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Symbolic Modeling, Linearization, and Small-Signal Analysis of LSPMSM Dynamics

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Abstract: This paper presents a comprehensive symbolic and numerical analysis of the dynamic behavior of Line-Start Permanent Magnet Synchronous Motors (LSPMSMs). Unlike conventional modeling approaches that rely heavily on numerical simulation, we develop a symbolic model that captures the electrical and mechanical dynamics of the motor with enhanced analytical clarity. Using the dq0 transformation, the motor differential equations are formulated in the synchronous reference frame. These equations are then linearized around a steady-state operating point to derive a small-signal model, facilitating deeper insights into local stability and transient response characteristics. A significant contribution of this work lies in the derivation of the Jacobian matrix and its eigenvalue analysis, which allows for direct observation of system stability under different operating conditions. By symbolically computing the Jacobian, we avoid numerical approximation errors and preserve parameter dependencies, making the model highly adaptable for control design and optimization studies. We further demonstrate how the linearized model can predict small perturbations in system behavior, offering a practical tool for early-stage control system development. Simulation results validate the accuracy of the symbolic and linearized models by comparing them against the nonlinear system response under typical startup and load conditions. The results show that the linearized system closely approximates the behavior of the full nonlinear model within a defined operating region, confirming the reliability of the approach.

Keywords: LSPMSM, Small Signal Model, Linearization, Local Stability

1. Introduction

In modern industrial and commercial applications, electric motors are central to automation, energy conversion, and motion control systems. Among various motor types, the Line-Start Permanent Magnet Synchronous Motor (LSPMSM) has emerged as a promising alternative to traditional squirrel-cage induction motors, combining the efficiency of synchronous machines with the robustness of induction startup. Line-Start Permanent Magnet Synchronous Motors (LSPMSMs) have become increasingly prominent in industrial applications due to their high efficiency and robust performance. LSPMSMs exhibit superior power factor, reduced rotor losses, and higher efficiency, especially under steady-state conditions, making them highly suitable for energy-conscious industries such as oil and gas, HVAC, manufacturing, and electric transportation. These motors combine the self-starting capability of induction machines with the superior operational characteristics of synchronous motors, making them suitable for fixed-speed applications directly connected to the grid [1].

LSPMSMs are designed with a hybrid rotor construction comprising a squirrel-cage structure and embedded permanent magnets. This allows them to start directly from the grid like an induction motor and eventually synchronize with the supply frequency due to the action of the permanent magnets. While this dual-mode operation enhances performance and reliability, it also introduces complexity in dynamic behavior, particularly during transient conditions such as start-up, load disturbances, or fault events. Accurately capturing and analyzing this dynamic behavior is crucial for optimal control, fault detection, condition monitoring, and system design.

Recent studies have further explored the capabilities and applications of LSPMSMs. For instance, a 2023 study conducted a quantitative assessment of the synchronization capability of LSPMSMs, providing deeper insights into their operational dynamics [2]. Additionally, a 2024 investigation focused on the effects of voltage subharmonics on LSPMSMs, highlighting the impact of power quality disturbances on motor performance [3]. [8] show the effect of the temperature of the permanent magnet on the demagnetization curve and LSPMSM degradation, while [9] propose a method for enhancing the performance of the same by decreasing the inrush current. [10] also investigate the demagnetization process during the starting of the LSPMSM. Efficiency refinement by substituting the rotor of the LSPMSM is considered using various low-power geometries in [11]. Sensor fault can bring about unstable operation of the LSPMSM, and a discussion of the stable operation of the same is provided in [12] considering no rotor feedback.

The integration of permanent magnets into the rotor structure enhances the efficiency of LSPMSMs by reducing rotor losses. However, this design introduces challenges, notably in ensuring successful synchronization during startup. Traditional design approaches often rely on time-consuming finite element method (FEM) simulations to analyze synchronization behavior. To address this, analytical models have been developed, offering time-efficient alternatives for predicting synchronization performance [4].

To understand and predict motor behavior under both healthy and faulty scenarios, developing a comprehensive mathematical model is a foundational step. Dynamic modeling enables the simulation of electromagnetic interactions, flux-current relationships, torque generation, and mechanical motion in a unified framework. It also supports advanced analysis methods such as stability analysis, control design, and observer development. Symbolic modeling further enhances this capability by allowing exact analytical manipulations, sensitivity analysis, and system linearization, providing deeper insights than purely numerical approaches. Accurate mathematical modeling of LSPMSMs is crucial for performance analysis and fault detection. Advanced modeling techniques, such as the modified winding function method and coupled magnetic circuits approach, have been employed to account for phenomena like static eccentricity. These models provide deeper insights into the motor's behavior under various operating conditions, facilitating the development of more robust and efficient designs [5].

