



Research Article

Analysis of Coupled Thermal and Concentration Gradients in Nanofluid Boundary-Layer Flow over a Stretching Surface

Chukwuemeka Paul Amadi^{1,*} , Emeka Amos² , Davies Iyai³

Department of Mathematics, Rivers State University, Port Harcourt, Nigeria

Abstract

This study presents an analysis of coupled thermal and concentration gradients in nanofluid boundary-layer flow over a stretching surface under the influence of magnetic, thermal, and mass transport effects. The governing nonlinear partial differential equations describing momentum, energy, and nanoparticle concentration are formulated for an electrically conducting nanofluid and solved numerically using the finite element method. Emphasis is placed on the roles of key dimensionless parameters including the magnetic field parameter, thermal and solutal Grashof numbers, Prandtl number, Brownian motion, thermophoresis, heat source, heat absorption, Lewis number, and chemical reaction rate on velocity, temperature, and concentration distributions within the boundary layer. The results indicate that the applied magnetic field retards the flow due to the Lorentz force, while buoyancy forces arising from thermal and concentration differences enhance fluid motion along the stretching surface. Thermal profiles are strongly influenced by internal heat generation, heat absorption, thermophoresis, and fluid thermal diffusivity, whereas nanoparticle concentration is governed by the combined effects of Brownian diffusion, thermophoretic transport, chemical reaction, and mass diffusivity. The analysis highlights the strong coupling between heat and mass transfer mechanisms in nanofluid boundary layers and demonstrates that controlling these parameters can effectively regulate transport processes in applications such as polymer extrusion, cooling of stretching sheets, coating processes, and advanced energy systems.

Keywords

Nanofluid, FEM, Thermal, Concentration Gradients, Heat and Mass Transfers, Heat Absorption

1. Introduction

The study of convective heat and mass transfer in nanofluid flow over stretching surfaces has attracted significant attention due to its wide-ranging applications in industrial manufacturing, energy systems, biomedical engineering, and thermal management, where the incorporation of nanoparticles enhances thermal conductivity and heat transfer performance,

thereby motivating extensive investigations into magnetohydrodynamic effects, magnetic field, internal heat generation, heat absorption coefficient, thermal radiation, chemical reactions, non-Newtonian behavior. Several researchers have contributed to understanding the complex interactions governing nanofluid transport phenomena. Weli *et al* [1] investigated the

*Correspondence: Chukwuemeka Paul Amadi (Chukwuemeka.amadi2@ust.edu.ng)

Received: 24 February 2026; Accepted: 9 March 2026; Published: 17 March 2026



Copyright: © The Author(s), 2026. Published by Science Publishing Group. This is an **Open Access** article, distributed under the terms of the Creative Commons Attribution 4.0 License (<http://creativecommons.org/licenses/by/4.0/>), which permits unrestricted use, distribution and reproduction in any medium, provided the original work is properly cited.

fluid flow and transport within a vertical channel featuring exponentially decaying suction and a mobile wall. The governing equations are derived under the assumptions of incompressible flow, incorporating buoyancy forces and viscous dissipation. A finite-difference scheme is developed and implemented for numerical analysis. The graphical representation of the results indicates that increases in the Brinkman number, suction parameter, and Prandtl number lead to elevated temperature distributions, while higher Thermal Grashof numbers, Mass Grashof numbers, and suction parameters enhance flow velocity. Sandeep and Kumar [2] investigated the heat and mass transfer magnetohydrodynamic (MHD) nanofluid flow containing conducting dust particles past an inclined permeable stretching sheet, considering radiation, non-uniform heat source/sink, volume fractions of nanoparticles and dust particles, and chemical reaction effects. Nandeppanavar *et al.* [3] investigated the stagnation point flow, as well as the heat and mass transfer characteristics of magnetohydrodynamic (MHD) nanofluid, induced by a porous stretching sheet within porous media, considering the influence of thermal radiation. Pal and Mandal [4] investigated the impact of nonlinearity in thermal radiation and a uniform magnetic field on the heat and mass transfer characteristics of Sisko nanofluid over a stretching sheet with convective boundary conditions. Amos and Uka [5] studied the hydromagnetic flow of nanofluids in the presence of radiation and non-uniform heat generation/absorption past an exponentially stretching sheet. Utilizing similarity transformations, the flow equations are converted into nonlinear coupled differential equations. These equations are subsequently solved using the method of asymptotic series. Ali *et al.* [6] provided a systematic review of carbon-based nanofluids, focusing specifically on carbon nanotubes, graphene, and nanodiamonds, and their applications in energy-related thermal systems. Tlili [7] formulated a mathematical model to comparatively study nanofluid and hybrid nanofluid flow between two eccentric pipes. Shafique *et al.* [8] investigated the transient magnetohydrodynamic (MHD) flow of a non-integer Maxwell fluid over a vertical plate, incorporating the effects of diffusion-thermo and heat absorption. The problem was modeled using the Caputo–Fabrizio fractional derivative, and the dimensionless governing equations were solved with the Laplace integral transform technique to obtain velocity, concentration, and temperature profiles, which were presented graphically for comparison and interpretation. Amos *et al.* [9] examined the free convection boundary layer flow of a rotating magnetohydrodynamic (MHD) fluid past a vertical porous plate, considering the effects of thermal radiation. They applied the perturbation technique to transform the dimensionless coupled governing boundary layer partial differential equations into ordinary differential equations, utilizing the Boussinesq and Rosseland approximations in their analysis. Karthik *et al.* [10] investigated the ternary nanofluid flow across a wedge providing valuable insights for optimizing heat exchangers, cooling mechanisms, and thermal control systems.

