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Research Article

Derivations on Some Algebras of Measurable Operators Affiliated with Real W*-algebras of Type I

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Abstract

It is well known that every derivation on a von Neumann algebra is inner, which reflects the strong rigidity of these algebras. In contrast, for general C*-algebras there may exist non-inner derivations, indicating a more complicated and diverse algebraic structure. This fundamental difference has stimulated extensive research on derivations on various classes of operator algebras. In recent years, increasing attention has been paid to derivations defined on algebras of unbounded operators, in particular on algebras of measurable, locally measurable, and τ-measurable operators associated with von Neumann algebras. Such algebras arise naturally within the framework of noncommutative integration theory and provide a rich setting for extending classical results from the theory of bounded operators. In particular, a complete description of derivations on these algebras has been established in a number of works when they are associated with type I von Neumann algebras, demonstrating that under appropriate assumptions the derivations possess strong regularity properties and admit explicit representations. The present article is devoted to the development of a real analogue of the results described above. More precisely, derivations on algebras of measurable, locally measurable, and τ-measurable operators associated with real type I von Neumann algebras are investigated. By carefully adapting the methods from the complex case and taking into account the specific algebraic and topological features of real operator algebras, a complete characterization of all derivations on the algebras under consideration is obtained. These results generalize known theorems for complex von Neumann algebras to the real setting and contribute to a deeper understanding of derivations on algebras of unbounded operators associated with real operator algebras.

Keywords

Derivations, Algebra of Measurable Operators, Locally Measurable Operator, τ -measurable Operator, Von Neumann Algebras

1. Introduction

Derivations on operator algebras on a Hilbert space have been widely studied. In particular, it is known that all derivations on a von Neumann algebra are inner, while on a C*-algebra there are non-inner derivations. Recently, considerable attention has been devoted to the study of derivations on algebras of unbounded operators. In papers [1-5] a full description of all derivations on algebras of measurable, locally measurable, and τ -measurable operators affiliated with von Neumann algebra of type I is given. Papers [6-9] study derivations on algebras of measurable and locally measurable operators affiliated with von Neumann algebras, focusing on their continuity and structural properties under natural measure-type topologies. In the present paper, a real analogue of these results is considered. More precisely, de-

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scriptions of derivations on some algebras of measurable, locally measurable, and τ -measurable operators affiliated with real von Neumann algebra of type I are obtained.

2. Derivations on Some Algebras of Measurable Operators

2.1. Preliminaries

Let M be a *-subalgebra of $B(H)$ of all bounded linear operators on complex Hilbert space H. The set $M' = \{x \in B(H): xy = yx, \forall y \in M\}$ is called the commutant of the *-algebra M. The set $Z(M) = M \cap M'$ is called the center of the algebra M. A *-subalgebra M with the property $M = M''$ is called W*-algebra, where M'' is the double commutant of M. It is equivalent to that, *-subalgebra M is weakly closed and consists identity (i.e. $\mathbf{1} \in M$). W*-algebras are also known as von Neumann Algebras. A W*-algebra M is called a *factor* if its center is trivial, i.e. $Z(M)$ coincides with $\{\lambda 1: \lambda \in \mathbb{C}\}$. M is called discrete, or of type *I*, if for every nonzero central projection z there exists a nonzero abelian projection q such that $q < z$. M is called of *type* I_∞ if M is a nonfinite (infinite) algebra of type *I*.

A functional $\tau: M_+ \to [0;\infty]$ is called a *trace* on M_+ if

1) $\tau(T+S) = \tau(T) + \tau(S)$ for all $T, S \in M_+$;
2) $\tau(\lambda T) = \lambda \tau(T)$ for every $T \in M_+$ and $\lambda \geq 0$ (with the convention $0 \cdot \infty = 0$);
3) $\tau(U^* TU) = \tau(T)$ for every $T \in M_+$ and every unitary operator $U \in M$.

A trace τ is called *finite* if $\tau(T) < \infty$ for every $T \in M_+$; *semifinite* if $\tau(T) = \sup\{\tau(S): 0 \leq S \leq T, \tau(T) < \infty\}$ for every $T \in M_+$; *faithful* if from $\tau(T) = 0$ with $T \in M_+$ it follows that $T = 0$; *normal* if from $T_\alpha \uparrow T$ with $T, T_\alpha \in M_+$ it follows that $\tau(T_\alpha) \uparrow \tau(T)$.

