to bank disintermediation. This could reduce bank lending capacity, affecting credit supply and financial stability [17]. A floating-rate CBDC allows negative interest rates when needed, overcoming the zero lower bound limitation of physical cash. This prevents excessive liquidity hoarding during deflationary periods [20].

2.2.2. Determinants of Optimal Wealth Allocation to Risky Assets

Wealth allocation, at its core, involves strategically distributing an investor's funds across various asset classes to achieve their financial objectives. This process is fundamental to successful financial planning, aiming to construct a port-

folio that balances potential returns with an acceptable level of risk [22]. A crucial component of wealth allocation is the consideration of risky assets, which encompass investments such as stocks, high-yield bonds, and alternative investments like private equity and hedge funds [23]. These assets are characterised by their potential to generate higher returns over time, offering the growth necessary to meet long-term financial goals such as retirement or significant capital accumulation [24]. Even though the allocation of wealth to risky assets is predicted by various factors. The study focused on factors within the succinct of the Merton model, including risk tolerance, returns on risky assets and volatility of returns.

Risk tolerance is a central determinant in the allocation of wealth to risky assets, representing the level of unpredictability, volatility, and potential loss an investor is both willing and able to accept in pursuit of their financial growth objectives [25]. Empirical studies have shown that an investor's willingness and ability to endure market volatility and potential losses significantly influence the proportion of risky assets in their portfolio [24], [26]. Thus, those with higher risk tolerance may allocate more to equities and other volatile investments, while risk-averse individuals might prefer a conservative mix. For instance, Yao and Rabbani (2021) on the interplay between investment risk tolerance, confidence levels, and portfolio risk established a positive correlation between an investor's risk tolerance and the proportion of risky assets in their portfolio. Further, Liu et al., utilising panel data from the Health and Retirement, demonstrated that persistent individual differences in risk tolerance account for a significant portion of the variation in portfolio choices [24].

Anticipated returns and risks associated with different asset classes influence allocation decisions [4]. For instance, if equities are expected to outperform bonds, an investor might increase their equity exposure. Changes in interest rates have a significant impact on the relative attractiveness of different asset classes, particularly the balance between fixed income and equities [27]. Rising interest rates can make bonds more appealing to investors as newly issued bonds offer higher yields, potentially drawing capital away from the stock market. Conversely, lower interest rate environments may favour equities as borrowing becomes cheaper for companies, potentially fuelling economic growth and boosting stock valuations [28].

Volatile returns on risky assets may push risk-averse investors to take refuge in risk-free assets. For instance, a study on the strategy of combining the highest expected returns assets with those having the lowest variance in the U.S. market supports the approach of investing in portfolios that include both high-return and low-volatility assets, suggesting that such combinations can lead to superior risk-adjusted returns [29]. Further, a study on how ambiguity aversion influences portfolio allocation between risky and ambiguous assets demonstrated that greater ambiguity aversion leads to a decreased optimal proportion invested in ambiguous assets [30].

3. Materials and Methods

3.1. The Data

This paper sought to establish the optimal wealth allocation to a floating interest rate CBDC and a risk-free asset. Using data collected from the Central Bank of Kenya and Kenya National Bureau of Statistics, the paper applied the Merton model to hypothetical (CBDC) and real-world data (inflation, interest, 91-day T-bill) to allocate investors' wealth to a floating interest CBDC and a risk-free asset. The interbank lending rate was used as a proxy for the CBDC floating rate to ensure that it does not go beyond the commercial bank deposit rate (less a constant 2%) as a way to avoid appposable bank run. The inflation rate was used as a proxy for the subjective discount rate, 91-day T-bill rate was used as a proxy for the risk-free interest rate [29].

3.2. Analytical Model

The Merton Model is a classical optimisation problem that involves determining optimal allocation of investor initial wealth to consumption (C_t) over time and investment ($W_t - C_t$). The investor can invest in a risk-free asset (such as treasury bonds and bills), earning a risk-free rate return (R_t) or a risky asset (such as shares) with return μ and volatility σ . The optimal allocation to risky assets is given by π , and allocation to risk-free assets is given by $1 - \pi$. The objective is to maximise the expected utility of consumption and terminal wealth over a given time horizon. The objective of an individual is to maximise their expected lifetime utility from consumption and end-of-period wealth. The objective function is given by:

$$\max_{\pi_t, C_t} E \left[\int_0^T e^{-\beta t} U(C_t) dt + e^{-\beta T} U(X_T) \right] \quad (1)$$

Where:

(C_t) is the utility function representing an individual's preferences for consumption at time t , β is the subjective discount rate reflecting how much future utility is discounted compared to current utility. X_T is the terminal wealth at time T . π_t = fraction of wealth invested in the risky asset, while C_t is consumption at time t . The objective of the individual is to determine how π_t and C_t should evolve to maximise the expected utility of consumption and terminal wealth, taking into account the stochastic nature of asset returns.

Utility of Consumption: From the objective function in Eq. (1), we can pick the first term to get the utility of consumption. Therefore, cumulative utility derived from Consumption over the investment horizon can be obtained by:

$$\int_0^T e^{-\beta t} U(C_t) dt \quad (2)$$

The utility function (C_t) is applied to the consumption rate

C_t at time t . Utility in the future is discounted at a rate β , which reflects the investor's preference for time. The individual investor consumes a portion C_t of their wealth at a time t . The change in the consumption rate over time can be represented by the differential equation.

$$dC_t = C_t \cdot d_t \quad (3)$$

Where C_t is the consumption rate, while d_t is very small change in time. Therefore, consumption changes over time.

