

Research Article

The Application of Homotopy Perturbation Method in Newtonian Fluids

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Abstract

The Homotopy Perturbation Method (HPM) has advanced as an efficient semi-analytical technique for solving nonlinear differential equation arising in fluid mechanics. In this study, the application of HPM to Newtonian fluid flow problems is investigated in order to obtain accurate approximate solutions with reduced computational effort. The governing momentum equations of a Newtonian fluid flow, with a focus on deriving and analyzing engineering parameters like skin friction, Nusselt number, and Sherwood number was formulated under appropriate boundary conditions. The nonlinear partial differential equations describing the momentum, Navier-stokes equation and concentration equation were transformed into a homotopy framework by embedding an auxiliary a parameter. The solution is constructed in form of a rapidly convergent series without the need for small perturbation parameters or linearization. The analytical results obtaining using (HPM) are discussed and, where possible, compared with exact or numerical solutions to validate the accuracy and convergence of the method. The impact of the material parameters from the basic hydrodynamic equations were noticed and the findings demonstrate that the Homotopy Perturbation Method provides a reliable, Straightforward, and computationally efficient approach for analyzing Newtonian fluid flow phenomena and can be readily extended to more complex transport and heat transfer problems in engineering and applied sciences, making it a valuable tool for solving a wide range of nonlinear fluid mechanics models.

Keywords

Homotopy Perturbation Method, Newtonian Fluids, Application Performance, Ordinary Differential Equations, Dimensionless Parameter

1. Introduction

The application of homotopy perturbation (HPM) technique in physical sciences and engineering, has remained the subject of numerous research projects in recent years. Scien-

tists can gain a deeper understanding of fundamental physical processes, optimise technological applications, and resolve urgent physical challenges by analysing the complexities of the homotopy

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perturbation method. Numerous pioneering scientists have established the foundation for the development of (HPM) in various fields. The homotopy perturbation method (HPM), postulated by Chinese mathematician “He” in year 1998, is an analytical technique for differential equations at first which also displayed the mainframe methods in applied mechanics and engineering [1]. Researchers determined and applied the homotopy perturbation method (HPM) to the non-linear (AC) Allen-Cahn equation and the time fractional dispersion equation to illustrate lubricant contamination in water. They introduced techniques for demonstrating applicability and accuracy in concentrations of pollution in the flow field are presented over time and space across various levels of fractional derivation. At lower levels of fractional derivatives, the behavior of pollution concentration is nonlinear, but as concentration diminishes. Their findings, obtained through the proposed method, are equated to exact outcomes, showing that it is effective, user-friendly, and cuts down on computation time [2]. A comparative analysis concerning the radiative impacts on 2D unsteady magnetohydrodynamic Al_2O_3 -water nanofluid flow occurring between squeezing plates by utilizing two analytical methods, both executed using Python programming employing Python programming to tackle these equations represents an innovative technique that yields precise and effective results was examine and the results show that with an increase in the Prandtl number, both the temperature and thermal characteristics of the nanofluid flow rise, while there is a decrease in concentration, in addition to the evaluation of heat transfer analysis and modeling in ordinary convection in-between the two infinite vertical parallel flat plates [3, 4]. The heat analysis of non-Newtonian Visco-elastic parallel flat plates was determined showcasing non-linear ordinary differential equations with boundary conditions was addressed by utilizing the (HMP) Homotopy Perturbation Method and (AGM) Akbari-Ganji's Method. The research evaluated how various physical parameters impact temperature distributions and the components of velocity in the circular, transverse, and axial directions, including the visco-elastic stricture, force parameter, Prandtl profile, Reynolds number, and captivating field influence. Representative and numerical computations of velocity components and temperature parameters were performed using the Python programming language. The precision of the outcomes achieved over the systematic method confirms the pertinence and effectiveness of the suggested strategy for analyzing forced current and thermal transfer in non-Newtonian liquids situated between revolving disks [5]. Thermal transfer properties in the squeezing motion of a non-Newtonian fluid, with a specific emphasis on the Casson fluid situated amid two flat spherical plates was investigated. The study encompasses the creation of nonlinear ordinary differential equations (ODEs), the application of conservation principles along with similarity transformations, and the following analytical solution found via the Akbari-Ganji's Method (AGM) and Differential Transform Method (DTM).

The finding explores how transformed factors affect flow features, illustrating the findings graphically and engaging in thorough discussions. The analysis includes a detailed review crosswise a variety of strictures, such as the Prandtl profile, squeeze number, Casson fluid parameter, and Eckert number. Importantly, the findings reveal that the speed of motion increases concerning both the casson fluid profile and the squeeze number [6]. The influence of radiation and chemical processes on the recovery of the ozone layer was established. The researcher used the fundamental hydrodynamic equations as ordinary differential equation through the use of similarity solutions and subsequently made non-dimensional. The vigor momentum equation, Navier-Stoken equation, and concentration equation were addressed applying the Homotopy perturbation method with adjusted boundary conditions. The solutions were examined using Wolfram Mathematica software version 12 and it was found that substituting the negatively affecting term in the energy with a indicated a recovery of the ozone layer. Comparable improvements in the Prandtl and Schmidt variables were observed. It was found that an increase in both the temperature profile and concentration led to a rise in the Gashof number which supported the motion of ozone layer healing. The effect of radiation on the temperature profile showed that when the heat transfer quantity reduced, it led to an upsurge in the radiation parameter, confirming that the ozone layer is undergoing recovery [7]. The basic principle of HMP is merging the homotopy technique and the traditional perturbation method is to transform a difficult solving problem into a simple one. Therefore, in recent years, this method has been applied extensively to various models in the field of physics and engineering including not only the differential equations, using HMP to solve N/MEMS oscillator, bifurcation of non-linear problems, quantum mechanical problem, nonlinear differential equations connecting to fractional operator with exponential kernel and for fractional-order advection-diffusion-reaction boundary value problems [8-17]. In order to elaborate current situation of the application of the HPM to the largest degree, researchers investigated the couple method of HPM with other mathematical methods, by combining Laplace transform method, solving reaction-diffusion equations and solution of fuzzy partial partial differential equation [18-25]. Newtonian equations with a combination of effective thermal conductivity and effective viscosity was determined with outcome of the material parameters on Nusselt number, skin friction, and Sherwood number been varying [26-28].