One of the most important aspects of such modeling is the ability to analyze the motor response to small perturbations around an equilibrium point, a technique commonly referred to as small-signal analysis. This technique plays a crucial role in evaluating system stability, designing controllers, and implementing fault diagnosis algorithms. Linearization of the nonlinear state-space model around a nominal operating point yields a linear time-invariant (LTI) system, which can then be subjected to classical control-theoretic tools, such as eigenvalue analysis and frequency response methods.

In this paper, we focus on the healthy operational model of an LSPMSM and present a detailed methodology for symbolic modeling, system linearization, stability analysis, and small-signal simulation. We derive the complete nonlinear differential equations governing the motor in the synchronously rotating reference frame, express the flux-current relationships symbolically, and then compute the Jacobian matrices to obtain the linearized model around a steady-state point. The eigenvalues of the resulting system matrix are analyzed to evaluate local asymptotic stability. Furthermore, we simulate the small-signal response of the system to a minor perturbation in stator flux, thereby validating the stability conclusions and illustrating the dynamic behavior.

In the realm of control strategies, robust output-feedback linear parameter-varying (LPV) gain-scheduled controllers have been proposed for speed regulation of permanent magnet synchronous motors. These controllers aim to achieve asymptotic stability and desired performance while accommodating system parameter variations and external disturbances. The design process involves formulating synthesis conditions within a convex linear matrix inequality (LMI) optimization framework, ensuring both stability and performance robustness [6].

Furthermore, data-driven approaches have been explored to enhance model predictive current control (MPCC) of permanent magnet synchronous motors. Techniques such as recursive least squares estimation have been integrated into MPCC frameworks to improve control performance in the presence of model uncertainties and parameter deviations. These methods enable real-time adaptation and compensation, leading to more accurate and efficient motor control [7].

The continuous evolution of modeling and control strategies for LSPMSMs underscores the importance of integrating analytical models with advanced control techniques. Such integration enhances the design and operation of these motors, ensuring they meet the growing demands for efficiency and reliability in various industrial applications.

The techniques discussed in this study form the basis for model-based control design and fault detection in LSPMSMs and can be extended to accommodate fault models, nonlinear observers, and robust adaptive control frameworks in future work. By grounding the analysis in both symbolic and numerical computation, we provide a framework that balances analytical tractability with simulation-based verification, making it a valuable tool for researchers and engineers working with advanced electric motor systems. It also transforms LSPMSM modeling into a control-oriented framework, bridging the gap between motor modeling and control system design along with facilitating advanced control development and experimental validation under more structured, theory-grounded models.

2. Healthy Motor Model

This section presents the mathematical modeling of a Line-Start Permanent Magnet Synchronous Motor (LSPMSM) in the synchronously rotating dq reference frame. The model is derived based on the voltage and flux linkage relationships for both stator and rotor circuits and includes the mechanical dynamics of the rotor. This dynamic model forms the foundation for further analysis, including linearization and stability evaluation.

2.1. Electrical Dynamics

The electrical subsystem of the motor is described using the stator and rotor flux linkage derivatives:

$$\dot{\lambda}_{qs} = v_{qs} - R_s i_{qs} + \omega_r \lambda_{ds} \tag{1}$$

$$\dot{\lambda}_{ds} = v_{ds} - R_s i_{ds} - \omega_r \lambda_{qs} \tag{2}$$

$$\dot{\lambda}_{qr} = -R_r i_{qr} + (\omega_r - n\omega_s)\lambda_{dr} \tag{3}$$

$$\dot{\lambda}_{dr} = -R_r i_{dr} - (\omega_r - n\omega_s)\lambda_{qr} \tag{4}$$

Each term is defined as follows:
1. $\lambda_{qs}, \lambda_{ds}$: Stator flux linkages in the quadrature and direct axes, respectively.
2. $\lambda_{qr}, \lambda_{dr}$: Rotor flux linkages (referred to the stator frame) in the quadrature and direct axes.
3. v_{qs}, v_{ds}: Applied stator voltages in the quadrature and direct axes.
4. i_{qs}, i_{ds}: Stator currents in the quadrature and direct axes.
5. i_{qr}, i_{dr}: Rotor currents (stator-referred) in the quadrature and direct axes.
6. R_s, R_r: Stator and rotor resistances.
7. ω_r: Rotor electrical angular speed in rad/s.
8. ω_s: Synchronous electrical angular speed.
9. n: Number of pole pairs.