Kumar *et al.* [11] the two-dimensional double-diffusive convective flow of a hybrid nanofluid in an inverted T-shaped porous medium was investigated. The governing equations were formulated using the generalized Darcy–Forchheimer–Brinkman model, along with heat and mass transport equations. Walegign *et al.* [12] investigated heat and mass transfer in a two-dimensional flow of electrically conducting, thermally radiative, and chemically reactive Maxwell nanofluid toward a vertical stretching and permeable sheet within a porous medium. Boundary layer approximation and appropriate transformations are employed to simplify the governing differential equations for computational ease. Gireesha *et al.* [13] investigated the nonlinear convective heat and mass transfer of Oldroyd-B nanofluid over a stretching sheet while considering the presence of a uniform heat source or sink showing the impact of physical parameters on velocity, temperature, and concentration profiles. Sadighi *et al.* [14] investigated the theoretical application of the first law of thermodynamics in MHD nanofluid flow under various conditions including an inclined magnetic field, radiation, heat source/sink, viscous dissipation, Joule heating, concentration power-law exponent, and chemical reaction, occurring on a porous stretching surface submerged within a permeable Darcian medium. Alkbar *et al.* [15] investigated the numerical analysis of convective heat transfer and flow characteristics of hybrid nanoparticles composed of molybdenum disulfide and graphene oxide within a circular domain featuring narrow edge fins. A novel fin configuration is integrated with the convective heat transfer analysis, considering thermal convection and magnetohydrodynamic effects for steady, incompressible, laminar flow. The governing partial differential equations are solved numerically using finite element simulations, and the impact of fin count on thermal efficiency is assessed for 4 and 10 fins. Streamlines, isotherms, and Nusselt number distributions are analyzed under key dimensionless parameters, revealing that an increased number of fins enhances heat transfer through improved fluid mixing and larger recirculation zones. Overall, the synergistic effects of hybrid nanofluid flow significantly improve heat absorption, flow behavior, and thermal efficiency. Akhtar *et al.* [16] studied the heat transfer characteristics of hybrid nanoparticles, electroosmosis with microbial effects, and the composition of nanofluids. The non-Newtonian nature of blood flow is analyzed through the Johnson–Segalman model, and the application of hybrid nanoparticles with electroosmosis promotes the practical study of biofluidic phenomena. Moreover, this analysis illustrated the significance of microorganisms on the rheological characteristics of flow dynamics. The physical phenomenon is modeled using the governing partial differential equations, which accurately describe the flow dynamics. Numerical solutions are computed using the finite element method, ensuring precise results for complex systems. The results of velocity, temperature, streamlines, microorganism and nanoparticle concentrations, and pressure gradients are plotted against pertinent parameters to examine their influence on blood flow rheology. Alkbar *et*

al, [17] investigated the effects of the Hall current on the transport of clean water-based nanofluids containing silver and copper nanoparticles through a vertical rotating channel, incorporating the Boussinesq approximation. Exact analytical solutions of the governing equations, subject to relevant physical boundary conditions, were obtained through non-dimensionalization techniques. The influence of key parameters including the Grashof number, radiation parameter, Prandtl number, frequency parameter, magnetic parameter, Hall parameter, and rotation parameter on primary and secondary velocity profiles, as well as temperature distribution, was presented graphically. The Hall effects and mixed convection in magnetohydrodynamic (MHD) nanofluid flow were analyzed in detail, and a comparative study of silver and copper nanoparticles was performed. Additionally, tabulated results illustrated the impact of these parameters on the convection rate and skin friction coefficient. The findings had significant implications for both geophysical fluid dynamics and engineering applications. Baig *et al*, [18] investigated the interaction between the impinging stagnation flow and the flow generated by the stretching surface of the cylinder was thoroughly examined. To implement the phase flow framework, water was considered as the base fluid with multi-walled carbon nanoparticles. Exact analytical solutions were obtained for this problem, and the results were evaluated graphically. A detailed heat transfer analysis, including graphical interpretation of the Nusselt number, was also conducted. Streamlines were presented to visualize the flow, highlighting the dominant effects of either the stretching-induced flow or the impinging stagnation flow resulting from the oncoming pressure. Alghamdi *et al*, [19] directed research attention toward hybrid nanofluids, in which two or more different types of nanoparticles are dispersed within a single base fluid. This hybridization strategy has attracted considerable interest because it exploits the synergistic interactions among different nanoparticles to further enhance heat transfer performance. Their investigations demonstrated that the careful selection of nanoparticle types and their concentrations can effectively modify key thermophysical properties, such as thermal conductivity, viscosity, and stability, thereby improving the overall efficiency of the base fluid. Consequently, hybrid nanofluids have been widely recognized for their potential applications in various industrial and engineering systems. Furthermore, the incorporation of nanoparticles into the base fluid has been reported to enhance heat transfer through several mechanisms, including increased thermal conductivity and intensified microscopic energy transport processes. Nadeem *et al*, [20] investigated the effects of concentration gradients, thermal buoyancy forces, and Brownian motion, which played a significant role in the transport phenomena. The fluid model also incorporated heat absorption, viscous dissipation, and thermal radiation effects to capture realistic physical conditions. By employing suitable similarity transformations, the governing partial differential equations were reduced to a coupled system of nonlinear ordinary differential equations. The resulting simplified

system was then solved numerically using appropriate computational techniques to obtain accurate solutions for the flow and heat transfer characteristics.

This study presents a novel analysis of transient laminar nanofluid flow over a vertically stretching sheet, accounting for buoyancy, MHD forces, heat source/sink, chemical reactions, and nanoparticle transport phenomena. The model captures the interactions between velocity, temperature, and concentration fields, revealing mechanisms for enhanced heat and mass transfer in industrial and biomedical applications.

2. Mathematical Formulation

Consider an unsteady, two-dimensional, incompressible nanofluid flow along a vertical surface under the influence of a transverse magnetic field. The flow is assumed to occur in a Cartesian coordinate system (x', y') , where x' is taken along the vertical plate and y' is normal to the plate. The velocity components in the x' and y' directions are denoted by u' and v' , respectively.

The nanofluid is assumed to be electrically conducting and subjected to a uniform magnetic field of strength B_0 applied normal to the flow direction. The effects of thermal buoyancy and concentration buoyancy are included through the Boussinesq approximation. Additionally, Brownian motion, thermophoresis, heat generation/absorption, viscous dissipation, and chemical reaction effects are considered in the present study.

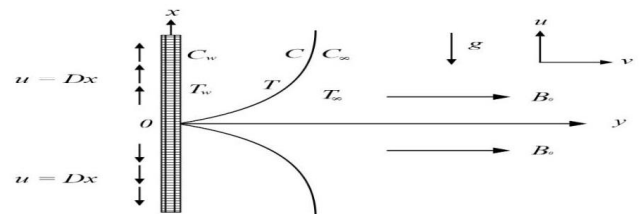


Figure 1. Schematic diagram of the flow.

Let T and C denote the temperature and nanoparticle concentration of the fluid, respectively. The quantities T_∞ and C_∞ represent the ambient temperature and nanoparticle concentration far away from the plate, while T_w and C_w denote the temperature and concentration at the surface of the plate. Under the boundary layer approximation and following Khan *et al*, [21] the governing equations describing the conservation of mass, momentum, energy, and nanoparticle concentration for the present problem are given as follows.

The continuity equation:

For an incompressible fluid, the conservation of mass is expressed as

$$\frac{\partial u'}{\partial x'} + \frac{\partial v'}{\partial y'} = 0 \quad (1)$$

This implies that u' and v' are the velocity components in the x' and y' directions respectively.