Thus, based on the aforementioned publications, this paper expands upon the previous research by using homotopy perturbation method in solving Newtonian fluid equations and to compare and contrast the effects of various material properties. Also, this paper exposes how the application of the HPM can be further investigated and applied in other fields.

2. Basic Idea of the Homotopy Perturbation Method

The basic concepts of Homotopy Perturbation Method, we consider the below nonlinear differential equation.

$$\frac{\partial^2 \theta}{\partial x^2} + Gr\phi = 0 \quad (1)$$

A homotopy is then constructed by introducing the convex homotopy functional $H(\theta, p)$ with an embedding parameter (homotopy parameter) under the boundary condition.

$$\theta = 1, \phi = 1 \text{ at } x = 0 \quad (2)$$

$$\phi = 0, \theta = 0, \text{ at } x = 1 \quad (3)$$

Applying Homotopy Perturbation Method with convex homotopy, equation (1) becomes

$$H(\theta, p) = (1-p) \left[\frac{\partial^2 \theta}{\partial x^2} \right] + p \left[\frac{\partial^2 \theta}{\partial x^2} + Gr\phi \right] = 0 \quad (4)$$

Assume solution of equation (4) implies

$$\begin{aligned} \theta &= \theta_0 + p\theta_1 + p^2\theta_2 + \dots \\ \phi &= \phi_0 + p\phi_1 + p^2\phi_2 + \dots \end{aligned} \quad (5)$$

Substituting equation (5) into equation (4).

$$\frac{\partial^2}{\partial x^2} (\theta_0 + p\theta_1 + p^2\theta_2 + \dots) = -pGr(\phi_0 + p\phi_1 + p^2\phi_2 + \dots) \quad (6)$$

$$\frac{\partial^2 \theta_0}{\partial x^2} + \frac{\partial^2 \theta_1}{\partial x^2} p + \frac{\partial^2 \theta_2}{\partial x^2} p^2 = -pGr\phi_0 - p^2Gr\phi_1 - p^3Gr\phi_2 \quad (7)$$

Comparing the coefficient of p^0, p^1, p^2, p^3 from equation (7).

$$p^0 \Rightarrow \frac{\partial^2 \theta_0}{\partial x^2} = 0 \quad (8)$$

$$p^1 \Rightarrow \frac{\partial^2 \theta_1}{\partial x^2} = -Gr\phi_0 \quad (9)$$

$$p^1 \Rightarrow \frac{\partial^2 \theta_2}{\partial x^2} = -Gr\phi_1 \quad (10)$$

$$p^3 \Rightarrow \frac{\partial^2 \theta_3}{\partial x^2} = -Gr\phi_2 \quad (11)$$

Equations (8) – (11) solution, can be written as

$$\theta_0 = Ax + B \quad (12)$$

$$\theta_1 = \frac{1}{2}\phi_0 x^2 + Cx + D \quad (13)$$

$$\theta_2 = \frac{1}{2}\phi_1 x^2 + Ex + F \quad (14)$$

$$\theta_3 = \frac{1}{2}\phi_2 x^2 + Gx + H \quad (15)$$

3. Physical Model and Governing Equations

The equations that describe the flow of Newtonian fluid through a flat boundary layer are concentration equation, Navier-Stokes equation and the energy equation which are also the governing partial differential equations that theoretically confirm the properties of chemical reaction and radiation respectively.

$$\frac{\partial v}{\partial t} \rho = -\nabla \rho + \mu \nabla^2 v + \rho g \quad (16)$$

$$\rho C_p \frac{\partial C}{\partial y} = D \frac{\partial^2 C}{\partial y^2} - K_0 r^2 C \quad (17)$$

$$\rho C_p \frac{\partial T}{\partial y} = K \frac{\partial^2 T}{\partial y^2} - \frac{\partial q_y}{\partial y} \quad (18)$$

If the fluid is incompressible, then equations (16) – (18) can be written as

$$-\frac{1}{\rho} \frac{\partial \rho}{\partial y} + \frac{\mu}{\rho} \frac{\partial^2 v}{\partial y^2} + \frac{1}{\rho} \rho g = 0 \quad (19)$$

$$\frac{D}{\rho C_p} \frac{\partial^2 C}{\partial y^2} - \frac{K_0 r^2 C}{\rho C_p} = 0 \quad (20)$$

$$\frac{K}{\rho C_p} \frac{\partial^2 T}{\partial y^2} - \frac{1}{\rho C_p} \frac{\partial q_y}{\partial y} = 0 \quad (21)$$

Where v is fluid velocity, T is temperature, ρ is fluid density, P is fluid pressure, μ is viscosity of fluid, K is radiation term,

C_p is specific heat at constant pressure, K_0 is chemical reaction term, Kr is thermal conductivity of fluid, C is fluid concentration, D is chemical molecular diffusivity.