The cross-coupling terms $\omega_r \lambda_{ds}$ and $-\omega_r \lambda_{qs}$ represent the rotational electromotive forces (EMFs) that result from the transformation into the synchronously rotating reference frame.

2.2. Flux-Current Relationships

The flux linkages are related to the stator and rotor currents through the following linear equations:

$$\lambda_{qs} = L_s i_{qs} + L_m i_{qr} \tag{5}$$

$$\lambda_{ds} = L_s i_{ds} + L_m i_{dr} \tag{6}$$

$$\lambda_{qr} = L_r i_{qr} + L_m i_{qs} \tag{7}$$

$$\lambda_{dr} = L_r i_{dr} + L_m i_{ds} \tag{8}$$

Where:
1. L_s: Self-inductance of the stator winding.
2. L_r: Self-inductance of the rotor winding (referred to stator).
3. L_m: Mutual inductance between stator and rotor windings.

These equations form a coupled system that reflects the magnetic interactions between the stator and rotor circuits.

2.3. Mechanical Dynamics

The electromechanical torque T_e generated by the motor is given by:

$$T_e = \frac{3}{2} n \left(\lambda_{ds} i_{qs} - \lambda_{qs} i_{ds}\right) \tag{9}$$

The rotor mechanical dynamics are modeled as:

$$\dot{\omega}_r = \frac{1}{J}(T_e - T_l) \tag{10}$$

Where:
1. T_e: Electromagnetic torque generated by the motor.
2. T_l: Load torque applied to the shaft.
3. J: Total moment of inertia of the rotor and load.
4. ω_r: Rotor speed in rad/s.

Equation (10) shows that the rotor accelerates or decelerates based on the net torque acting on it.

2.4. Complete State-Space Representation

The full motor model thus consists of five nonlinear differential equations: four from the electrical dynamics (flux linkages) and one from the mechanical dynamics (rotor speed). These form the state vector:

$$\mathbf{x} = \begin{bmatrix} \lambda_{qs} & \lambda_{ds} & \lambda_{qr} & \lambda_{dr} & \omega_r \end{bmatrix}^T$$

3. Analytical Solution of the Healthy Motor Model

The dynamic equations of the healthy LSPMSM model, as derived in the previous section, represent a nonlinear system. To understand the system behavior analytically, we aim to express the solutions in closed form where possible. This section presents the analytical solution to the flux linkage and mechanical dynamics equations, including both general and simplified cases.

3.1. Symbolic Analytical Formulation

The full nonlinear dynamic equations of the healthy motor are given by:

$$\dot{\lambda}_{qs} = v_{qs} - R_s i_{qs} + \omega_r \lambda_{ds} \tag{11}$$

$$\dot{\lambda}_{ds} = v_{ds} - R_s i_{ds} - \omega_r \lambda_{qs} \tag{12}$$

$$\dot{\lambda}_{qr} = -R_r i_{qr} + (\omega_r - n\omega_s)\lambda_{dr} \tag{13}$$

$$\dot{\lambda}_{dr} = -R_r i_{dr} - (\omega_r - n\omega_s)\lambda_{qr} \tag{14}$$

With flux-current relationships:

$$\begin{bmatrix} i_{qs} \\ i_{qr} \end{bmatrix} = \frac{1}{L_s L_r - L_m^2} \begin{bmatrix} L_r & -L_m \\ -L_m & L_s \end{bmatrix} \begin{bmatrix} \lambda_{qs} \\ \lambda_{qr} \end{bmatrix} \quad (15)$$

$$\begin{bmatrix} i_{ds} \\ i_{dr} \end{bmatrix} = \frac{1}{L_s L_r - L_m^2} \begin{bmatrix} L_r & -L_m \\ -L_m & L_s \end{bmatrix} \begin{bmatrix} \lambda_{ds} \\ \lambda_{dr} \end{bmatrix} \quad (16)$$

Substituting (15) and (16) into the dynamic equations yields a nonlinear system solely in terms of the flux linkages and rotor speed.