$\frac{\partial u'}{\partial x'}$ represents the rate of change of the u' component with respect to x' direction. This is only within a certain framework, like a coordinate transformation. Where u' and v' represents the transformed component of velocity.

Note that the continuity equation expresses the principle of conservation of mass in a fluid flow.

The Momentum equation:

The momentum equation along the x' -direction, including viscous diffusion, buoyancy forces due to temperature and concentration differences, and the magnetic field effect, is given by

$$\frac{\partial u'}{\partial t'} + u' \frac{\partial u'}{\partial x'} + v' \frac{\partial u'}{\partial y'} = \frac{\partial}{\partial y'} \left(\frac{\partial u'}{\partial y'} \right) + g\beta(T - T_\infty) + g\beta(C - C_\infty) - \frac{\sigma B_0^2 u'}{\rho} \quad (2)$$

The first term on the left-hand side represents the unsteady acceleration, while the second and third terms denote the convective acceleration of the fluid. The first term on the right-hand side represents viscous diffusion. The second and third terms correspond to thermal and solutal buoyancy forces, respectively. The last term accounts for the resistive Lorentz force produced by the applied magnetic field.

The Energy equation:

$$\frac{\partial T'}{\partial t'} + u' \frac{\partial T'}{\partial x'} + v' \frac{\partial T'}{\partial y'} = \frac{\partial}{\partial y'} \left(\alpha \frac{\partial T'}{\partial y'} \right) + \frac{Q_0}{\rho c_p} (T' - T_\infty) + \tau \left[D_B \left(\frac{\partial T'}{\partial y'} \frac{\partial C'}{\partial y'} \right) + \frac{D_T}{T_\infty} \left(\frac{\partial T'}{\partial y'} \right)^2 \right] + \frac{Q'}{\rho c_p} (C' - C_\infty) \quad (3)$$

The left-hand side represents the transient and convective heat transport within the fluid. On the right-hand side, the first term denotes thermal diffusion, the second term represents internal heat generation or absorption, and the third term accounts for the effects of Brownian motion and thermophoresis based on the nanofluid model. The final term represents heat generation due to concentration differences.

The Concentration equation:

The transport equation for nanoparticle concentration, incorporating Brownian diffusion, thermophoretic diffusion, and chemical reaction effects, is given as

$$\frac{\partial C'}{\partial t'} + u' \frac{\partial C'}{\partial x'} + v' \frac{\partial C'}{\partial y'} = D_B \frac{\partial^2 C'}{\partial y'^2} + \frac{D_T}{T_\infty} \frac{\partial^2 T'}{\partial y'^2} + K_r (C' - C_\infty) \quad (4)$$

The first term on the right-hand side represents Brownian diffusion of nanoparticles, the second term accounts for thermophoretic diffusion caused by temperature gradients, and the

last term describes the effect of chemical reaction on nanoparticle concentration.

Subject to the following initial and boundary conditions

$$t' = 0, u' = Dx, v' = 0, T' = T_\infty, C' = C_\infty \text{ everywhere.}$$

$$t' \geq 0, u' = 0, v' = 0, T' = T_\infty, C' = C_\infty \text{ at } x' = 0.$$

$$u' = Dx, v' = 0, T' = T_\infty, C' = C_\infty \text{ at } y' = 0.$$

$$u' = 0, v' = 0, T' = T_w, C' = C_w \text{ as } y' \rightarrow \infty. \quad (5)$$

In equations (1) – (5), primes represent dimensional variables. The above system of equations represents the coupled behavior of velocity, temperature, and nanoparticle concentration fields in the presence of magnetic field effects, buoyancy forces, Brownian motion, thermophoresis, and chemical reaction.

We introduce the following nondimensional variables

$$x' = \frac{xU_0}{v}, y' = \frac{yU_0}{v}, u' = \frac{u}{U_0}, v' = \frac{v}{U_0}, t' = \frac{tU_0^2}{v}, \theta = \frac{T' - T_\infty}{T_w - T_\infty}, \phi = \frac{C' - C_\infty}{C_w - C_\infty}, K_r' = \frac{K_r v}{U_0^2}$$

$$M = \frac{\sigma B_0^2}{\rho U_0^2}, G_r = \frac{g\beta(T_w - T_\infty)v}{U_0^3}, G_m = \frac{g\beta(C_w - C_\infty)v}{U_0^3}, Q = \frac{Q_0}{\rho c_p} \left(\frac{v}{U_0^2} \right), Q_1 = \frac{Q'}{\rho c_p} \left(\frac{v}{U_0^2} \right), Pr = \frac{v}{\alpha},$$

$$Le = \frac{v}{D_B}, N_b = \frac{(\rho c) p D_B (C_w - C_\infty)}{(\rho c) f v}, N_t = \frac{(\rho c) p D_T (T_w - T_\infty)}{(\rho c) f T_\infty v}. \quad (6)$$

We apply the nondimensional variables in the expressions in (6) into equations (1) - (5) and obtain the following partial differential equations with their corresponding initial and boundary conditions.

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \frac{\partial^2 u}{\partial y^2} + G_r \theta + G_m \phi - Mu \quad (7)$$

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \frac{\partial^2 u}{\partial y^2} + G_r \theta + G_m \phi - Mu \quad (8)$$

$$\frac{\partial \theta}{\partial t} + u \frac{\partial \theta}{\partial x} + v \frac{\partial \theta}{\partial y} = \frac{1}{Pr} \frac{\partial^2 \theta}{\partial y^2} + Q\theta + N_b \frac{\partial \theta}{\partial y} \frac{\partial \phi}{\partial y} + N_t \left(\frac{\partial \theta}{\partial y} \right)^2 + Q_1 \phi \quad (9)$$

$$\frac{\partial \phi}{\partial t} + u \frac{\partial \phi}{\partial x} + v \frac{\partial \phi}{\partial y} = \frac{1}{Le} \left[\frac{\partial^2 \phi}{\partial y^2} + \frac{N_t}{N_b} \frac{\partial^2 \theta}{\partial y^2} \right] + K_r' \phi \quad (10)$$

$$t \leq 0, u = 0, v = 0, \theta = 0, \phi = 0 \text{ everywhere.}$$

$$t \geq 0, u = 0, v = 0, \theta = 0, \phi = 0 \text{ at } x = 0$$

$$u = 1, v = 0, \theta = 1, \phi = 1 \text{ at } y = 0$$

$$u = 0, v = 0, \theta = 0, \phi = 0 \text{ at } y \rightarrow \infty \quad (11)$$

where the parameter M represents the magnetic parameter, which quantifies the influence of the applied magnetic field on the fluid motion. The term G_r denotes the thermal Grashof number, accounting for the buoyancy force generated due to temperature differences within the fluid. Similarly, G_m corresponds to the solutal Grashof number, which represents the buoyancy effects arising from concentration gradients. The parameter Q signifies the heat source parameter, describing the internal heat generation within the fluid, while Q_1 represents the heat absorption coefficient, which accounts for heat absorption effects in the system. Furthermore, Pr denotes the Prandtl number, expressing the ratio of momentum diffusivity to thermal diffusivity. The parameter Le is the Lewis number, defined as the ratio of thermal diffusivity to mass diffusivity. In addition, N_b represents the Brownian motion parameter, which characterizes nanoparticle diffusion due to random motion, whereas N_t denotes the thermophoresis parameter, describing the migration of nanoparticles induced by temperature gradients.