Utility of Terminal Wealth: The second term in the objective function (1) is the utility of terminal wealth, which can be obtained by

$$\int_0^T e^{-\beta t} U(X_T) \quad (4)$$

The X_T is the total wealth at time T , reflecting the results of investment and consumption decisions over time. The utility of terminal wealth (X_T) is also discounted back to the present based on the investor's preference. Considering a single agent investing in simple portfolio consisting of one risk-free asset, such as treasury bond (B_t) and one risky asset, such as stock (S_t). The differential equation for risk risk-free asset (B_t) is given by:

$$dB_t = rB_t \cdot dt, 0 \leq t \quad (5)$$

Where t is time and r is the interest rate per year and constant. Additionally, the price of the risky asset (S_t) is modelled as a geometric Brownian motion that follows a stochastic differential equation.

$$dS_t = \mu S_t \cdot dt + \sigma S_t dB_t \quad (6)$$

The parameter μ is the drift term, capturing the growth of the asset price. The term σdW_t is a stochastic component capturing the random fluctuations of risky asset prices. Further, σ denotes the volatility of risky asset prices and μ denotes expected return. Finally, B_t models the inherent randomness in the evolution of the stock price, thus unpredictable.

3.3. Wealth Dynamics

The wealth dynamics is given by a stochastic differential equation in Eq. (7):

$$dX_t = \left(X_t \pi_t \frac{dS_t}{S_t} + X_t (1 - \pi_t) \frac{dB_t}{B_t} - C_t dt \right) \quad (7)$$

Substituting the first order deference of Eq. (5) and Eq. (6) into Eq. (7), wealth equation becomes:

$$dX_t = (X_t \pi_t (\mu \cdot dt + \sigma dW_t) + X_t (1 - \pi_t) r \cdot dt - C_t dt) \quad (8)$$

Rearranging terms in Eq. (8) gives Eq. (9):

$$dX_t = (X_t [r + (\mu - r)\pi_t]dt + \pi_t \sigma W_t) - C_t dt \quad (9)$$

Where: π_t = fraction of wealth invested in the risky asset, $0 \leq \pi_t \leq 1$, X_t = investor's wealth at time t , W_t = Brownian motion, C_t = the consumption rate C_t , $0 \leq C_t$, r = represents the risk-free rate, μ = denotes the expected return of the risky asset, σ is its volatility. In an empirical study, we apply a deterministic model without considering market uncertainty and randomness in the risky asset, since we use historical data. Wealth dynamics describe how investors' wealth evolves as a function of consumption and investment decisions. The evolution of wealth over time is governed by the deterministic differential Eq. (10).

$$\frac{dX_t}{dt} = X_t [\pi_t(\mu - r) + r] - C_t \quad (10)$$

This equation (10) describes the dynamics of wealth accumulation, where the total return is composed of two components: the return from the risky asset, which is proportional to the allocation πt and the excess return $\mu - r$, and the return from the risk-free asset at rate r . The consumption term C_t represents the outflow of wealth due to spending, which reduces total wealth over time. The total wealth at time t is given by Eq. (11).

$$X_t = [\pi_t X_{t-1} (1 + R_{CBDC})] + [(1 - \pi_t)(X_{t-1})R_f - C_t] \quad (11)$$

3.4. Optimal Allocation to Risky Asset

Merton applied the Hamilton-Jacobi-Bellman (HJB) equation to optimal portfolio allocation, where an investor allocates wealth between risky and risk-free assets as given in Eq. (12).

$$pV(W) = \max_{0 < (\pi, c) < 1} [U(c) + V_W \cdot (\pi(\mu - R_f) + R_f W - c) + \frac{1}{2} V_{WW} \pi^2 \sigma^2] \quad (12)$$

Where:

p = discount rate

V_W = first derivative of $V(W_t)$ (marginal value of wealth)

V_{WW} = second derivative of $V(W_t)$ (risk aversion effect)

The optimal allocation to the risky asset (π^*) satisfies the rule:

$$\frac{\partial U}{\partial \pi} = 0$$

To maximise the right-hand side, take the derivative with respect to π :

$$\frac{\partial}{\partial \pi} = [V_W(\mu - R_f)\pi + \frac{1}{2} V_{WW} \pi^2 \sigma^2] = V_W(\mu - R_f) + V_{WW} \pi^2 \sigma^2 = 0 \quad (13)$$

Solving for π^* in Eq. (13) gives Eq. (14):

$$\pi^* = - \frac{V_W(\mu - R_f)}{V_{WW} \sigma^2} \quad (14)$$

Since risk aversion is defined as:

$$\gamma = - \frac{V_{WW}}{V_W}$$

Substituting γ into the equation Eq. (14) generates Eq. (15).

$$\pi = \frac{(\mu - R_f)}{\gamma \sigma^2} \quad (15)$$

Therefore, solving the Hamilton-Jacobi-Bellman (HJB) equation, the optimal portfolio allocation to risky assets is given by:

$$\pi^* = \frac{(\mu - R_f)}{\gamma \sigma^2} \quad (16)$$

Where: $\mu - R_f$ = excess return of the risky asset, σ^2 = variance of the risky asset's returns, γ = risk aversion parameter, π^* = optimal portfolio allocation to risky assets. This Merton share formula shows that: If the *risk-adjusted excess return* ($\mu - r$) is high, investors allocate more to the risky asset. If investors are *highly risk-averse* (γ is large), they allocate less to the risky asset. If the risky asset's volatility increases, the optimal risky asset allocation decreases.

3.5. Optimal Consumption

Beginning from the Hamilton-Jacobi-Bellman (HJB) equation to optimal portfolio allocation, where an investor allocates wealth between risky and risk-free assets, as shown in Eq. (12) and

Since $U(c) = \frac{c^{1-\gamma}}{1-\gamma}$, substituting into Eq. (12) gives Eq. (17).

$$pV(W) = \max_{0 < (\pi, c) < 1} [\frac{c^{1-\gamma}}{1-\gamma} + V_W(\pi(\mu - R_f) + R_f W - c) + \frac{1}{2} V_{WW} \pi^2 \sigma^2] \quad (17)$$

Taking the first derivative with respect to C and solving for optimal consumption, the optimal consumption solution in the Merton portfolio problem is given in Eq. (18):

$$C_t^* = \frac{p}{\gamma} X_t \quad (18)$$

Where: C_t^* = optimal consumption at time t as a fraction of wealth. The investor consumes a fixed proportion of wealth at all times; γ is the coefficient of relative risk aversion; p is the rate of time preference capturing the impact of the discount rate, and X_t is the wealth at time t .