4. Dimensional Analysis

To reduce the number of governing parameters and to identify the dominant physical mechanisms in the flow system, dimensional analysis is employed, therefore equations (19) – (21) are transmute into dimensionless form.

$$Sc^{-1} = \frac{D}{\rho C_p}, Gr = Gr_T \theta + Gr_C \phi, K_0 = \frac{Kr^2 \mu}{\rho C_p}, Re^{-1} = \frac{\mu}{V_0 p}$$

$$R = \frac{16\varepsilon}{3\alpha(\rho C_p)} T_a^3, Pr^{-1} = \frac{1}{\rho C_p} K, \rho = \frac{1}{\rho} \frac{\partial \rho}{\partial y}, \theta = \frac{T - T_a}{T_b - T_a}, \phi = \frac{C - C_a}{C_b - C_a}$$

Where Pr represents Prandtl profile, Re implies Reynolds profile, Sc is Schmidt profile, $Gr_T \theta$ is thermal Grashofs number, $Gr_C \phi$ is modified Grashofs number, $\Re$ is dimensionless radiation term, θ is dimensionless temperature, C is dimensionless concentration.

Transforming the dimensionless parameters into equations (19) – (21) gives

$$\frac{1}{Re} \frac{\partial^2 u}{\partial y^2} - P + Gr = 0 \quad (22)$$

$$\frac{1}{Pr} \frac{\partial^2 \theta}{\partial y^2} - R\theta = 0 \quad (23)$$

$$\frac{1}{Sc} \frac{\partial^2 \phi}{\partial y^2} - K_0 \phi = 0 \quad (24)$$

The boundary conditions for equations (22) – (24) implies $u = 1, \phi = 1, \theta = 1$ at $y = 0$

$$Re^{-1} U_{(y)}'' - pP_{(y)} + pGr_T \theta_0 + p^2 Gr_T \theta_1 + p^3 Gr_T \theta_2 + pGr_C \phi_0 + p^2 Gr_C \phi_1 + p^3 Gr_C \phi_2 = 0 \quad (33)$$

$$Pr^{-1} \theta_{(y)}'' - pR\theta_0 - p^2 R\theta_1 - p^3 R\theta_2 = 0 \quad (34)$$

$$Sc^{-1} \phi_{(y)}'' + pK_0 \phi_0 + p^2 K_0 \phi_1 + p^3 K_0 \phi_2 = 0 \quad (35)$$

To compare the coefficient of p^0, p^1, p^2, p^3 the result implies

$$p^0: Re^{-1} U_{(y)}'' - P_x = 0 \quad (36)$$

$$p^0: Pr^{-1} \theta_0'' = 0 \quad (37)$$

$$u = 0, \phi = 0, \theta = 0 \text{ at } y = 1 \quad (25)$$

5. Method of Solution

Applying Homotopy Perturbation Method to solve equations (22) – (24) implies

$$H(u, p) = (-p+1) \left[\frac{1}{Re} U_{(y)}'' \right] + P \left[\frac{1}{Re} U_{(y)}'' - P_{(y)} + Gr_T \theta + Gr_C \phi \right] = 0 \quad (26)$$

$$H(\theta, p) = (1-p) \left[\frac{1}{Pr} \theta_{(y)}'' \right] + P \left[\frac{1}{Pr} \theta_{(y)}'' - R\theta_{(y)} \right] = 0 \quad (27)$$

$$H(\phi, p) = (1-p) \left[\frac{1}{Sc} \phi_{(y)}'' \right] + P \left[\frac{1}{Sc} \phi_{(y)}'' - K_0 \phi_{(y)} \right] = 0 \quad (28)$$

The assume solution for equations (26) – (28) is written as

$$U = U_0 + pU_1 + p^2 U_2 + \dots$$

$$\theta = \theta_0 + p\theta_1 + p^2 \theta_2 + \dots \quad (29)$$

$$\phi = \phi_0 + p\phi_1 + p^2 \phi_2 + \dots$$

Equations (26) – (28) gives the form respectively

$$H(u, p) = Re^{-1} U_{(y)}'' - pP_{(y)} + pGr_T \theta_{(y)} + pGr_C \phi_{(y)} = 0 \quad (30)$$

$$H(u, p) = Pr^{-1} \theta_{(y)}'' - p\theta_{(y)} R = 0 \quad (31)$$

$$H(u, p) = Sc^{-1} \phi_{(y)}'' + pK_0 \phi_{(y)} = 0 \quad (32)$$

Substituting equation (29) into equations (30) – (32) the result is

$$p^0: Sc^{-1} \phi_0'' = 0 \quad (38)$$

$$p^1: Re^{-1} U_2'' + Gr_C \phi_0 + Gr_T \theta_0 = 0 \quad (39)$$

$$p^1: Pr^{-1} \theta_1'' - R\theta_0 = 0 \quad (40)$$

$$p^1: Sc^{-1} \phi_1'' + K_0 \phi_0 = 0 \quad (41)$$

$$p^2: Re^{-1} U_2'' + Gr_C \phi + Gr_T \theta_{11} = 0 \quad (42)$$

$$p^2: Pr^{-1} \theta_1'' - R\theta_0 = 0 \quad (43)$$

$$p^2: Sc^{-1}\phi_1'' + K_0\phi_1 = 0 \quad (44)$$

$$p^3: Re^{-1}U_3'' + Gr_T\theta_2 + Gr_C\phi_2 = 0 \quad (45)$$

$$p^3: Pr^{-1}\theta_3'' - R\theta_2 = 0 \quad (46)$$

$$p^3: Sc^{-1}\phi_3'' + K_0\phi_2 = 0 \quad (47)$$

From equation (29) the modified boundaries conditions imply

$$\begin{aligned} U_0 &= U_1 = U_2 = U_3 = 1 \\ \theta_0 &= \theta_1 = \theta_2 = \theta_3 = 1 \\ \phi_0 &= \phi_1 = \phi_2 = \phi_3 = 1 \\ U_0 &= U_1 = U_2 = U_3 = 0 \\ \theta_0 &= \theta_1 = \theta_2 = \theta_3 = 0 \\ \phi_0 &= \phi_1 = \phi_2 = \phi_3 = 0 \end{aligned} \quad (48)$$