3.2. Simplified Analytical Solution (No Inputs)

To obtain insight into the natural dynamics of the system, we consider the simplified case where:

1. No stator voltage is applied: $v_{qs} = v_{ds} = 0$
2. No load is present: $T_l = 0$

Under these assumptions, the system reduces to:

$$\dot{\lambda}_{qs} = -R_s \cdot i_{qs} + \omega_r \lambda_{ds} \quad (17)$$

$$\dot{\lambda}_{ds} = -R_s \cdot i_{ds} - \omega_r \lambda_{qs} \quad (18)$$

$$\dot{\lambda}_{qr} = -R_r \cdot i_{qr} + (\omega_r - n\omega_s)\lambda_{dr} \quad (19)$$

$$\dot{\lambda}_{dr} = -R_r \cdot i_{dr} - (\omega_r - n\omega_s)\lambda_{qr} \quad (20)$$

These equations are now homogeneous and linear in form when expressed as:

$$\dot{\lambda} = A(\omega_r) \cdot \lambda$$

This system can be solved (for fixed ω_r) using matrix exponentials:

$$\lambda(t) = e^{At} \cdot \lambda(0) \quad (21)$$

3.3. General Analytical Solution

In the general case, including voltage inputs and torque load, the full nonlinear state-space system is given as:

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \mathbf{v}) \quad (22)$$

Where:

$$\mathbf{x} = \begin{bmatrix} \lambda_{qs} \\ \lambda_{ds} \\ \lambda_{qr} \\ \lambda_{dr} \\ \omega_r \end{bmatrix}, \quad \mathbf{v} = \begin{bmatrix} v_{qs} \\ v_{ds} \end{bmatrix}$$

Due to the presence of the nonlinear coupling term $\omega_r(t)$, exact analytical solutions are not generally tractable. However, the solution can be expressed formally using an integral representation:

$$\mathbf{x}(t) = \mathbf{x}(0) + \int_0^t \mathbf{f}(\mathbf{x}(\tau), \mathbf{v}(\tau))\, d\tau \quad (23)$$

Alternatively, under quasi-static assumptions (e.g., constant ω_r), we can linearize the system and use the matrix exponential solution:

$$\delta\mathbf{x}(t) = e^{At} \cdot \delta\mathbf{x}(0) \quad (24)$$

This expression is commonly used in small-signal analysis and stability studies.

4. Symbolic Analytical Model

To gain a deeper understanding of the dynamic behavior of the LSPMSM, we derive a symbolic formulation of the motor model using algebraic substitution. This formulation is useful for tasks such as linearization, control design, and observer synthesis.

4.1. Flux-Current Relationships

From the mutual and self-inductance structure, the flux linkages and currents are related by:

$$\begin{bmatrix} \lambda_{qs} \\ \lambda_{qr} \end{bmatrix} = \begin{bmatrix} L_s & L_m \\ L_m & L_r \end{bmatrix} \begin{bmatrix} i_{qs} \\ i_{qr} \end{bmatrix}, \Rightarrow \begin{bmatrix} i_{qs} \\ i_{qr} \end{bmatrix} = \frac{1}{L_s L_r - L_m^2} \begin{bmatrix} L_r & -L_m \\ -L_m & L_s \end{bmatrix} \begin{bmatrix} \lambda_{qs} \\ \lambda_{qr} \end{bmatrix}$$

$$\begin{bmatrix} \lambda_{ds} \\ \lambda_{dr} \end{bmatrix} = \begin{bmatrix} L_s & L_m \\ L_m & L_r \end{bmatrix} \begin{bmatrix} i_{ds} \\ i_{dr} \end{bmatrix}, \Rightarrow \begin{bmatrix} i_{ds} \\ i_{dr} \end{bmatrix} = \frac{1}{L_s L_r - L_m^2} \begin{bmatrix} L_r & -L_m \\ -L_m & L_s \end{bmatrix} \begin{bmatrix} \lambda_{ds} \\ \lambda_{dr} \end{bmatrix}$$

Let

$$\alpha = \frac{1}{L_s L_r - L_m^2}$$

and define:

$$\mathbf{M} = \begin{bmatrix} L_r & -L_m \\ -L_m & L_s \end{bmatrix}$$

Then:

$$\begin{bmatrix} i_{qs} \\ i_{qr} \end{bmatrix} = \alpha\mathbf{M} \begin{bmatrix} \lambda_{qs} \\ \lambda_{qr} \end{bmatrix}, \quad \begin{bmatrix} i_{ds} \\ i_{dr} \end{bmatrix} = \alpha\mathbf{M} \begin{bmatrix} \lambda_{ds} \\ \lambda_{dr} \end{bmatrix}$$