3.6. Merton Model for Wealth Allocation to Floating Rate CBDC

Using Merton's model in Eq. (16), the optimal holding of a floating interest CBDC follows a similar logic. Assuming the floating interest CBDC is a risky asset (exposed to risks such as fluctuating interest rate, possibility of negative interest rate, redemption restrictions, cybersecurity threats) that is pegged on the money market rate, the investor allocates wealth to risk-free assets (such as bonds and bills) and floating interest rate CBDC. Assuming further that investors are risk-averse. If an investor chooses between CBDC and risk-free assets, the optimal allocation to CBDC is given in Eq. (19).

$$\pi^* = \frac{R_{cbdc} - R_f}{\gamma \sigma^2} \quad (19)$$

Where: σ^2 = Variance of returns on CBDC (capturing uncertainty), R_{cbdc} = is the return (interest rate) on CBDC, R_f = is the risk-free assets returns (i.e. treasury bill), γ = Risk aversion coefficient. If $R_{cbdc} > R_f$, more wealth is allocated to CBDC and vice versa. Higher risk aversion (γ) reduces CBDC allocation if CBDC is perceived as risky. If the CBDC volatility increases, the optimal CBDC allocation decreases. Equation 16 implies that if CBDC interest rises relative to interest on T-bills, a rational investor increases CBDC allocation. Further, if the volatility of CBDC rises, allocation to it falls. The analytical framework adopted Eq. (19) assumes one representative, moderately risk-averse investor. Therefore, it applies to a theoretical investor profile and not all possible investor types.

3.7. Application of the Merton Model to Hypothetical and Real Data

We applied historical monthly data to the model in a deterministic fashion rather than continuous-time modelling as espoused in the model. In empirical finance papers like this one, CBDC returns, inflation, interbank lending rate, risk risk-free rate of return are observed at discrete intervals (such as daily, monthly, or quarterly) and not continuously. To operationalise continuous-time models like Merton's (1969), researchers apply discrete-time approximations that mimic the continuous process over small intervals. This is justified under the Euler-Maruyama approximation, which states that a continuous-time diffusion process can be accurately represented by discrete increments when the time step (Δt) is small [31]. Therefore, using monthly data is a finite difference approximation of the continuous process, capturing the essential stochastic behaviour of asset returns over manageable intervals. Further, although Merton's model is inherently stochastic, empirical studies often adopt deterministic estimation techniques to test its equilibrium implications. In such cases (like in this case), the model is interpreted not as a literal stochastic simulation, but as a deterministic mapping between observable economic factors and expected returns. This is

consistent with the model's equilibrium nature, where expectations are taken as conditional means of stochastic processes [32]. Therefore, the deterministic application focuses on the expected value relationships implied by the stochastic model, rather than simulating the underlying random paths.

3.7.1. Risk-Free Rate of Return

We adopted the interest on the 91-day treasury bill (TB) rate in Kenya as the risk-free rate of return. The interest rates on 91-day T-Bills serve as a benchmark for the risk-free rate due to their short duration and government backing. For instance, a study on the effects of interest rates on stock market returns in Kenya utilised T-Bill rates as a proxy for the risk-free rate [33]. The per annum rate was downscaled to a monthly rate by dividing the annual rate by 12 months. The rate was then applied as the return on short-term treasury financial asset.

3.7.2. Investor Subjective Discounting Rate

Investor subjective discounting rate reflects the rate at which investors discount future cash flows based on personal risk assessments and return expectations. We adopted the difference between risk risk-free rate of return and the inflation rate as the investor's time preference in line with Campbell, who explored real interest rates as indicators of investor expectations [34]. The investor's subjective discounting rate was calculated as:

$$p = R_f - INF_t \quad (20)$$

Where p is the subjective discounting rate, R_f is risk risk – free rate of return, and INF is the inflation rate.

3.7.3. Returns on Floating Interest Rate CBDC

We assumed that the Central Bank of Kenya has adopted a floating interest CBDC that is pegged to the interbank lending rate among commercial banks in Kenya. A constant policy spread (positive or negative) gives the central bank control over relative attractiveness vis-à-vis bank deposits. A negative spread reduces deposit flight risk; a small positive can encourage CBDC adoption but risks disintermediation [35]. We added a constant to the interbank lending rate to keep the rate above the bank deposit rate to make CBDC attractive for adoption. The floating interest on CBDC was calculated as:

$$R_{CBDC} = i_{\text{Interbank Lending Rate}} + C \quad (21)$$

Where C is the constant, arbitrarily taken as 3 percent. The interbank rate is given per day, so we generated monthly averages. Given that the rate was in annual terms, we divided the annual rate by 12 months to generate monthly equivalents. Although pegging the CBDC rate to the interbank lending rate restricts the central bank's ability to use the CBDC inde-

pendently as a new monetary policy instrument [36], its adoption in this study is justified given that the interbank rate is already the primary policy signalling instrument that reflects the stance of monetary policy. Linking the CBDC rate to it reinforces transmission consistency, ensuring that changes in monetary stance (such as tightening or easing) are uniformly transmitted to both digital and traditional forms of money [37]. Further, interbank rates may fluctuate due to transient liquidity shocks, potentially transmitting volatility to CBDC returns. However, in well-developed interbank markets, the central bank can typically stabilise short-term rate fluctuations through open market operations and standing facilities. Therefore, consequently, pegging the CBDC rate to the target interbank rate (policy rate) and not the observed volatile rate ensures predictable and policy-controlled adjustments [38]. Finally, even though pegging may not improve monetary transmission, since the CBDC merely mirrors existing interest rate structures, in economies with weak banking intermediation or cash dominance, pegging the CBDC rate to the interbank lending rate can strengthen the transmission mechanism by extending the reach of monetary policy directly to households and firms holding CBDC wallets [39]. Additionally, other possible benchmark rates, such as commercial deposit rates and bond yields, were considered and dropped in this study. The Commercial Bank Deposit Rates, interest rates paid by commercial banks on customer deposits, vary by bank, deposit size, and maturity, leading to rate fragmentation and lack of a uniform policy anchor [40]. Further, if CBDC rates follow average deposit rates, banks may manipulate deposit pricing to influence CBDC attractiveness, causing unintended disintermediation or arbitrage [36]. Further, the use of sovereign bond yields as a possible pegging rate for CBDC was considered. Bond yield is the yield on government securities of varying maturities, reflecting long-term borrowing costs and market expectations. Bond yields are subject to market speculation, fiscal risk, and global capital flows, making them unstable as short-term policy anchors [41]. Additionally, central banks could face a conflict between fiscal and monetary goals, as bond yields also reflect government borrowing conditions.