6. Coefficient Analysis of Homotopy Perturbation Method

Integrate equations (36) – (38) twice with respect to y with boundary condition in equations (2) and (3), the results yield.

$$U_0 = -\frac{Re P_x y^2}{2} + \frac{(Re P_x - 2A)y}{2} + 1 \quad (49)$$

$$\theta_0 = -y + 1 \quad (50)$$

$$\phi_0 = -y + 1 \quad (51)$$

Solving equations (39) – (41) by making use of the suitable boundary conditions from equation (33), it gives.

$$U_1 = -\frac{Re Gr_T \theta_0 y^2}{2} - \frac{1}{2} Re Gr_C \phi_0 y^2 + \frac{(Re Gr_T \theta_0 + Re Gr_C \phi_0 - 2)y}{2} + 1 \quad (52)$$

$$\theta_1 = -\frac{Pr R \theta_0}{2} y^2 - \left(\frac{Pr R \theta_0}{2} + 1 \right) y + 1 \quad (53)$$

$$\phi_1 = -\frac{1}{2} Sc K_0 \phi_0 y^2 + \left(\frac{1}{2} Sc K_0 \phi_0 + 1 \right) y + 1 \quad (54)$$

Solving equations (42) – (44) by using the appropriate boundary conditions from equation (33), the result implies.

$$U_2 = -\frac{1}{2} Re Gr_T \theta_1 y^2 - \frac{1}{2} Re Gr_C \phi_1 y^2 + \frac{(Re Gr_T \theta_1 + Re Gr_C \phi_1 - 2)y}{2} + 1 \quad (55)$$

$$\theta_2 = -\frac{Pr R \theta_1}{2} y^2 - \left(\frac{Pr R \theta_1}{2} + 1 \right) y + 1 \quad (56)$$

$$\phi_2 = -\frac{1}{2} Sc K_0 \phi_1 y^2 + \left(\frac{1}{2} Sc K_0 \phi_1 + 1 \right) y + 1 \quad (57)$$

Solving equations (45) – (47) with the use of the appropriate boundary conditions from equation (33), the results yield.

$$U_3 = -\frac{1}{2} Re Gr_T \theta_2 y^2 - \frac{1}{2} Re Gr_C \phi_2 y^2 + \left(\frac{Re Gr_T \theta_2 + Re Gr_C \phi_2 - 2}{2} \right) y + 1 \quad (58)$$

$$\begin{aligned} \theta_3 &= -\frac{Pr R \theta_2}{2} y^2 - \left(\frac{Pr R \theta_2}{2} + 1 \right) y + 1 \quad (59) \\ \phi_3 &= -\frac{Sc K_0 \phi_2}{2} y^2 + \left(\frac{Sc K_0 \phi_2 + 1}{2} \right) y + 1 \quad (60) \end{aligned}$$

Equations (49) – (60) give the expressions for the temperature, velocity and concentration as

$$\begin{aligned} U &= U_0 + U_1 + U_2 + U_3 + \dots \\ \theta &= \theta_0 + \theta_1 + \theta_2 + \theta_3 + \dots \\ \phi &= \phi_0 + \phi_1 + \phi_2 + \phi_3 + \dots \end{aligned} \quad (61)$$

Recasting equation (61) the results are

$$\begin{aligned} U &= 4 + \frac{1}{8} \text{Re} \left[\frac{K_0^2 \text{Sc}^2 \text{Gr}_C}{\text{Pr}^2 R^2 \text{Gr}_T} \right] y^7 - \frac{1}{2} \text{Re} \left[\frac{K_0^2 \text{Sc}^2 \text{Gr}_C}{\text{Pr}^2 R^2 \text{Gr}_T} \right] y^6 + \frac{3}{4} \left[\frac{K_0^2 \text{Sc}^2 - \frac{2}{3} \text{Sc} K_0}{R \text{Pr} \text{Gr}_T} \text{Gr}_C + \right] \text{Re} y^5 - \frac{1}{2} \text{Re} \left[\frac{K_0^2 \text{Sc}^2 - 3K_0 \text{Sc}}{R \text{Pr} \text{Gr}_T} \text{Gr}_C + \right] y^4 \\ &+ \frac{1}{8} \text{Re} \left[\frac{K_0^2 \text{Sc}^2 - 3K_0 \text{Sc}}{\text{Gr}_T} \text{Gr}_C + \right] y^3 - \frac{1}{2} \text{Re} \left[\frac{-K_0 \text{Sc} + 6}{R \text{Pr} \text{Gr}_T - P_x + 6 \text{Gr}_T} \text{Gr}_C + \right] y^2 + \frac{1}{8} \left[-32 + \frac{4P_x + 12 \text{Gr}_C}{+12 \text{Gr}_T} \right] \text{Re} y \end{aligned} \quad (62)$$

$$\theta = -\frac{1}{8} \left[\frac{\text{Pr}^3 R^3 y^6 - 3 \text{Pr}^3 R^3 y^5 + 3 \text{Pr}^3 R^3 y^4 - \text{Pr}^3 R^3 y^3 + 4 \text{Pr}^2 R^2 y^4}{-8 \text{Pr}^2 R^2 y^3 + 4 \text{Pr}^2 R^2 y^2 + 12 \text{Pr} R y - 12 \text{Pr} R y + 32} \right] y + 1 \quad (63)$$

$$\phi = \frac{1}{8} \left[\frac{K_0^3 \text{Sc}^3 y^6 - 3K_0^3 \text{Sc}^3 y^5 + 3K_0^3 \text{Sc}^3 y^4 - K_0^3 \text{Sc}^3 y^3 - 4K_0^2 \text{Sc}^2 y^4}{+8K_0^2 \text{Sc}^2 y^3 - 4K_0^2 \text{Sc}^2 + 12K_0 \text{Sc} y - 12K_0 \text{Sc} y - 32} \right] y - 1 \quad (64)$$

7. Application to Engineering

The engineering application of fluid dynamics from equations (62) – (64) can be denoted as the Nusselt profile, skin friction, and the Sherwood number.