3. Method of Solution

These equations represent the conservative laws of momentum, energy, and particle species conservation, which are conventionally expressed as partial differential equations (PDEs). They illustrate the temporal evolution of these quantities while accounting for various transport mechanisms.

We are adopting a numerical approach called finite element method (FEM) to solve equation (8) to (10) respectively with their corresponding boundary condition in equation (11).

Discretizing the partial differential equations and their corresponding initial and boundary condition, we split the time domain into steps using the backward Euler scheme (semi-implicit means treating linear terms implicitly and the nonlinear terms explicitly). Then we let;

$u^n(y)$, $\theta^n(y)$, $\phi^n(y)$ be known at time step n .

$u^{n+1}(y)$, $\theta^{n+1}(y)$, $\phi^{n+1}(y)$ be solved at time step $n + 1$.

Δt be the time step size.

$\{\psi_i(y)\}_{i=1}^N$ with corresponding nodal values u^{n+1} , θ^{n+1} , ϕ^{n+1} .

We apply the Galerkin projection to test each equation by multiplying $\psi_i(y)$ and integrate over the domain Ω using integration by parts.

Momentum Equation

$$\int_{\Omega} \psi_j \left(\frac{u^{n+1} - u^n}{\Delta t} + u^n \frac{\partial u^n}{\partial x} + v^n \frac{\partial u^n}{\partial y} \right) dy = - \int_{\Omega} \frac{\partial \psi_j}{\partial y} \frac{\partial u^{n+1}}{\partial y} dy + \int_{\Omega} \psi_j (G_r \theta^n + G_m \phi^n - M u^{n+1}) dy \quad (12)$$

Energy Equation

$$\int_{\Omega} \psi_i \left(\frac{\theta^{n+1} - \theta^n}{\Delta t} + u^n \frac{\partial \theta^n}{\partial x} + v^n \frac{\partial \theta^n}{\partial y} \right) dy = - \frac{1}{Pr} \int_{\Omega} \frac{\partial \psi_j}{\partial y} \frac{\partial \theta^{n+1}}{\partial y} dy + \int_{\Omega} \psi_j \left(Q \theta^{n+1} + N_b \frac{\partial \theta^n}{\partial y} \frac{\partial \phi^n}{\partial y} + N_t \left(\frac{\partial \theta^n}{\partial y} \right)^2 + Q_1 \phi^n \right) dy \quad (13)$$

Concentration Equation

$$\int_{\Omega} \psi_j \left(\frac{\phi^{n+1} - \phi^n}{\Delta t} + u^n \frac{\partial \phi^n}{\partial x} + v^n \frac{\partial \phi^n}{\partial y} \right) dy = - \frac{1}{Le} \int_{\Omega} \frac{\partial \psi_j}{\partial y} \frac{\partial \phi^{n+1}}{\partial y} dy + \frac{N_t}{N_b Le} \int_{\Omega} \frac{\partial \psi_j}{\partial y} \frac{\partial \theta^n}{\partial y} dy + \int_{\Omega} \psi_j K_r \phi^{n+1} dy \quad (14)$$

Let β = mass matrix

K = Stiffness matrix

u , θ , ϕ = nodal value vectors.

F_u , F_{θ} , F_{ϕ} vectors from sources and explicit terms.

Therefore, equation (12), (13) and (14) becomes.

$$\beta \frac{u^{n+1} - u^n}{\Delta t} + A_u^n = -K u^{n+1} + \beta (G_r \theta^n + G_m \phi^n - M u^{n+1}) \quad (15)$$

$$\beta \frac{\theta^{n+1} - \theta^n}{\Delta t} + A_{\theta}^n = -\frac{1}{Pr} K \theta^{n+1} + \beta (Q \theta^{n+1} + N_b \theta^n \phi^n + N_t (\theta^n)^2 + Q_1 K \phi^{n+1}) \quad (16)$$

$$\beta \frac{\phi^{n+1} - \phi^n}{\Delta t} + A_{\phi}^n = -\frac{1}{Le} K \phi^{n+1} + \frac{1}{Le} \frac{N_t}{N_b} K \theta^n + \beta K_r \phi^{n+1} \quad (17)$$

where A_u^n , A_{θ}^n , A_{ϕ}^n are vectors from the explicit advection and nonlinear terms at time n .

$y \in [0, y_{max}]$ in the direction with N nodes such that $y_{max} \gg 1$ approximates $y \rightarrow \infty$.

$t = 0, \Delta t, 2\Delta t, \dots$

$u(y_i, 0) = 0$, $v(y_i, 0) = 0$, $\theta(y_i, 0) = 0$, $\phi(y_i, 0) = 0$ for $i = 1, \dots, N$

$u(y_i, 0) = 1$, $v(y_i, 0) = 1$, $\theta(y_i, 0) = 1$, $\phi(y_i, 0) = 1$ at $y = 0$

$u(y_i, 0) = 0$, $v(y_i, 0) = 0$, $\theta(y_i, 0) = 0$, $\phi(y_i, 0) = 0$ as $y \rightarrow \infty$ (18)

4. Result

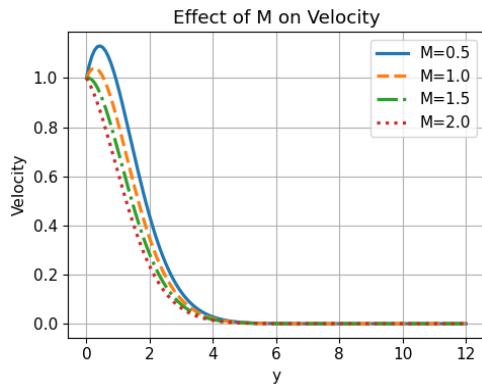


Figure 2. Effect of Magnetic field M on velocity of some parameter values $G_r=1.5$, $G_m=1.0$, $Pr=0.72$, $Q=0.2$, $N_b=0.1$, $N_t=0.1$, $Q_1=0.2$, $Le=2.0$, $Kr=0.3$.