3.7.4. Variance of CBDC Interest Rate

We calculated the variance of the hypothesised returns of CBDC using the variance formulae.

$$\sigma^2 = \frac{\sum_{i=1}^n (x_i - \mu)(x_i - \mu)}{N} \quad (22)$$

3.7.5. Risk Aversion and Initial Wealth

An investor, call him Otieno, has a risk aversion coefficient of 3. We, following the Merton model, assume a risk-averse investor. Empirical studies and theoretical models suggest that the risk aversion coefficient for low risk aversion ranges from $0 \leq \gamma \leq 1$. Investors with low risk aversion are willing to

take high risks for high returns. Moderate Risk Aversion coefficient ranges between $1 \leq \gamma \leq 4$, and most investors fall in this range, balancing risk and return [42]. High Risk Aversion coefficient ranges between $4 \leq \gamma \leq 10$, describing investors who are highly risk-averse and prefer low-volatility assets such as bonds [41]. Extreme Risk Aversion coefficient ranges above 10 ($\gamma > 10$). These investors avoid almost all risk and prefer safe assets like T-bills or bank deposits [43]. The paper hypothetically assumed that the investor was moderately risk-averse, and thus the risk aversion coefficient was arbitrarily chosen at a value of 3, falling between 1 and 4. Finally, the paper adopted an initial allocable wealth of Ksh. 200,000 to be allocated between the floating interest CBDC and a short-term bond paying risk-free rate of return.

4. Results

4.1. Preliminary Descriptive Statistics of Model Input Variables

We presented the descriptive statistics of the Merton model input, including time preference for money, hypothesised CBDC floating rate, risk-free rate of return (The interest rates on 91-day T-Bills), the variance of the returns of the hypothesised CBDC, the inflation rate and the interbank lending rate in Kenya. All the input variables are presented in monthly rates (Table 1). The mean monthly rate of return for the hypothesised CBDC (R_{CBDC}) in Kenya was 0.010189 percent with a standard deviation of 0.003916 percent around the mean. The minimum value was -0.0004 percent and the maximum value was 0.0102 percent. The mean monthly interbank rate was 0.555 percent with a standard deviation of 0.1648 percent around the mean. The minimum monthly interbank rate was 0.226 percent while the maximum monthly interbank rate was 0.79916 percent. Further, the mean monthly inflation rate (INF) was 0.7689 percent with a standard deviation of 0.3047 percent around the mean. The minimum monthly inflation rate was 0.36344 percent while the maximum monthly inflation rate was 1.1426 percent. The mean monthly risk-free rate of return (R_f) was 0.009 percent with a spread of 0.002861 percent around the mean. The minimum monthly risk-free rate of return was $2.72E-05$, while the maximum monthly risk-free rate of return was 0.07684 percent. Additionally, the mean monthly subjective time preference was 0.004353 percent with a standard deviation of 0.003916 percent around the mean. Minimum subjective time preference was -0.00045 percent while the maximum subjective time preference was 0.010207 percent. Finally, the mean variance of the hypothesised returns of CBDC was 0.004984 percent with a spread of 0.01723 percent. The minimum and maximum variance of the hypothesised returns of CBDC was $2.72E-05\%$ and 0.076848 percent respectively.

Table 1. Descriptive Statistics of Model Input Variables.

	p	R_{CBDC}	R_f	σ^2_{CBDC}	INF	Inter Bank Rate
Mean	0.004353	0.010189	0.009905	0.004984	0.768900942	0.55525463
Standard Dev.	0.003916	0.003047	0.002861	0.01723	0.304718283	0.164837079
Kurtosis	-1.71062	-1.79256	-1.6894	15.16194	-1.792564692	-0.644432649
Skewness	0.171171	-0.154	0.088175	4.027696	-0.153996335	-0.392880313
Minimum	-0.00045	0.006134	0.006043	2.72E-05	0.363446825	0.226666667
Maximum	0.010207	0.013927	0.013898	0.076848	1.142654167	0.799166667
Count	36	36	36	36	36	36

p = subjective time preference, R_{CBDC} = hypothesised CBDC floating rate, R_f = risk-free rate of return, σ^2_{CBDC} = variance of hypothesised CBDC floating rate, INF = inflation rate

4.2. Numerical Findings

In this paper, we applied data to the Merton model components in an effort to establish terminal wealth, consumption behaviours, and portfolio allocation between floating interest CBDC and risk-free asset. By employing hypothesised and real-world data, the paper emphasises the effect of variables such as asset returns, volatility, and interest rates on portfolio results, while also highlighting the trade-offs involved in consumption versus investment choices. Monthly data on Merton model components were applied beginning from January 2022.