The skin friction is the friction at the surface of a fluid and a solid in comparative motion. Friction between a fluid and the exterior of a solid moving through its enclosing surface. Therefore, the Skin friction can be attained by differentiating equation (62) with respect to ‘y’ limit to zero.

$$\tau = \left[\frac{dU}{dy} \right]_{y=0} = -4 + \frac{1}{2} \text{Re} P_x + \frac{3}{2} \text{Re} \text{Gr}_\theta + \frac{3}{2} \text{Re} \text{Gr}_\phi \quad (65)$$

The Nusselt number is the proportion of convection heat transfer to fluid conduction heat transfer under the same conditions. In fluid motions, the Nusselt number is the ration of convective to conductive heat transfer at a boundary in a fluid. Thus, the Nusselt number can be gotten from equation (63).

$$\text{Nu} = -\left[\frac{d\theta}{dy} \right]_{y=0} = 4 + \frac{3}{2} \text{Pr} R \quad (66)$$

The Sherwood number is a dimensionless number used in mass transfer operation; it represents the proportion of the convective mas transfer to the rate of diffusive mass transport. Hence, the Sherwood number can be obtained from equation (64).

$$\text{Sh} = -\left[\frac{dC}{dy} \right]_{y=0} = 4 + \frac{3}{2} \text{Sc} K_0 \quad (67)$$

8. Results and Discussion

For clearer insight into the physical problem and application of Newtonian fluids equations (65), (66), together with the modified boundary conditions in equations (2), (3) are numerically solved using the Homotopy Perturbation Method. The resulting computational fluid dynamics outcomes are illustrated through the graph in Figures 1 – 16.

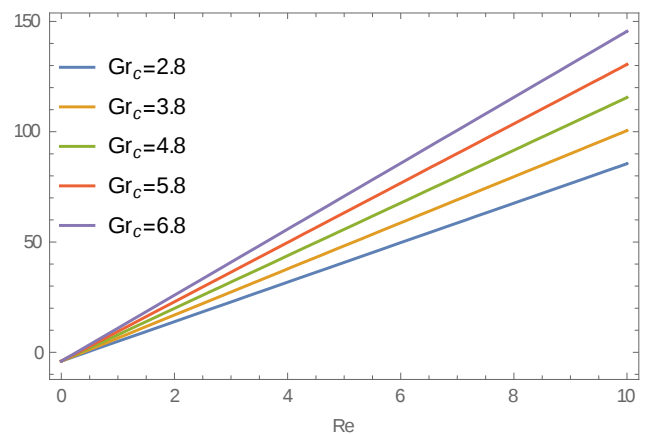


Figure 1. Skin friction (τ) against boundary layer of Reynolds profile (Re) with Grashof's number due to concentration (Gr_c) varying.

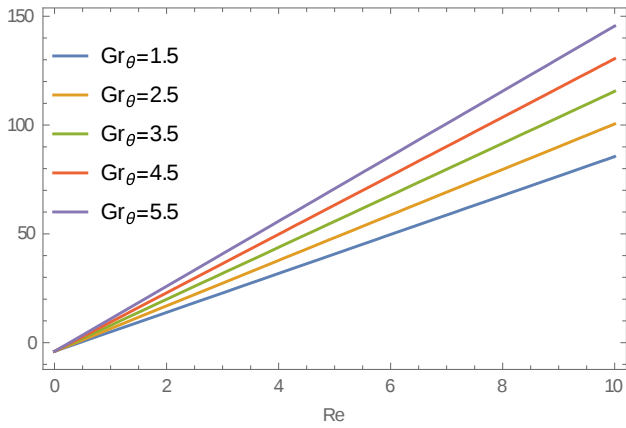


Figure 2. Skin friction (τ) against boundary layer of Reynolds profile (Re) with Grashofs number due to temperature (Gr_θ) varying.

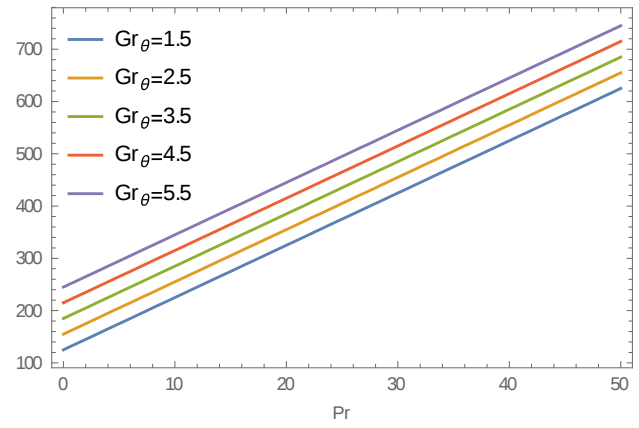


Figure 5. Skin friction (τ) against boundary layer of Prandtl number (Pr) with Grashofs number due to temperature (Gr_θ) varying.

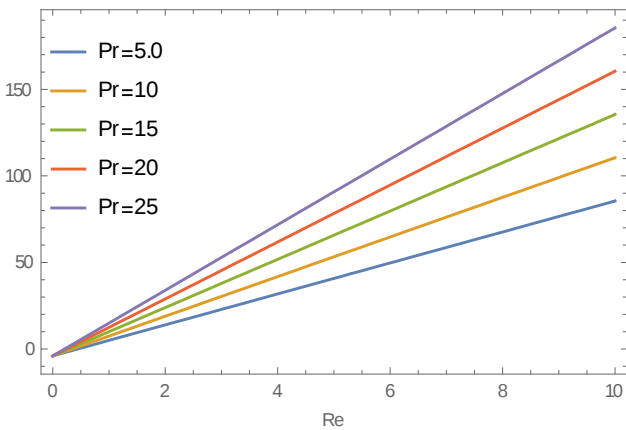


Figure 3. Skin friction (τ) against boundary layer of Reynolds number (Re) with Prandtl number (Pr) varying.