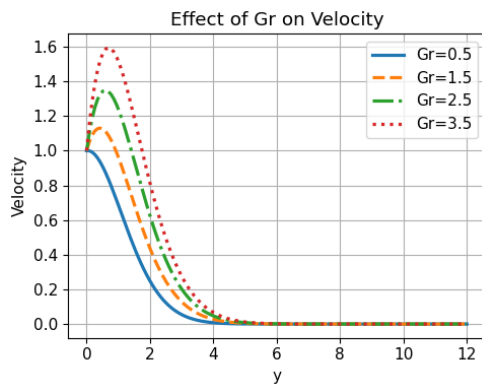


Figure 3. Effect G_r on velocity of some parameter values, $G_m=1.0$, $M=0.5$, $Pr=0.72$, $Q=0.2$, $N_b=0.1$, $N_t=0.1$, $Q_1=0.2$, $Le=2.0$, $Kr=0.3$.

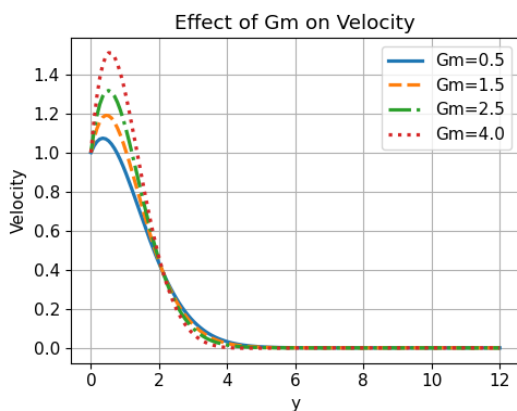


Figure 4. Effect of Mass grashof number G_m on velocity of some parameter values $M=0.5$, $G_r=1.5$, $Pr=0.72$, $Q=0.2$, $N_b=0.1$, $N_t=0.1$, $Q_1=0.2$, $Le=2.0$, $Kr=0.3$.

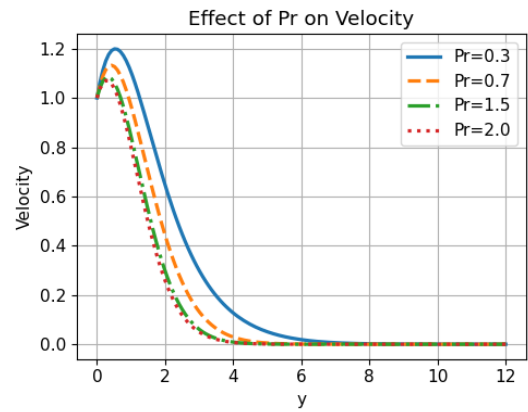


Figure 5. Effect of Prantl number Pr on velocity of some parameter values $M=0.5$, $G_r=1.5$, $G_m=1.0$, $Q=0.2$, $N_b=0.1$, $N_t=0.1$, $Q_1=0.2$, $Le=2.0$, $Kr=0.3$.

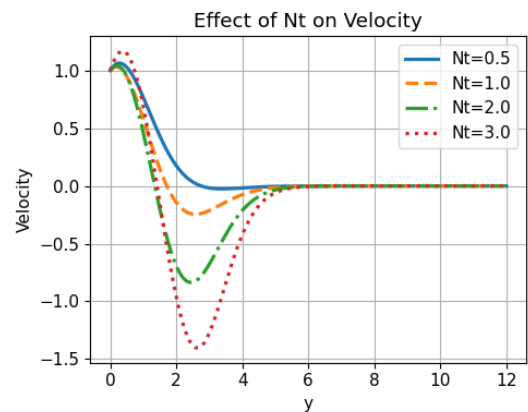


Figure 6. Effect of thermophoresis N_t on velocity of some parameter values $M=0.5$, $G_r=1.5$, $G_m=1.0$, $Pr=0.72$, $Q=0.2$, $N_b=0.1$, $N_t=0.1$, $Q_1=0.2$, $Le=2.0$, $Kr=0.3$.

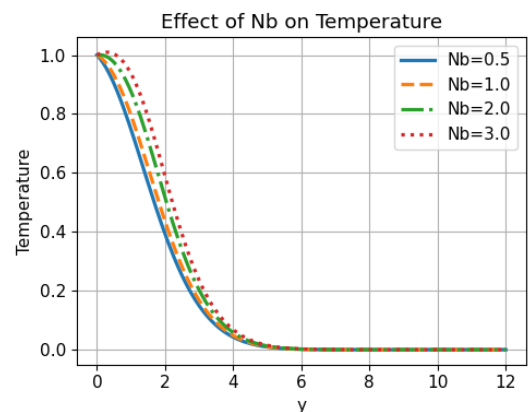


Figure 7. Effect of brownian motion N_b on velocity of some parameter values $M=0.5$, $G_r=1.5$, $G_m=1.0$, $Pr=0.72$, $Q=0.2$, $N_t=0.1$, $Q_1=0.2$, $Le=2.0$, $Kr=0.3$.

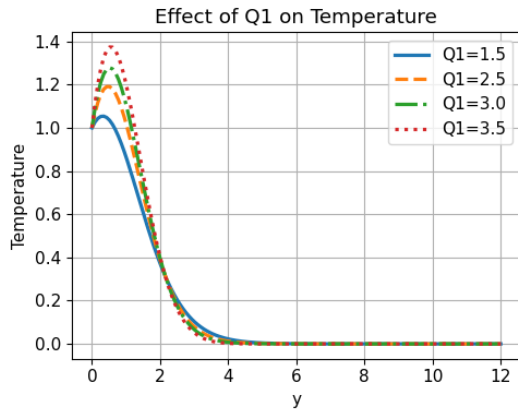


Figure 8. Effect of heat absorption Q_1 on the temperature of some parameter values $M=0.5$ $G_r=1.5$, $G_m=1.0$, $Pr=0.72$, $Q=0.2$, $N_b=0.1$, $N_t=0.1$, $Le=2.0$, $Kr=0.3$.