4.2.1. Optimal Wealth Allocation to Floating Interest Rate CBDC

We utilised the Merton Optimal Investment-Consumption Model to analyse hypothetical data. Optimal allocation to floating interest rate CBDC by an investor was determined based on Eq. (19):

$$\pi^* = \frac{(R_{CBDC} - R_f)}{\gamma \sigma^2}$$

For the first month of January 2022, the study calculated optimal investment in floating interest rate CBDC based on the following:

$$R_{CBDC} = 0.6134468\%; R_f = 0.6099833\%; \gamma = 3, \text{ and } \sigma^2 = 0.028988\%$$

Therefore, the optimal allocation of wealth to a hypothetical floating interest rate CBDC in January 2022 was calculated as:

$$\pi^* = \frac{(0.006134 - 0.006099)}{3(0.0002898)} = 0.039826743 \text{ or } 3.98\%$$

For the second month of February 2022, the study calculated optimal investment in floating interest rate CBDC based on the following:

$$R_{CBDC} = 0.6438408\%; R_f = 0.606375\%; \gamma = 3; \text{ and } \sigma^2 = 0.021769\%$$

The optimal allocation of wealth to floating interest rate CBDC in February 2022 was calculated as:

$$\pi^* = \frac{(0.006438408 - 0.00606375)}{3(0.00021769)} = 0.573688392 \text{ or } 57.36\%$$

The findings showed that an increase in the monthly interest rate on CBDC from 0.6134468 percent in January 2022 to 0.6438408 percent in February 2022, accompanied by reduced volatility in returns from CBDC from 0.028988 percent to 0.021769 percent resulted in increased wealth allocation to floating interest rate CBDC from 3.98 percent in January 2022 to 57.36 percent in February 2022. Therefore, increasing interest in and declining volatility of CBDC resulted in more wealth allocation to the CBDC by the risk-averse investor. The finding agrees with Kozak, who noted that increasing interest rates on bonds makes bonds more appealing to investors, potentially drawing capital away from the stock market [28]. The study thus went ahead and calculated the optimal allocation of wealth to floating interest rate CBDC for all 36 months from January 2022 to December 2024, as presented in Figure 3. The study constrained the value between 1 and 0 to meet the demands of the Merton model. A value of 1 implied that all wealth was invested in a hypothetical floating interest rate CBDC, while a value of 0 implied that all the wealth was invested in risk-free asset.

4.2.2. Optimal Monthly Consumption

The optimal consumption solution in the Merton portfolio problem was derived based on the formulae in Eq. (18):

$$C_t^* = \frac{p}{\gamma} X_t$$

For the first month (January 2022), the study calculated optimal consumption as:

$$\gamma = 3; p = 0.001608167; \text{ and } X_t = \text{Ksh. } 200,000.$$

Therefore, the optimal consumption in January 2022 was:

$$C_t^* = \frac{0.001608167}{3} * 200,000 = \text{Ksh. } 107.21$$

For the second month (February 2022), the study calculated optimal consumption as:

$$\gamma = 3; p = 0.001830417; \text{ and } X_t = \text{Ksh. } 201,113.03$$

Therefore, the optimal consumption in February 2022 was:

$$C_t^* = \frac{0.001830417167}{3} * 201,113.03 = \text{Ksh. } 122.70$$

The findings showed that the increased subjective discounting rate from 0.1608167 percent in January 2022 to 0.1830417 percent in February 2022, accompanied by increased wealth to Ksh 201,113.03 resulted in increased optimal monthly consumption from Ksh. 107.21 in January 2022 to Ksh. 122.70 in February 2022. Thus, increasing the investor's subjective discounting rate resulted in increased current consumption. The study went ahead and calculated the optimal consumption for all 36 months from January 2022 to December 2024 as presented in Figure 2. The study con-

strained the value to a minimum of zero, implying no consumption taking place to meet the demands of the Merton model.

4.2.3. Wealth Dynamics

The total wealth at the end of January 2022 was arrived at using deterministic wealth dynamics formulae given as:

$$X_t = [\pi_t X_{t-1} (1 + R_{CBDC})] + [(1 - \pi_t)(X_{t-1})r_f] - C_t$$

Where

Therefore, total wealth at the end of January 2022 was given by:

$$X_t = [(0.0398)(200,000)(1 + 0.006)] + [(1 - 0.0398)(200,000)(0.006)] - 107.21$$

Therefore, $X_t = \text{Ksh. } 201,113.03$

We went ahead and calculated the total wealth for all 37 months from January 2022 to December 2024 as given in Figure 1.

4.3. Trends for Wealth Dynamics, Optimal Consumption and Allocation to CBDC

The section presents the trend for wealth dynamics, optimal consumption and allocation to floating interest Rate CBDC. Total wealth per month, monthly optimal consumption and optimal weights to floating interest CBDC was examined based on time series line graphs.

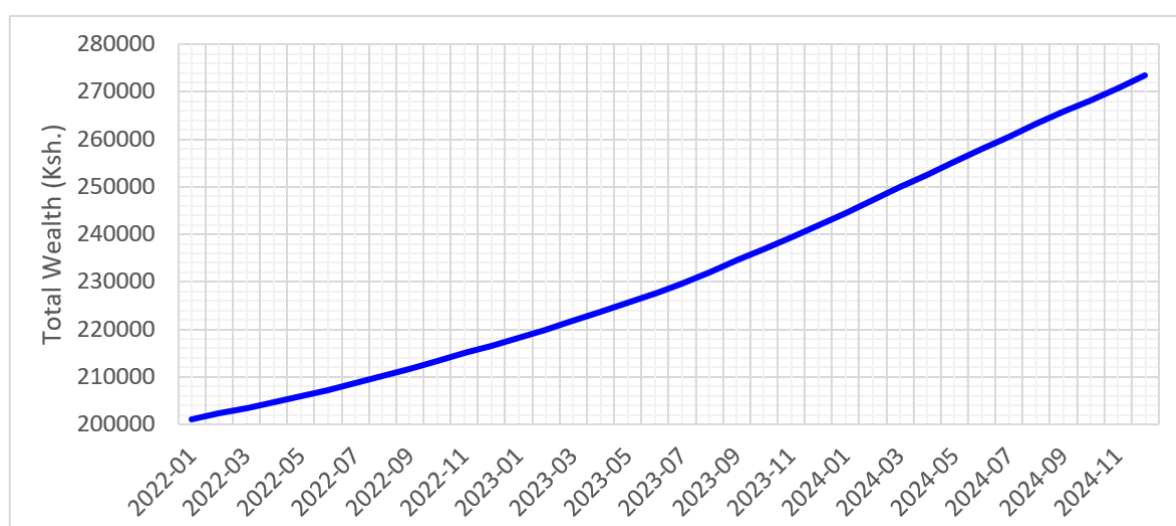


Figure 1. Wealth Evolution (January 2022 - December 2024).