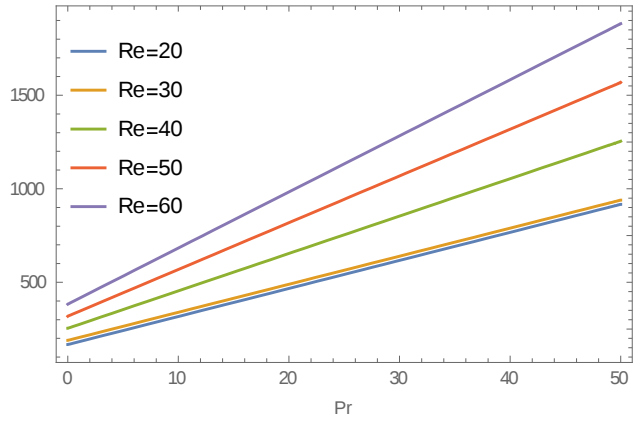


Figure 6. Skin friction (τ) against boundary layer of Prandtl number (Pr) with Reynolds number (Re) varying.

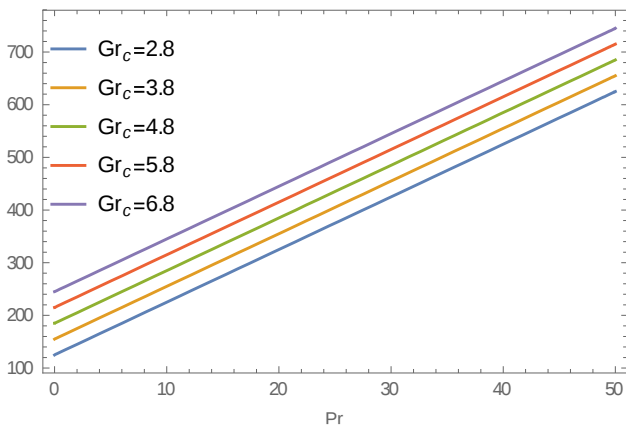


Figure 4. Skin friction (τ) against boundary layer of Prandtl number (Pr) with Grashofs number due to concentration (Gr_c) varying.

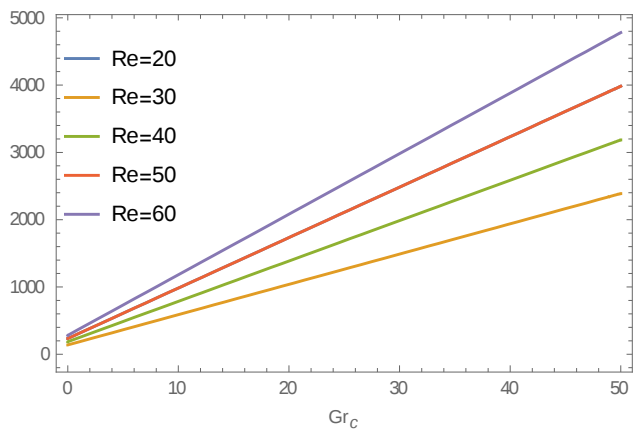


Figure 7. Skin friction (τ) against boundary layer of Grashofs number due to concentration (Gr_c) with Reynolds number (Re) varying.

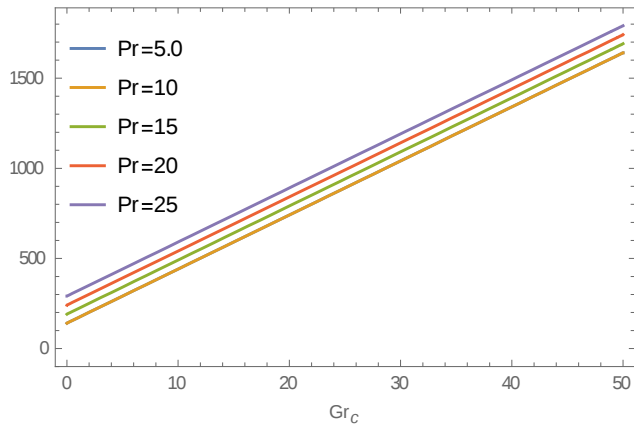


Figure 8. Skin friction (τ) against boundary layer of Grashof's number due to concentration (Gr_c) with Prandtl number (Pr) varying.

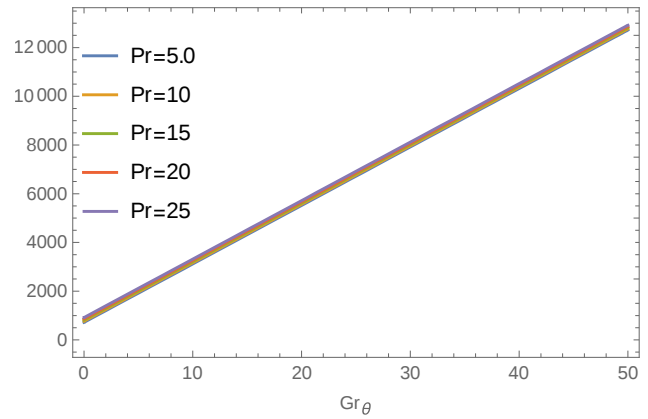


Figure 11. Skin friction (τ) against boundary layer Grashof's number due to temperature (Gr_θ) with Prandtl number (Pr) varying.