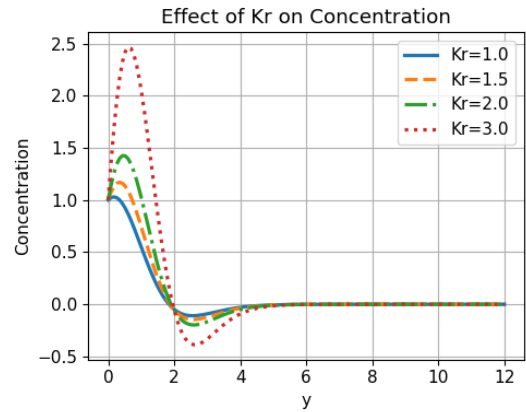


Figure 11. Effect of chemical reaction Kr on the concentration of some parameter values $M=0.5$ $G_r=1.5$, $G_m=1.0$, $Pr=0.72$, $Q=0.2$, $N_b=0.1$, $N_t=0.1$, $Q_1=0.2$, $Le=2.0$.

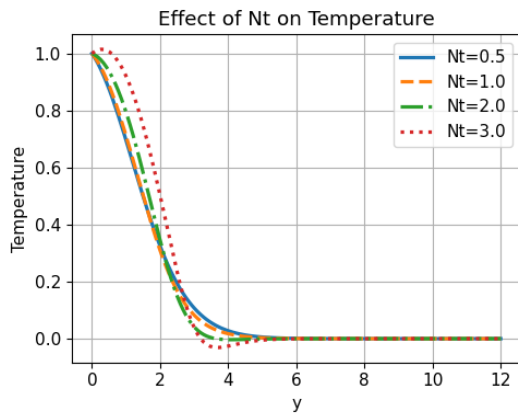


Figure 9. Effect of thermophoresis N_t on the temperature of some parameter values $M=0.5$ $G_r=1.5$, $G_m=1.0$, $Pr=0.72$, $Q=0.2$, $N_b=0.1$, $Q_1=0.2$, $Le=2.0$, $Kr=0$.

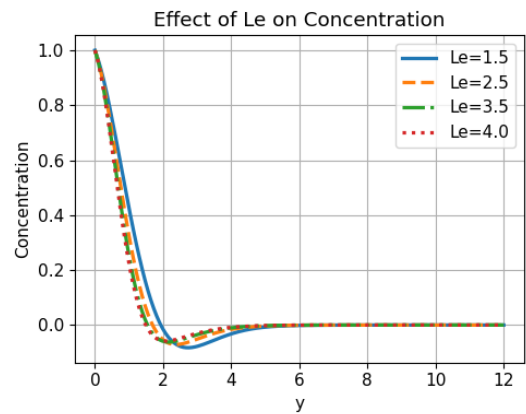


Figure 12. Effect of Magnetic field Le on the concentration of some parameter values $M=0.5$ $G_r=1.5$, $G_m=1.0$, $Pr=0.72$, $Q=0.2$, $N_b=0.1$, $N_t=0.1$, $Q_1=0.2$, $Kr=0$.

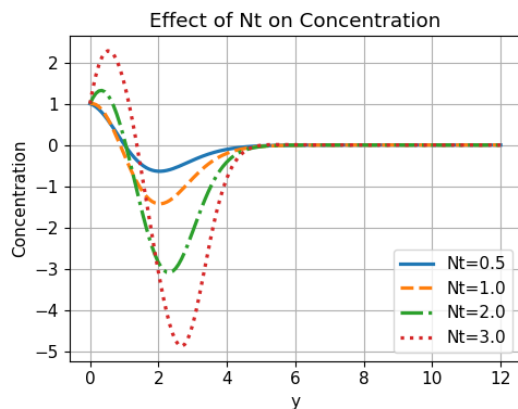


Figure 10. Effect of Thermophoresis N_t on the concentration of some parameter values $M=0.5$ $G_r=1.5$, $G_m=1.0$, $Pr=0.72$, $Q=0.2$, $N_b=0.1$, $Q_1=0.2$, $Le=2.0$, $Kr=0.3$.

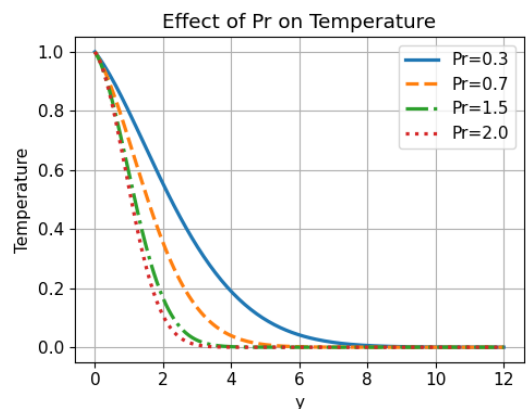


Figure 13. Effect of Prantl number Pr on temperature of some parameter values $M=0.5$ $G_r=1.5$, $G_m=1.0$, $Q=0.2$, $N_b=0.1$, $N_t=0.1$, $Q_1=0.2$, $Le=2.0$, $Kr=0.3$.

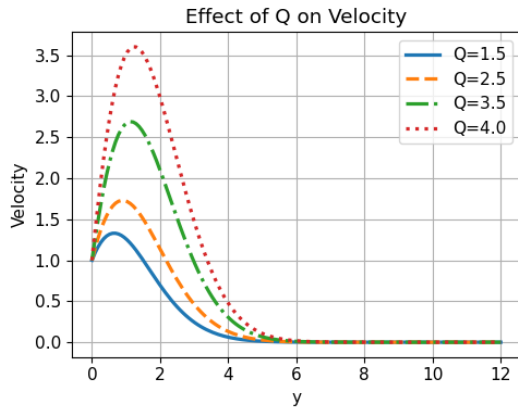


Figure 14. Effect of heat source Q on velocity of some parameter values $M=0.5$, $G_r=1.5$, $G_m=1.0$, $Pr=0.72$, $N_b=0.1$, $N_t=0.1$, $Q_1=0.2$, $Le=2.0$, $Kr=0.3$.

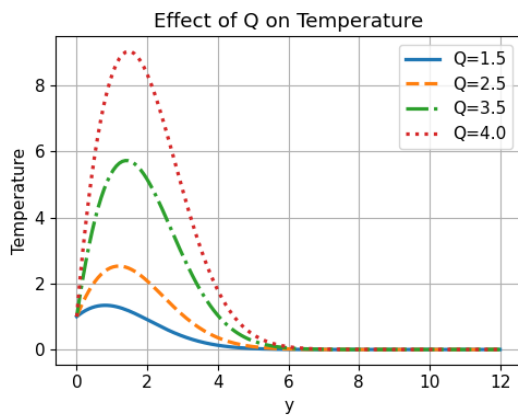


Figure 15. Effect of heat source Q on the temperature of some parameter values $M=0.5$, $G_r=1.5$, $G_m=1.0$, $Pr=0.72$, $N_b=0.1$, $N_t=0.1$, $Q_1=0.2$, $Le=2.0$, $Kr=0.3$.