As presented in Figure 1, the total investor wealth had increased from the initial Ksh. 200, 000 in January 2022 to Ksh. 273,498.49 at the end of December 2024. The growth in

wealth was from investment in hypothetical floating interest rate CBDC and treasury bonds.

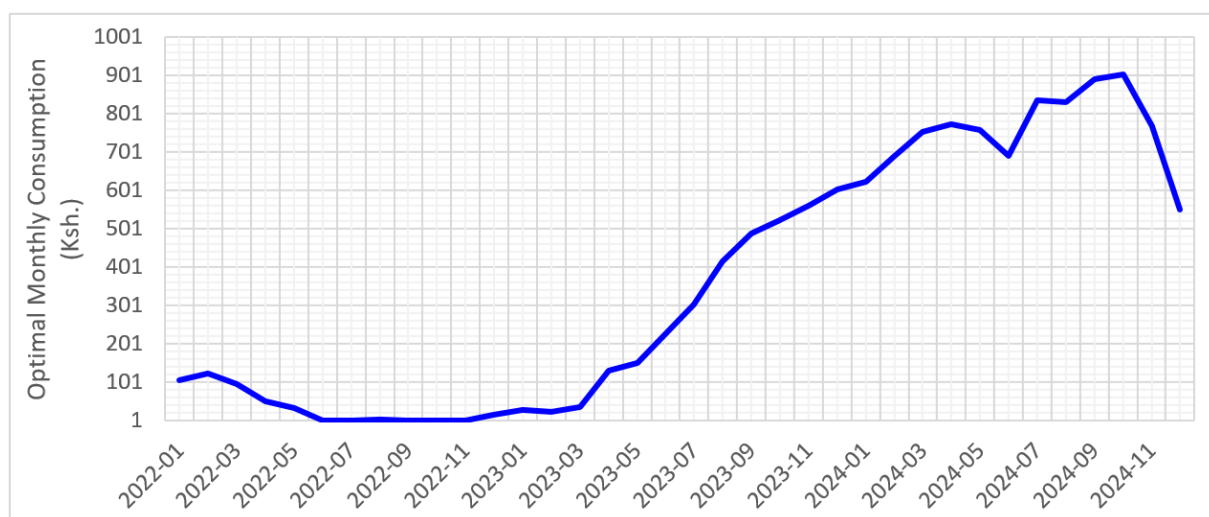


Figure 2. Optimal Monthly consumption (January 2022 - December 2024).

The trend for optimal monthly consumption initially rose to Ksh.122.7 in February 2022 before falling and remaining Ksh. 0 between June 2022 to December 2022 (Figure 2). Thereafter, the consumption began to rise steadily as total wealth grew,

reaching a maximum of Ksh. 904 in October 2024. The cumulative consumption in the study window reached Ksh. 13,001.46.

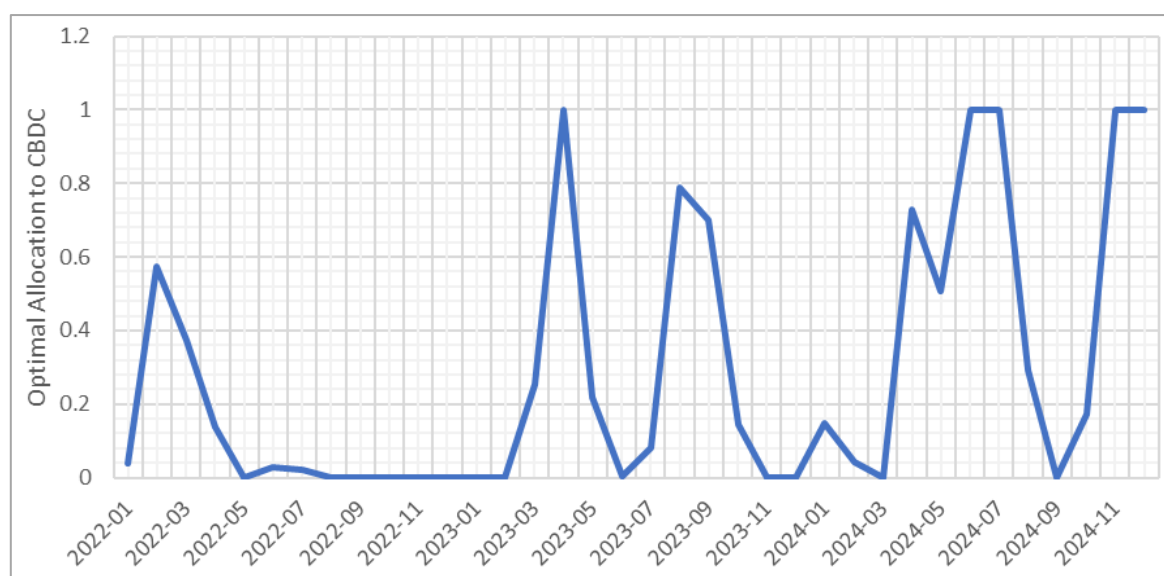


Figure 3. Optimal Weights to Floating Interest CBDC.

The study applied controls to optimal asset allocation to a hypothetical floating interest rate CBDC by ensuring that the minimum weight was zero and the maximum weight was 1. This was achieved via the use of the IF function in Excel, where negative weights were corrected to zero and weights greater than 1 were corrected to 1. Figure 3 shows that the optimal allocation to a hypothetical floating interest rate CBDC changed in the data collection window. The Merton model adjusted CBDC weights dynamically based on considerations of risk and time horizon.

4.4. Summary for Total Wealth, Monthly Consumption and Optimal Allocation to CBDC

The mean monthly optimal consumption based on the Merton model for the investor was Ksh. 361.1517 with a standard deviation of Ksh.333.285 around the mean in the study period. The minimum optimal consumption was Ksh. 0, and the maximum monthly optimal consumption was Ksh.