A linear subspace $D \subset H$ is said to be *affiliated* with algebra M (notation: $D\eta M$) if $U(D) \subset D$ for every unitary operator $U \in M'$.

If D is a closed linear subspace of H and P_D denotes the orthogonal projection onto D, then $D\eta M$ if and only if $P_D \in P(M)$, where $P(M)$ is complete lattice of all projections from M.

A linear subspace $D \subset H$ is called *strongly dense* in H with respect to M if $D\eta M$ and there exists a sequence of projections $\{P_n\}_{n=1}^\infty \subset P(M)$ such that $P_n \uparrow I$, $P_n(H) \subset D$, $P_n^\perp$ is a finite projection for each $n = 1, 2, \ldots$.

In this case one says that the strongly dense linear subspace D is *determined* by the sequence of projections $\{P_n\}_{n=1}^\infty$.

From the condition $P_n \uparrow I$ it follows directly that every strongly dense linear subspace is dense in H.

A linear operator T with domain $D(T)$, acting in the Hilbert space H, is called *affiliated* with M (notation: $T\eta M$) if $UT \subset TU$ for every unitary $U \in M'$, that is, $D(T)\eta M$ and $UT\xi = TU\xi$ for all $\xi \in D(T)$.

It is easy to see that a bounded linear operator $T \in B(H)$ is affiliated with M if and only if $T \in M$. A closed linear operator T acting in a Hilbert space H is called *measurable* with respect to algebra M if $T\eta M$ and its domain $D(T)$ is strongly dense in H.

We denote by $S(M)$ the set of all operators that are measurable with respect to algebra M. Clearly, $M \subset S(M)$.

A closed linear operator T acting in a Hilbert space H is called *locally measurable* with respect to algebra $M \subset B(H)$ if $T\eta M$ and there exists a sequence of central projections $\{Z_n\}_{n=1}^\infty \subset P(Z(M))$ such that $Z_n \uparrow I$, $TZ_n \in S(M)$ for all $n = 1, 2, \ldots$.

We denote by $LS(M)$ the set of all linear operators that are locally measurable with respect to algebra M. It is clear that $S(M) \subset LS(M)$, and $S(M) = LS(M)$ if M is a factor.

2.2. Derivations on the Algebra $LS(R)$

Let A be an algebra over the complex numbers. A linear operator $D: A \to A$ is called a derivation if it satisfies the Leibniz rule $D(xy) = D(x)y + xD(y)$ for all $x, y \in A$. Every element $a \in A$ defines a derivation D_a on A by $D_a(x) = ax - xa$, $x \in A$. Such derivations D_a are called inner derivations.

If the element a that defines the derivation D_a belongs to a larger algebra B that contains A, then D_a is called a spatial derivation.

In the special case when A is commutative, all inner derivations are zero, so they are trivial. One of the main problems in the theory of derivations is to determine when derivations are automatically inner or spatial, and whether non-inner derivations exist, especially nontrivial derivations on commutative algebras.

Recall that, real *-subalgebra $R \subset B(H)$ is called real W*-algebra, if it is weakly closed, $\mathbf{1} \in R$, and $R \cap iR = \{0\}$. Real W*-algebras are also known as real von Neumann Algebras. Let R be a real von Neumann algebra and $M = R + iR$. We consider the algebra $S(R)$ of all measurable operators affiliated with a real von Neumann algebra R. It is proved that (see [10]) $S(M)=S(R)+iS(R)$, analogically it is proven that $LS(M)=LS(R)+iLS(R)$. Below, we give a full description of derivations on the algebra $LS(R)$ of all locally measurable operators affiliated with a type I real von Neumann algebra R.

It is clear that if a derivation D on $LS(R)$ is inner, then it is Z-linear, that is, $D(\lambda x)=\lambda D(x)$ for all $\lambda \in Z$, $x\in LS(R)$, where Z is the center of R. The following result shows that the opposite statement is also true.