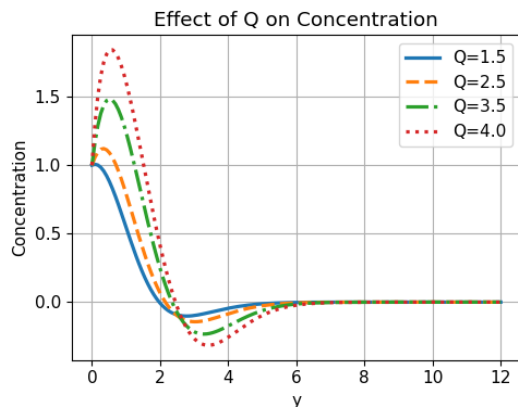


Figure 16. Effect of heat source Q on the concentration of some parameter values $M=0.5$, $G_r=1.5$, $G_m=1.0$, $Pr=0.72$, $N_b=0.1$, $N_t=0.1$, $Q_1=0.2$, $Le=2.0$, $Kr=0.3$.

5. Discussion of Results

The influence of the governing parameters on the flow characteristics is illustrated in Figures 1-6 through the velocity distributions. The application of a transverse magnetic field parameter M is observed to significantly reduce the velocity due to the resistive Lorentz force, which opposes the motion of the electrically conducting nanofluid and suppresses the momentum boundary layer. In contrast, increases in the thermal Grashof number G_r and the solutal Grashof number G_m enhance the velocity profiles because both parameters represent buoyancy forces arising from temperature and concentration differences, respectively. Stronger buoyancy promotes fluid acceleration near the surface and thickens the momentum boundary layer. Conversely, an increase in the Prandtl number Pr leads to a reduction in velocity, as higher Pr fluids possess greater viscous effects and lower thermal diffusivity, which weaken convective motion. The thermophoresis parameter N_t and Brownian motion parameter N_b produce secondary but noticeable influences on velocity by altering nanoparticle transport, fluid viscosity, and temperature gradients within the boundary layer.

Figures 8, 9, 13, and 15 present the temperature field behavior under varying thermal parameters. The heat absorption parameter Q_1 is found to reduce the temperature distribution because energy is removed from the system, resulting in a thinner thermal boundary layer. In contrast, increasing the thermophoresis parameter N_t elevates the temperature since thermophoretic forces transport nanoparticles from hotter to cooler regions, thereby enhancing thermal energy distribution. The temperature decreases with increasing Prandtl number Pr , which is attributed to the reduction in thermal diffusivity that limits heat penetration into the fluid. On the other hand, the presence of an internal heat source Q significantly increases the temperature field by supplying additional thermal energy, leading to a thicker thermal boundary layer and intensified heat transfer.

The concentration distributions depicted in Figures 10-12 and 16 reveal the effects of mass transport parameters. An increase in the thermophoresis parameter N_t enhances nanoparticle concentration away from the heated surface due to particle migration induced by temperature gradients. However, the chemical reaction parameter K_r reduces the concentration profile, indicating that stronger destructive reactions consume nanoparticles and diminish species diffusion. Similarly, increasing the Lewis number Le , which represents the ratio of thermal diffusivity to mass diffusivity, decreases the concentration boundary layer thickness because higher values correspond to weaker mass diffusion. The heat source parameter also influences concentration indirectly through coupled thermal mass diffusion effects.

6. Conclusion

The study demonstrates that momentum, heat, and mass transfer in electrically conducting nanofluids are strongly influenced by magnetic forces, buoyancy, thermal gradients, and nanoparticle transport phenomena such as thermophoresis, Brownian motion, and chemical reactions. Magnetic fields retard flow, buoyancy enhances velocity, and thermal and species transport parameters govern heat and concentration distributions. Strategic tuning of these factors can significantly improve nanofluid performance, offering practical benefits for industrial heat exchangers, microfluidics, biomedical systems, and energy applications. These results provide a robust framework for designing tailored nanofluids with enhanced thermal and mass transfer properties under complex operational conditions.

7. Limitation

This study has several limitations. The analysis is purely theoretical and numerical, lacking experimental validation, which limits direct real-world applicability. The flow is assumed laminar and two-dimensional, neglecting turbulence, three-dimensional effects, and flow separation. The nanofluid is treated as a single-phase homogeneous fluid with constant thermophysical properties, ignoring particle-fluid interactions, agglomeration, and temperature-dependent variations. Magnetic, chemical reaction, and thermal radiation effects are simplified, with the magnetic field considered uniform and reaction/radiation treated linearly. Future studies incorporating turbulence, multi-phase effects, variable properties, and experimental validation are needed to fully capture realistic hybrid nanofluid behavior.

Abbreviations

FEM	Finite Element Method
MHD	Magnetohydrodynamics

Author Contributions

Chukwuemeka Paul Amadi: Writing – original draft, Visualization, Methodology, Conceptualization, Resources

Emeka Amos: Writing – review & editing, Writing – original draft, Supervision

Conflicts of Interest

The authors declare there is no conflicts of interest.