904.3419. The mean optimal allocation to the hypothesised floating rate CBDC was 0.284804248 (28.48%) with a spread of 0.368196921(36.8%) around the mean. The minimum and maximum mean optimal allocation to the hypothesised floating rate CBDC was 0(0.0%) and 1(100%), respectively.

Finally, the mean total wealth per month was Ksh. 232,249.86 with a spread of Ksh. 22,414.69 around the mean. The minimum total wealth was Ksh. 201113.03 and the Maximum total wealth was 273,498.49.

Table 2. Summary for Total wealth, Monthly consumption and Optimal allocation to CBDC.

	Total wealth (Ksh)	Monthly consumption (Ksh)	Optimal allocation to CBDC
Mean	232249.8646	361.1517	0.284804248
Standard Deviation	22414.69107	333.285	0.368196921
Kurtosis	-1.170347118	-1.62865	-0.406201408
Skewness	0.329571113	0.276889	1.072155619
Minimum	201113.0314	0	0
Maximum	273498.4936	904.3419	1
Count	36	36	36

4.5. Sensitivity Analysis of Optimal Wealth Allocation to Floating Interest Rate CBDC

All the earlier calculations were based on a constant risk aversion coefficient of 3. In the interest of establishing how wealth allocation to hypothetical CBDC rate and risk-free

assets changed if investor risk aversion changed, we undertook a sensitivity analysis by varying the risk aversion coefficient between 1 and 4. The finding in Figure 4 showed that at any point in time, the more the investor became risk averse from a risk aversion coefficient of 1 to 4, holding other variables constant on the Merton model adopted, the allocation to risky assets fell.

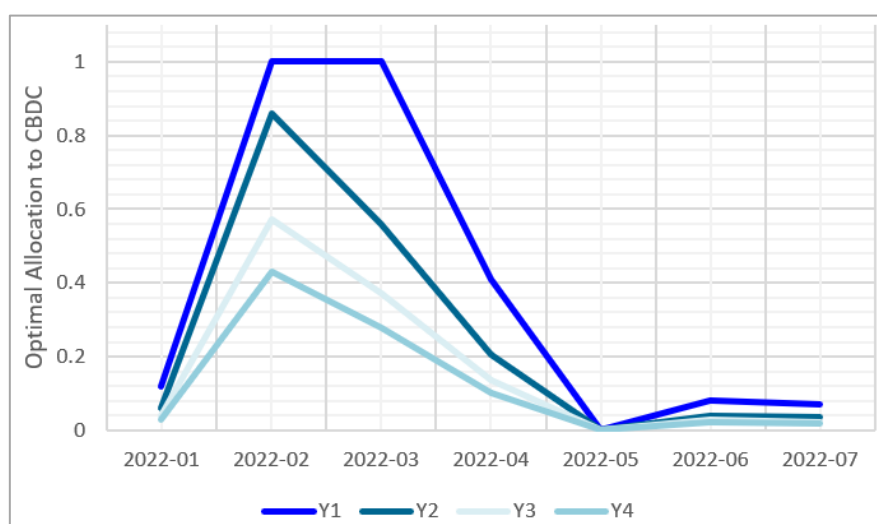


Figure 4. Optimal Weights to Floating Interest CBDC at varying risk aversion.

5. Discussion

The findings revealed that an increase in the monthly in-

terest rate on CBDC from 0.6134468 percent in January 2022 to 0.6438408% in February 2022, coupled with a decline in return volatility from 0.028988 percent to 0.021769 percent, led to a substantial rise in wealth allocation to the

floating interest rate CBDC from 3.98 percent in January 2022 to 57.36 percent in February 2022. This suggests that an increase in interest rates and a reduction in volatility made the CBDC more attractive to risk-averse investors. This finding aligns with Kozak's findings, who observed that rising interest rates on bonds enhance their appeal to investors, potentially diverting capital from the stock market [28]. The study further computed the optimal wealth allocation to the floating interest rate CBDC over 36 months, from January 2022 to December 2024, as illustrated in Figure 3. Figure 3 showed that weight allocation to floating interest rate CBDC varied between 0 and 1 in conformity with the requirements of the Merton model, where a value of 1 indicated that all wealth was invested in the hypothetical floating interest rate CBDC, while a value of 0 signified full investment in a risk-free asset. The Merton model, therefore, dynamically adjusted the CBDC weights based on changing risk conditions and investment time horizons. It is worth noting that the study uses hypothetical CBDC returns derived from interbank rates. While logical, it is still an indirect measure, and the results are not real-world tested. Therefore, the findings are illustrative rather than predictive. Further, the findings indicated that an increase in the subjective discount rate from 0.1608167 percent in January 2022 to 0.1830417 percent in February 2022, coupled with a rise in total wealth to Ksh. 201,113.03, led to a corresponding increase in optimal monthly consumption from Ksh. 107.21 in January 2022 to Ksh. 122.70 in February 2022. This suggests that a higher subjective discount rate, reflecting greater preference for present consumption, resulted in increased current consumption by the investor. In the Merton (1969) model, a higher subjective discount rate signifies that the investor places greater weight on present utility relative to future utility. This parameter directly affects the optimal consumption decision over time by altering the intertemporal trade-off between current and future consumption. A higher discount rate implies that the investor perceives future utility as less valuable, either because of impatience, uncertainty about the future, or shorter planning horizons. To maximise lifetime utility under such preferences, the investor optimally shifts consumption toward the present period, even if it reduces future consumption potential. Subsequently, the study computed the optimal consumption levels for the entire 36-month period from January 2022 to December 2024, as illustrated in Figure 2. In alignment with the assumptions of the Merton (1969) model, the consumption values had a minimum of zero, indicating periods when no consumption occurred. The trend in optimal monthly consumption initially rose to Ksh. 122.70 in February 2022, then declined to Ksh. 0 between June 2022 and December 2022 (Figure 2). Thereafter, consumption increased steadily as total wealth expanded, peaking at Ksh. 904 in October 2024. Overall, the cumulative consumption over the study period amounted to Ksh. 13,001.46. The monthly consumption values (e.g., Ksh. 107 in Jan 2022, rising to Ksh. 900 in 2024) are very small compared to the

wealth base, which looks unrealistic. However, it is important to note that the subjective discount rate (ρ) was relatively low in the study window; the investor therefore placed more value on future consumption compared to present consumption. This intertemporal preference resulted in reduced current consumption as the investor saves more today to smooth consumption over time [44, 45].