Theorem 3.1. *Let R be a type I real von Neumann algebra with the center Z. Then every Z-linear derivation D on the algebra $LS(R)$ is inner.*

Proof. Since $LS(R,\tau)+iLS(R,\tau)=LS(R+iR,\tau)$, the derivation D can be extended by Z_c-linearity to derivation $\bar{D}$ on $LS(R+iR)$ as

$$\bar{D}(x+iy)=D(x)+iD(y),\ \ x,y\in LS(R),$$

where $Z_c=Z+iZ$ is a center of $LS(R+iR)$. Since every derivation on $LS(R+iR)$ is inner (see [2], Theorem 2.1), then derivation $\bar{D}$ is inner, i.e. there is an element $z=a+ib\ \ (a,b\in LS(R))$, such that $\bar{D}(x+iy)=[z,x+iy]=z(x+iy)-(x+iy)z,\ \forall x,y\in LS(R)$. Then we have $D(x)=\bar{D}(x)=[z,x]=[a+ib,x]=[a,x]+i[b,x].$

Since $x\in LS(R)$ and $D(x)\in LS(R)$, then $[b,x]=0$. Hence $D(x)=[a,x]$, i.e. D is inner. The theorem is proven.

Theorem 3.2. If R is a type I_∞ real von Neumann algebra, then any derivation on the algebra $LS(R)$ is inner.

The proof of the theorem 3.2 is carried out similarly to the proof of Theorem 3.1.

Suppose that A isa commutative algebra and suppose that $M_n(A)$ is the algebra of $n\times n$ matrices over A. If $e_{i,j},\ i,j=\overline{1,n}$, are the matrix units in $M_n(A)$, then every element $x\in M_n(A)$ has the form

$$x=\sum_{i,j=1}^{n}\lambda_{i,j}e_{i,j},\qquad \lambda_{i,j}\in A,\quad i,j=\overline{1,n}.$$

Let $\delta: A\to A$ be a derivation. Setting

$$D_\delta\left(\sum_{i,j=1}^{n}\lambda_{i,j}e_{i,j}\right)=\sum_{i,j=1}^{n}\delta(\lambda_{i,j})e_{i,j}\tag{1}$$

we obtain a well-defined linear operator D_δ on the algebra $M_n(A)$. Moreover D_δ is a derivation on the algebra $M_n(A)$ and its restriction onto the center of the algebra $M_n(A)$ coincides with the given δ.

Now let R be a homogeneous real von Neumann algebra of type I_n, $n\in\mathbb{N}$, with the center Z. Since $Z_c=Z+iZ$ is the center of M, it follows that $S(Z_c)=S(Z)+iS(Z)$ and $LS(Z_c)=LS(Z)+iLS(Z)$. Let $\delta: S(Z)\to S(Z)$ be a derivation and D_δ be a derivation on the algebra $M_n(S(Z))$ defined by (1). Extend D_δ to $M_n(S(Z_c))=M_n(S(Z))+iM_n(S(Z))$ by setting

$$\bar{D}_\delta\left(\sum_{i,j=1}^{n}\lambda_{i,j}e_{i,j}\right)=\sum_{i,j=1}^{n}\bar{\delta}(\lambda_{i,j})e_{i,j}\tag{2}$$

where $\bar{\delta}: M\to M$ is the extension of $\delta: R\to R$ to M, defined by $\bar{\delta}(x+iy)=\delta(x)+i\delta(y)$. It is clear that D_δ is a derivation on $M_n(S(Z))$, and $\bar{D}_\delta$ is a derivation on $M_n(S(Z_c))$.

The next lemma explains the form of derivations on the algebra of measurable operators in homogeneous type I_n, $n\in\mathbb{N}$, real von Neumann algebras.

Lemma 3.3. Let R be a homogeneous real von Neumann algebra of type I_n, $n\in\mathbb{N}$. Each derivation D on the algebra $LS(R)$ has a unique representation as a sum

$$D=D_a+D_\delta$$

in this representation, D_a is an inner derivation given by an element $a\in LS(R)$, and D_δ is a derivation of type (1) coming from a derivation δ on the center $S(Z)$.

Proof. The proof of the this lemma and lemma 3.3 in [2] is very similar. Here, we give a sketch of the proof. Let D be an arbitrary derivation on the algebra $LS(R)\cong M_n(S(Z))$. Now consider the restriction of this derivation δ onto the center $S(Z)$ of this algebra, and let D_δ be the derivation on the algebra $M_n(S(Z))$ constructed as in (1). Put $D_1=D-D_\delta$. Then for any $\lambda\in S(Z)$ we have $D_1(\lambda)=0$, i.e. D_1 is identically zero on $S(Z)$. Therefore D_1 is Z-linear and by Theorem 3.1 we obtain that D_1 is an inner derivation, and thus $D_1=D_a$ for an appropriate $a\in M_n(S(Z))$. Therefore $D=D_a+D_\delta$. If $D=D_{a_1}+D_{\delta_1}=D_{a_2}+D_{\delta_2}$, then $D_{a_1}-D_{a_2}=D_{\delta_2}-D_{\delta_1}$. Since $D_{a_1}-D_{a_2}$ is identically zero on the center of the algebra