References

- [1] Weli, A., Amadi, C. P., & Nwaigwe, C. (2019). Numerical investigation of transport in a Couette flow with unsteady suction. *IOSR Journal of Mathematics*, 15(6), 74-83. <https://doi.org/10.9790/5728-1506067483>
- [2] Sandeep, N., & Kumar, J. (2016). Heat and mass transfer in nanofluid flow over an inclined stretching sheet with volume fraction of dust and nanoparticles. *Journal of Applied Fluid Mechanics*, 9(5), 2205-2215. <https://doi.org/10.18869/acadpub.jafm.68.236.25282>
- [3] Nandeppanavar, M. M., Abel, M. S., & Kemparaju, M. C. (2017). Stagnation Point flow, heat and mass transfer of MHD nanofluid due to porous stretching sheet through porous media with effect of thermal radiation. *Journal of Nanofluids*, 6(1), 38-47.
- [4] Pal, D., & Mandal, G. (2019). Magnetohydrodynamic heat and mass transfer of Sisko nanofluid past a stretching sheet with nonlinear thermal radiation and convective boundary condition. *Journal of Nanofluids*, 8(4), 852-860.
- [5] Amos, E., & Uka, U. A. (2022). Hydromagnetic nanofluid flow over an exponentially stretching sheet in the presence of radiation and nonuniform heat generation. *IOSR Journal of Mathematics*, 18(1), 31-43.
- [6] Ali, N., Bahman, A. M., Aljuwayhel, N. F., Ebrahim, S. A., Mukherjee, S., & Alsayegh, A. (2021). Carbon-based nanofluids and their advances towards heat transfer applications—a review. *Nanomaterials*, 11(6), 1628. <https://doi.org/10.3390/nano11061628>
- [7] Tlili, I. (2024). Impact of thermal conductivity on the thermophysical properties and rheological behavior of nanofluid and hybrid nanofluid. *Mathematical Sciences*, 18(4), 493-501.
- [8] Shafique, A., Ramzan, M., Amir, M., Abbas, S., Nazar, M., Ali, E.,... & Garalleh, H. A. (2025). Heat and mass transfer effects on an unsteady MHD boundary layer flow of fractionalized fluid. *Radiation Effects and Defects in Solids*, 180(5-6), 708-720.
- [9] Amos, E., Amadi, C. P., & Nwaigwe, C. (2019). Free Convection Boundary Layer Flow of a Rotating MHD Fluid past a Vertical Porous Medium with Thermal Radiation. *International Journal of Applied Science and Mathematical Theory*, 5(2).
- [10] Kumar, K., Sarkar, J., & Mondal, S. S. (2024). Analysis of ternary hybrid nanofluid in microchannel-cooled cylindrical Li-ion battery pack using multi-scale multi-domain framework. *Applied Energy*, 355, 122241. <https://doi.org/10.1016/j.apenergy.2024.122241>
- [11] Kumar, S., Murthy, S. K., Kumar, B. R., & Parmar, D. (2024). Multi-physics study on double-diffusion convective flow in an inverted T-shaped porous enclosure: ANN-based parametric estimation. *Physics Letters A*, 518, 129681. <https://doi.org/10.1016/j.physleta.2023.129681>
- [12] Walelign, T., Haile, E., Kebede, T., & Awgichew, G. (2021). Heat and mass transfer in stagnation point flow of Maxwell nanofluid towards a vertical stretching sheet with effect of induced magnetic field. *Mathematical Problems in Engineering*, 2021(1), 6610099. <https://doi.org/10.1155/2021/6610099>

- [13] Gireesha, B. J., Kumar, K. G., Ramesh, G. K., & Prasanna-kumara, B. C. (2018). Nonlinear convective heat and mass transfer of Oldroyd-B nanofluid over a stretching sheet in the presence of uniform heat source/sink. *Results in Physics*, 9, 1555-1563. <https://doi.org/10.1016/j.rinp.2018.03.010>
- [14] Sadighi, S., Afshar, H., Jabbari, M., & Ashtiani, H. A. D. (2023). Heat and mass transfer for MHD nanofluid flow on a porous stretching sheet with prescribed boundary conditions. *Case Studies in Thermal Engineering*, 49, 103345. <https://doi.org/10.1016/j.csite.2023.103345>
- [15] Akbar, N. S., Akhtar, S., Shaiq, S., Hussain, M. F., Muhammad, T., Farooq, M., & Habib, M. B. (2025). Mixed convective heat transfer enhancement in hybrid nanofluid flow through complex-finned tube cavities. *Dynamics of Atmospheres and Oceans*, 101612. <https://doi.org/10.1016/j.dynatmoce.2025.101612>
- [16] Akhtar, S., Akbar, N. S., Akram, J., Hussain, M. F., & Muhammad, T. (2025). Heat transfer analysis on blood flow dynamics of hybrid nanoparticles in an asymmetric domain under electroosmotic and microbial effects: A Johnson-Segalman model approach. *International Journal of Heat and Fluid Flow*, 115, 109879. <https://doi.org/10.1016/j.ijheatfluidflow.2025.109879>
- [17] Akbar, N. S., Butt, A. W., Akhtar, S., & Tripathi, D. (2024). Hall effect on mixed convection MHD flow of nanofluid with clean, copper and Ag water through a vertical rotating channel. *ZAMM-Journal of Applied Mathematics and Mechanics/Zeitschrift für Angewandte Mathematik und Mechanik*, 104(12), e202200387. <https://doi.org/10.1002/zamm.202200387>
- [18] Baig, M. N. J., Salamat, N., Duraihem, F. Z., Akhtar, S., Nadeem, S., Alzabut, J., & Saleem, S. (2023). Exact analytical solutions of stagnation point flow over a heated stretching cylinder: A phase flow nanofluid model. *Chinese Journal of Physics*, 86, 1-11. <https://doi.org/10.1016/j.cjph.2023.05.001>
- [19] Alghamdi, M., Akbar, N. S., Hussain, M. F., Akhtar, S., & Muhammad, T. (2023). Thermodynamic study of hybrid nanofluid to explore synergistic effects of multiple ferromagnetic nanoparticles in co-axial disks for magnetized fluid. *Tribology International*, 188, 108867. <https://doi.org/10.1016/j.triboint.2023.108867>
- [20] Nadeem, S., Akhtar, S., & Abbas, N. (2020). Heat transfer of Maxwell base fluid flow of nanomaterial with MHD over a vertical moving surface. *Alexandria Engineering Journal*, 59(3), 1847-1856. <https://doi.org/10.1016/j.aej.2019.09.012>
- [21] Khan, M. S., Wahiduzzaman, M., Sazad, M. A. K., & Uddin, M. S. (2012). Finite difference solution of MHD free convection heat and mass transfer flow of a nanofluid along a Stretching sheet with Heat generation effects. *Indian J. Theor. Physics*, 60(4), 285-306.

Biography

Chukwuemeka Paul Amadi received his B.Sc. degree in Mathematics and Computer Science from Rivers State University of Science and Technology, Nigeria, and his M.Sc. degree in Applied Mathematics from Rivers State University, Port Harcourt, Nigeria. He is currently a Lecturer I in the Department of Mathematics at Rivers State University. His research interests include fluid dynamics, magnetohydrodynamics, nanofluid modeling, and numerical simulations. He is a registered member of the Mathematical Association of Nigeria (MAN) and the Nigerian Mathematical Society (NMS). He has published several research articles in reputable local and international journals, contributing to the advancement of applied mathematics research.

Emeka Amos earned a B.Sc. in Pure and Applied Mathematics, an M.Sc. in Radiation Physics, and a Ph.D. in Applied Mathematics from Rivers State University of Science and Technology, Port Harcourt, and a second M.Sc. in Applied and Industrial Mathematics from the Federal University of Technology, Owerri. A Professor of Mathematics at Rivers State University, he is a former Head of Department and currently Associate Dean of the Faculty of Science. His research focuses on fluid dynamics, blood-flow and magnetohydrodynamic modeling, nanofluids, and numerical simulation. A member of the Mathematical Association of Nigeria and the Nigerian Mathematical Society, he has published widely in local and international journals, supervised numerous M.Sc. and Ph.D. students, and serves as an external examiner for universities in Nigeria and abroad.