In such a case, the model predicts that consumption will grow gradually as wealth and investment returns accumulate. Additionally, the model imposed non-negativity and upper-bound constraints; the resulting consumption values are thus small fractions of wealth. Finally, as presented in Figure 1, the total investor wealth had increased from the initial Ksh. 200, 000 in January 2022 to Ksh. 273,498.49 at the end of December 2024. The growth in wealth was from investment in hypothetical floating interest rate CBDC and treasury bonds. The finding in Figure 4 also showed that at any point in time, the more the investor became risk-averse from a risk aversion coefficient of 1 to 4, holding other variables constant on the Merton model adopted, the allocation to risky assets fell. This implies that, risk aversion coefficient and optimal allocation to the hypothesised mildly risky CBDC were inversely related. Economically, this implies that more risk-averse investors prefer safer portfolios and allocate a greater share to the risk-free asset. Conversely, investors with lower risk aversion tolerate more uncertainty for potentially higher returns [46].

6. Conclusions

This paper applies the Merton model allocation of wealth to a hypothetical floating interest CBDC and risk-free assets. If a hypothetical floating interest CBDC is considered a risky asset, akin to money market funds or corporate bonds, the optimal allocation of wealth to a hypothetical interest-bearing CBDC happens where the first derivative of the expected utility function is equal to zero. The optimal wealth allocation to a hypothetical floating interest CBDC against risk-free assets depends on the interest rate on CBDC, CBDC returns volatility and the degree of investor risk aversion, holding the risk-free rate of return constant. Consequently, whenever a hypothetical CBDC offers a higher interest rate (R_{CBDC}) than alternative risk-free assets (R_f) such as treasury bills, investors should shift wealth to the hypothetical CBDC, and vice versa is true. Specifically, if the interest rate on the CBDC is equal to or lower than risk-free rate of return, investors should allocate all allocatable wealth to risk-free assets and not hold any CBDC. If CBDC has a higher risk (σ^2), such as floating interest, withdrawal restrictions or fees, investors should hold fewer units of interest-bearing CBDCs and more of risk-free assets, and vice versa is true. Additionally, optimal monthly consumption was a function of the coefficient of relative risk aversion, the rate of time preference (subjective discounting rate) and accumulated total wealth. Thus, whenever total wealth increased or risk aversion reduced or subjective discounting rate increased while holding other factors constant,

the individual investor consumed more as a proportion of total wealth. Finally, the total wealth of an individual risk-averse investor based on the Merton model evolved dynamically. This theoretical study was based on hypothesised values of floating interest rate CBDC, given that at the moment, no country has offered interest-bearing CBDC. Therefore, the results offer theoretical information for further discussion in the design of CBDC as far as the interest rate is concerned. The paper illustrated how individual investors can apply the Merton 1969 model to make decisions on wealth allocation to mildly risky hypothesised floating interest rate CBDC and risk-free assets such as treasury bills in emerging economies such as Kenya. The paper may serve as an insight to capital market authorities towards policy formulation and regulatory notes on institutional investors' optimal holding of floating interest CBDC vis-à-vis risk-free assets. The central banks, being the prime issuers of CBDC, would find this study insightful when setting or adjusting monetary policies by considering the possible impact on investor portfolios. Consequently, the study provides insights to the central banks regarding the design of CBDC, especially when remuneration is needed on the digital currency to achieve various monetary policy objectives. In applying the study findings, it is worth noting that the study uses hypothetical CBDC returns derived from interbank rates, and the results are not real-world tested; and findings are illustrative rather than predictive. The Merton model adopted is based on the assumption of Brownian movement in returns of floating interest rate CBDC. In reality, a floating interest rate especially pegged on the interbank lending rate has continuous time aspects as well as discrete aspects. Future studies can thus present a model version that captures both continuous time and discrete data. Additionally, the paper was based on the assumption of one individual investor's utility function. In reality, there are as many utility functions as the number of individual and institutional investors. Future studies can also experiment on other possible designs of remunerated CBDC, such as fixed rate and graduated rate, to observe how such different interest regimes on CBDC affect investors' optimal holdings of CBDC. The impact of optimal holding of CBDC by investors on monetary instruments and financial stability should also be investigated in future studies.

Abbreviations

CBDC	Central Bank Digital Currency
CBK	Central Bank of Kenya
T-bill	Treasury Bill
IMF	International Monetary Fund
ECB	European Central Bank
e-CNY	China's Digital Yuan
MPT	Modern Portfolio Theory
MVO	Mean-variance optimisation
SOFR	Secured Overnight Financing Rate
EONIA	Euro Overnight Index Average

SIBOR Shanghai Interbank Offered Rate

Author Contributions

Michael Ochieng Obuya: Conceptualization, Data curation, Formal Analysis, Methodology, Writing – original draft

Samuel Owino Onyuma: Project administration, Supervision, Validation, Writing – review & editing

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Data Availability Statement

The data is available from the corresponding author upon reasonable request.

Conflicts of Interest

The authors declare no conflicts of interest.

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Biography



His research interests are in area of derivatives finance, financial markets, AI applications in finance, and behavioural finance.



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Michael Ochieng Obuya: Derivatives finance, Behavioural finance, AI application in financial markets, Crypto-Finance, Financial markets, and Empirical finance.

Samuel Owino Onyuma: Securities market development, Market efficiency, Governance structures, Market competition and integration, and Market microstructure in developing economies.