$M_n(S(Z))$, this implies that $D_{\delta_2} - D_{\delta_1}$ is also identically zero on the center of $M_n(S(Z))$. This means that $\delta_1 = \delta_2$, and therefore $D_{a_1} = D_{a_2}$, i.e. the decomposition of D is unique. The proof is complete.

Remark 3.4. Under the conditions of the Lemma 3.3 we can extend the derivation D to $LS(M)$ by $\overline{D}(x+iy) = D(x) + iD(y),\ \forall x, y \in LS(R)$. By Lemma 3.3 (see [2]), $\overline{D}$ has the form $\overline{D} = \overline{D}_a + \overline{D}_\delta$, where $\overline{D_a} = [a, \cdot]$ with $a \in LS(M)$, and D_δ is the derivation defined in (2). From the proof of the Lemma 3.3, it is easy to see that $\overline{D_\delta}\,|_{LS(Z)} = D_\delta$, and for $a = a_1 + ia_2$ with $a_i \in LS(R)$ we obtain $\overline{D}_\delta\,|_{LS(R)} = D_{a_1}$, i.e. we have $D = D_{a_1} + D_\delta$.

Now let R be any finite real von Neumann algebra of type I with the center Z. There exists a family of central projections from R which we denote $\{z_n\}_{n\in F}$, $F \subseteq \mathbb{N}$, with $\sup_{n\in F} z_n = 1$ such that the algebra R is *-isomorphic to the C*-product of real von Neumann algebras $z_n R$ of type I_n respectively, $n \in F$, i.e. $R \cong \bigoplus_{n\in F} z_n R$.

Analogically to Proposition 1.1 (see [2]), one can prove that

$$LS(R) \cong \prod_{n\in F} LS(z_n R).$$

Let D be a derivation on $LS(R)$, and δ be its restriction onto the center $S(Z)$. Since δ maps each $z_n S(Z) \cong Z(LS(z_n R))$ into itself, δ generates a derivation δ_n on $z_n S(Z)$ for each $n \in F$.

Let D_{δ_n} be the derivation on the matrix algebra $M_n(z_n Z(LS(R))) \cong LS(z_n R)$ defined as in (1).

Put

$$D_\delta(\{x_n\}_{n\in F}) = \{D_{\delta_n}(x_n)\}, \quad \{x_n\}_{n\in F} \in LS(R) \tag{3}$$

It is easy to see that the map D_δ is a derivation on $LS(R)$.

Lemma 3.3 now leads to the following result:

Lemma 3.5. *Let R be a finite real von Neumann algebra of type I. Each derivation D on the algebra $LS(R)$ has a unique representation as a sum*

$$D = D_a + D_\delta$$

where D_a is an inner derivation implemented by an element $a \in LS(R)$, and D_δ is a derivation given as (3).

We shall now consider derivations on the algebra $LS(R)$ of locally measurable operators with respect to an arbitrary type I real von Neumann algebra R.

Suppose that M is a type I real von Neumann algebra. There exists a central projection $z_0 \in M$ such that

1) $z_0 R$ is a finite real von Neumann algebra;

2) $z_0^\perp R$ is a real von Neumann algebra of type I_∞.

Take a derivation D on $LS(R)$ and suppose that δ is its restriction onto its center $Z(S)$. By Theorem 3.2 $z_0^\perp D$ is inner and thus we have $z_0^\perp \delta \equiv 0$, i.e. $\delta = z_0 \delta$.

Let D_δ be the derivation on $z_0 LS(R)$ defined as in (3) and consider its extension D_δ on

$$LS(R) = z_0 LS(R) \oplus z_0^\perp LS(R)$$

which is defined as

$$D_\delta(x_1 + x_2) := D_\delta(x_1), \quad x_1 \in z_0 LS(R), \quad x_2 \in z_0^\perp LS(R) \tag{4}$$

The following theorem gives the general form of derivations on the algebra $LS(R)$.

By combining Lemma 3.5 with Theorem 3.2, we obtain the following result.