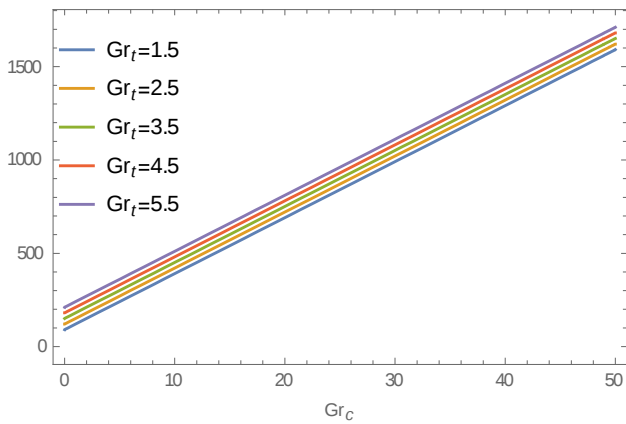


Figure 9. Skin friction (τ) against boundary layer of Grashof's number due to concentration (Gr_c) with Grashof's number due to temperature (Gr_θ) varying.

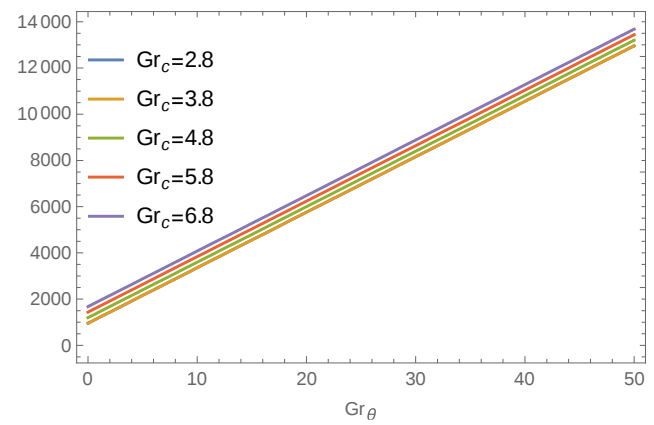


Figure 12. Skin friction (τ) against boundary layer Grashof's number due to temperature (Gr_θ) with Grashof's number due to concentration (Gr_c) varying.

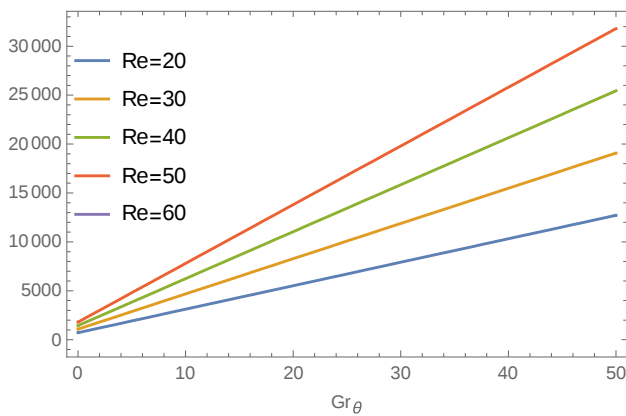


Figure 10. Skin friction (τ) against boundary layer Grashof's number due to temperature (Gr_θ) with Reynolds number (Re) varying.

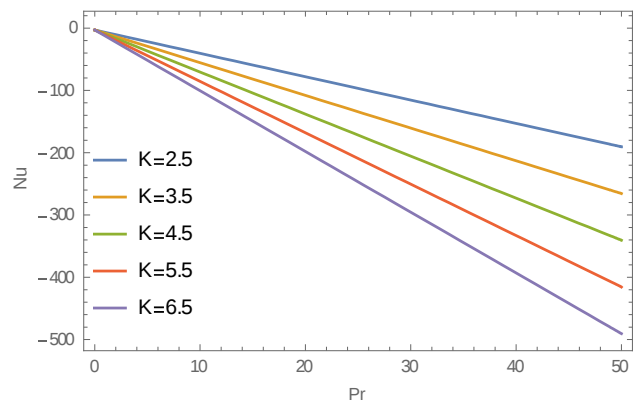


Figure 13. Nusselt number (Nu) against boundary layer Prandtl number (Pr) with Chemical reaction term (K) varying.

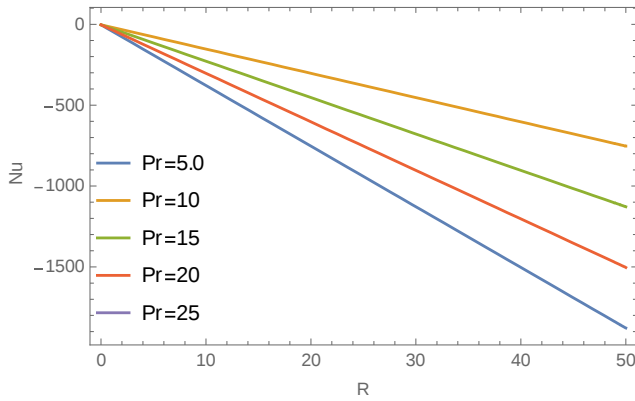


Figure 14. Nusselt number (Nu) against boundary layer Radiation (R) with Prandtl number (Pr) varying.

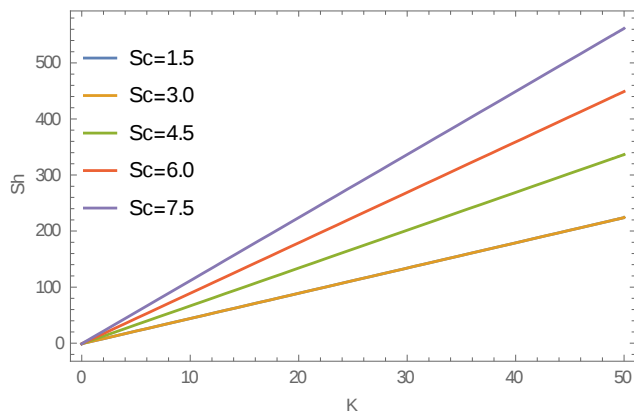


Figure 15. Sherwood number (Sh) against boundary layer Chemical reaction term (K) with Schmidt number (Sc) varying.