Theorem 3.6. *Let R be a type I real von Neumann algebra. Each derivation D on $LS(R)$ has a unique representation as a sum*

$$D = D_a + D_\delta$$

where D_a is an inner derivation implemented by an element $a \in LS(R)$, and D_δ is a derivation of the form (4), generated by a derivation δ on the center of $LS(R)$.

2.3. Derivations on the Algebra *S(R)*

In this section, we present a description of derivations on the algebra $S(R)$ of measurable operators associated with a type I real von Neumann algebra R.

Let A be an arbitrary subalgebra of $LS(R)$ which contains R.

Take a derivation $D: A \to LS(R)$ and let us show that D can be extended to a derivation D on the whole $LS(R)$.

Since R is of type I, for an arbitrary element $x \in LS(R)$ there exists a sequence $\{z_n\}$ of mutually orthogonal central projections with $\vee_{n\in\mathbb{N}} z_n = 1$ and $z_n x \in R$ for all $n \in \mathbb{N}$. Set

$$\tilde{D}(x) = \sum_{n\ge 1} z_n D(z_n x) \tag{5}$$

Since every derivation $D: A \to LS(R)$ is identically zero on central projections of R, the equality (5) yields a well-defined derivation $\tilde{D}: LS(R) \to LS(R)$ which coincides with D on A.

More specifically, from Z-linearity of D on A, implies Z-linearity of $\tilde{D}$, and by Theorem 3.1 the derivation $\tilde{D}$ is inner on $LS(R)$. Therefore D is a spatial derivation on A, i.e. there exists an element $a \in LS(R)$ such that $D(x) = ax - xa, \quad \forall x \in A$.

Hence, we arrive at the following result.

Theorem 4.1. Let R be a type I real von Neumann algebra with the center Z, and let A be an arbitrary subalgebra in $LS(R)$ containing R. Then any Z-linear derivation $D: A \to LS(R)$ is spatial and implemented by an element of $LS(R)$.

Corollary 4.2. Let R be a type I real von Neumann algebra with the center Z and let D be a Z-linear derivation on $S(R)$ or $S(R,\tau)$. Then D is spatial and implemented by an element of $LS(R)$.

Lemma 4.3. If R is a type I_∞ real von Neumann algebra, then every derivation $D: R \to S(R)$ has the form $D(x) = ax - xa, \quad \forall x \in R$, for an appropriate $a \in S(R)$.

Proof. Derivation D extends to derivation $\overline{D}$ on M as $\overline{D}(x+iy) = D(x) + iD(y), \quad x, y \in R$. By Lemma 3.4 (see [2]) derivation $\overline{D}$ on M is inner, i.e. there is an element $z = a + ib, \ (a, b \in R)$, such that $\overline{D}(x+iy) = [z, x+iy] = z(x+iy) - (x+iy)z, \quad \forall x, y \in R$. Then we have $D(x) = \overline{D}(x) = [a,x] + i[b,x]$. Since $x \in R$ and $D(x) \in R$, then $[b,x] = 0$. Hence $D(x) = [a,x]$, i.e. D is inner. The Lemma is proven.

Analogically, we prove the next lemma.

Lemma 4.4. *Let* R be a type I real von Neumann algebra with the cen*ter* Z. *Then every* Z*-linear* derivation D *on the* algebra $S(R)$ *is inne*r. In particul*ar, if* R is of type I_∞ *then* every derivation *on* $S(R)$ is inn*er.*

Finally, combining Lemmas 3.5 and 4.4, we obtain the main result of this section.

Theorem 4.5. *Let* R *be a type I real von Neumann algebra. Then every derivation* D *on the algebra* $S(R)$ has a unique representation as a sum

$$D = D_a + D_\delta$$

*w*here D_a is inner and implemented by an element $a \in S(R)$, and D_δ is the derivation of the form (3) generated by a derivation δ on the center of $S(R)$.

Let τ be a faithful normal semifinite trace on a von Neumann algebra M. A linear subspace $D \subset H$ is called τ-dense if $D \eta M$ and for every $\varepsilon > 0$ there exists a projection $P \in P(M)$ such that $P(H) \subset D$ and $\tau(I - P) \le \varepsilon$.

From Proposition 3.15 (see [5]) it follows that every τ-dense subspace $D \subset H$ is strongly dense.

A closed linear operator T acting in H is called τ-measurable with respect to algebra M if $T \eta M$ and $D(T)$ is τ-dense in H.

We denote by $S(M,\tau)$ the set of all τ-measurable operators. Clearly, $M \subset S(M,\tau) \subset S(M)$. And $T \in S(M,\tau)$ if and only if $T \in S(M)$ and $D(T)$ P(T) is τ-dense in H.

Now, we consider a general form of derivations on the algebra $S(R,\tau)$ of τ-measurable operators affiliated with a type I real von Neumann algebra R and a faithful normal semifinite trace τ. Similarly to the proof of Theorem 2.1 (see [2]) it is proved the following theorem.

Theorem 4.6. Let R be a type I real von Neumann algebra with the center Z and a faithful normal semi-finite trace. Then every Z-linear derivation D on the algebra $S(R,\tau)$ is inner. As a special case, if R is of type I_∞, then every derivation on $S(R,\tau)$ is inner.

In a similar way to (3), and formulae (12) from [2] we can define a derivation D_δ on $S(R,\tau)$. Now, similarly to Lemma 3.5 one can prove the following.

Lemma 4.7. *Let* R be a finite real von Neumann algebr*a* of type I with a faithful normal semi-finite trace τ. *Each derivation* D on the algeb*ra* $S(R,\tau)$ can be uniquely represented in the form $D = D_a + D_\delta$ *where* D_a *is* a*n inner* *d*erivation implemented by an ele*ment* $a \in S(R,\tau)$*, and* D_δ is a derivation given *as* (5).

Finally, Theorem 4.6 and Lemma 4.7 imply the following result.

Theorem 4.8. Let R be a type I real von Neumann algebra with a faithful normal semi-finite trace τ. Then every derivation D on the algebra $S(R,\tau)$ can be uniquely represented in the form $D = D_a + D_\delta$ where D_a is an inner derivation implemented by an element $a \in S(R,\tau)$, and D_δ is the derivation from lemma *4.7*.

If we consider the measure topology t_τ on the algebra $S(R,\tau)$ (see Section 1), then it is clear that every non-zero derivation of the form D_δ is discontinuous in t_τ. Therefore, the above Theorem 4.8 implies the following.

Corollary 4.9. Let R be a type I real von Neumann algebra with a faithful normal semi-finite trace τ. A derivation D on the algebra $S(R,\tau)$ is inner if and only if it is continuous in the measure topology.

3. Materials and Methods

The article employs methods from the theory of bounded and unbounded operator algebras. In proving the main results, the method of passing to the enveloping complex algebra is also used. In doing so, the results obtained in the complex case are applied.

4. Results

Descriptions of derivations on certain algebras of measurable, locally measurable, and τ -measurable operators, affiliated with real von Neumann algebra of type I, are obtained.

5. Discussion

Derivations on operator algebras on a Hilbert space have been widely studied. More specifically, it is known that all derivations on a von Neumann algebra are inner, while on a C*-algebra there are non-inner derivations. Recently, considerable attention has been devoted to the study of derivations on algebras of unbounded operators. In 2008-2013, a complete description of all derivations on algebras of measurable, locally measurable, and τ -measurable operators affiliated with von Neumann algebra of type I is obtained. Here, a real analogue of these results is considered. That is, descriptions of derivations on some algebras of measurable, locally measurable, and τ -measurable operators affiliated with real von Neumann algebra of type I are obtained.

6. Conclusions

A real analogue of the study of derivations on the algebra of unbounded operators is considered. In particularly, descriptions of derivations on certain algebras of measurable, locally measurable, and τ -measurable operators, affiliated with real von Neumann algebra of type I, are obtained.

Abbreviations

B(H) Algebra of all Bounded Linear Operators Acting on a Complex Hilbert Space H
P(M) Complete Lattice of all Projections from M

Author Contributions

Abdugafur Rakhimov: Conceptualization, Data curation, Formal Analysis, Investigation, Methodology, Supervision, Validation, Writing – original draft, Writing – review & editing

Ulugbek Karimov: Formal Analysis, Investigation, Methodology, Validation, Writing – original draft, Writing – review & editing

Conflicts of Interest

The authors declare no conflicts of interest.

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Research Field

Abdugafur Rakhimov: Functional analysis, Theory of operator algebras, Theory of unbounded operators, Topology, Fuzzy topology

Ulugbek Karimov: Functional analysis, Theory of operator algebras, Theory of unbounded operators, Topology.