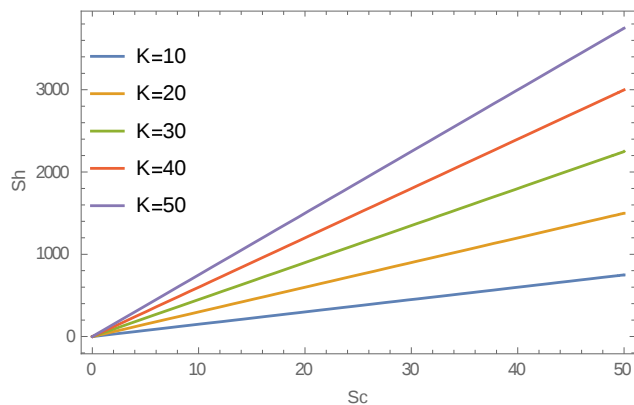


Figure 16. Sherwood number (Sh) against boundary layer Schmidt number (Sc) with Chemical reaction term (K) varying.

Figure 1: showcased the Skin friction against boundary layer of Reynolds profile with Grashofs number due to concentration varying, which indicates the onset of normal convection and intensity of heat transfer rates in Newtonian fluids flow. Figure 2 and Figure 3 also indicates the same outcomes as in Figure 1 which showcased an increase in the solutal

Grashof number enhances buoyancy due to concentration variation, which strengthens the Reynold velocity profile and increases skin friction through a higher wall velocity gradient. This result to greater viscous resistance at the surface and a more energetic boundary layer driven by mass diffusion effects.

Figure 4: displayed Skin friction against boundary layer of Prandtl number varying Grashofs number due to concentration (Gr_c) and Grashof number due to temperature in Figure 5, increase in grashof number increases skin friction due to enhanced buoyancy and convection which influences the surface shear stress of the Newtonian fluids flow. Figure 6 results that decrease in Reynolds number flaunt in laminar flow with lower skin friction also high Reynolds number results to turbulent flow with higher skin friction. This result to the higher solutal Grashof numbers amplify buoyancy-driven flow, enhancing momentum transform near the wall and increasing viscous resistance, which indicates a more energetic and convectively active boundary layer.

Figure 7: indicates Skin friction against boundary layer of Grashofs number due to concentration with Reynolds number varying result as combined inertial and buoyancy forces steepen the wall velocity gradient and enhance momentum transport in the boundary layer which also affirmed slightly as in Figure 6.

Figure 8 and Figure 9 results to the thermal boundary layer thickness, influencing skin friction. Decrease in Prandtl number in thicker thermal boundary layers, potentially reduces skin friction while High Prandtl number results to thinner thermal boundary layers, potentially increases skin friction.

Figure 10: gives Skin friction against boundary layer Grashofs number due to temperature with Reynolds number varying showcasing increase in Grashof number due to temperature increases skin friction which also results to the flow modification of Newtonian fluids flow.

Figure 11: presented the Skin friction against boundary layer Grashofs number due to temperature with Prandtl number varying displayed how the thins of the thermal boundary layer and can increase local gradients, its combined effect with a high ' Gr_T ' leads to a complex balance between viscous dissipation and buoyant acceleration and Figure 12: Skin friction against boundary layer Grashofs number due to temperature with Grashofs number due to concentration varying results higher grashof numbers due to concentration indicate stronger solutal buoyancy forces, potentially increasing skin friction.

Figure 13: exhibited Nusselt number against boundary layer Prandtl number with chemical reaction term varying and Figure 14: Nusselt number against boundary layer radiation with Prandtl number upshot the Nusselt number which also displayed the rate of chemical reaction can influence heat generation/absorption, affecting Nusselt number.

Figure 15: displayed the Sherwood number against boundary layer Chemical reaction term with Schmidt number varying and Figure 16: Sherwood number against boundary layer Schmidt number with Chemical reaction term varying which

showcased that increase in chemical reaction term led to increase in mass transfer rates, resulting in higher Sherwood number.

9. Conclusion

This work interest on the application of homotopy perturbation method in applied science and engineering. Dimensionless parameters were determined to a very large extent which are Grashof's number, the Prandtl number, Schmidt number and the Reynolds number, the impact of several substantial physical strictures in the application of engineering when solving the skin friction, the Sherwood profile and the Nusselt number from the three Newtonian fluids equation, we noticed the following important discoveries from the study.

- 1) Higher chemical reaction rates steepen concentration gradients at the wall, increasing the mass transfer rate.
- 2) Lower Prandtl numbers reduce Nusselt number by thickening the thermal boundary layer, while stronger chemical reactions enhance it by steepening temperature gradients.
- 3) An upsurge in grashof number increases skin friction due to enhanced buoyancy and
- 4) convection which influences the surface shear stress of the Newtonian fluids flow. Future studies could extend this method to non-Newtonian fluids, transient flows, or different geometries.
- 5) Decrease in Prandtl number in thicker thermal boundary layers, potentially reduces skin friction while High Prandtl number results to thinner thermal boundary layers, potentially increases skin friction.
- 6) This study demonstrates that HPM is a robust analytical tool for probing the parameter space of complex heat and mass transfer flows, providing fast and physically intuitive results that can guide more costly numerical simulations.

Abbreviations

HMP	Homotopy Perturbation Method
AGM	Akbari-Ganji's Method
DTA	Differential Transform Method
CFD	Computational Fluid Dynamics
N/MEMS	Nano/Micro-Electro-Mechanical System

Conflicts of Interest

The authors declare no conflicts of interest.

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