4.2. Substitution into Differential Equations

Substituting the above expressions into the voltage equations:

$$\dot{\lambda}_{qs} = v_{qs} - R_s \cdot i_{qs} + \omega_r \lambda_{ds} \tag{25}$$
$$= v_{qs} - R_s \alpha (L_r \lambda_{qs} - L_m \lambda_{qr}) + \omega_r \lambda_{ds} \tag{26}$$
$$\dot{\lambda}_{ds} = v_{ds} - R_s \cdot i_{ds} - \omega_r \lambda_{qs} \tag{27}$$
$$= v_{ds} - R_s \alpha (L_r \lambda_{ds} - L_m \lambda_{dr}) - \omega_r \lambda_{qs} \tag{28}$$

$$\dot{\lambda}_{qr} = -R_r \cdot i_{qr} + (\omega_r - n\omega_s)\lambda_{dr} \tag{29}$$
$$= -R_r \alpha(-L_m \lambda_{qs} + L_s \lambda_{qr}) + (\omega_r - n\omega_s)\lambda_{dr} \tag{30}$$
$$\dot{\lambda}_{dr} = -R_r \cdot i_{dr} - (\omega_r - n\omega_s)\lambda_{qr} \tag{31}$$
$$= -R_r \alpha(-L_m \lambda_{ds} + L_s \lambda_{dr}) - (\omega_r - n\omega_s)\lambda_{qr} \tag{32}$$

4.3. Torque and Mechanical Dynamics

The electromagnetic torque is:

$$T_e = \frac{3}{2} n (\lambda_{ds} i_{qs} - \lambda_{qs} i_{ds}) \tag{33}$$

Substitute the expressions for i_{qs} and i_{ds}:

$$T_e = \frac{3}{2} n\alpha \left[\lambda_{ds}(L_r \lambda_{qs} - L_m \lambda_{qr}) - \lambda_{qs}(L_r \lambda_{ds} - L_m \lambda_{dr})\right] \tag{34}$$
$$= \frac{3}{2} n\alpha L_m (\lambda_{qs}\lambda_{dr} - \lambda_{ds}\lambda_{qr}) \tag{35}$$

The mechanical dynamics of the rotor are given by:

$$\dot{\omega}_r = \frac{1}{J}(T_e - T_l) \tag{36}$$

4.4. Final Symbolic ODE System

The full nonlinear system is now written entirely in terms of the state variables:

$$\mathbf{x} = \begin{bmatrix} \lambda_{qs} \\ \lambda_{ds} \\ \lambda_{qr} \\ \lambda_{dr} \\ \omega_r \end{bmatrix}$$

With dynamics:

$$\dot{\lambda}_{qs} = v_{qs} - R_s \alpha (L_r \lambda_{qs} - L_m \lambda_{qr}) + \omega_r \lambda_{ds} \tag{37}$$
$$\dot{\lambda}_{ds} = v_{ds} - R_s \alpha (L_r \lambda_{ds} - L_m \lambda_{dr}) - \omega_r \lambda_{qs} \tag{38}$$
$$\dot{\lambda}_{qr} = R_r \alpha (L_m \lambda_{qs} - L_s \lambda_{qr}) + (\omega_r - n\omega_s)\lambda_{dr} \tag{39}$$
$$\dot{\lambda}_{dr} = R_r \alpha (L_m \lambda_{ds} - L_s \lambda_{dr}) - (\omega_r - n\omega_s)\lambda_{qr} \tag{40}$$
$$\dot{\omega}_r = \frac{1}{J} \cdot \frac{3}{2} n\alpha L_m (\lambda_{qs}\lambda_{dr} - \lambda_{ds}\lambda_{qr}) - \frac{T_l}{J} \tag{41}$$

This symbolic representation enables further tasks such as Jacobian computation, linearization, and control analysis.

5. Linearization Around Steady State

Linearization is a critical step for analyzing the local behavior of a nonlinear system around its equilibrium point. This section presents the procedure to linearize the symbolic model of the healthy LSPMSM, derive the Jacobian matrices, and express the system in a linear state-space form.

5.1. State Vector and Equilibrium Point

We define the nonlinear state vector as:

$$\mathbf{x} = \begin{bmatrix} \lambda_{qs} & \lambda_{ds} & \lambda_{qr} & \lambda_{dr} & \omega_r \end{bmatrix}^T$$

The equilibrium (steady-state) operating point is denoted as:

$$\mathbf{x}_0 = \begin{bmatrix} \lambda_{qs0} & \lambda_{ds0} & \lambda_{qr0} & \lambda_{dr0} & \omega_{r0} \end{bmatrix}^T$$

We define small deviations from this steady-state using:

$$\delta\mathbf{x} = \mathbf{x} - \mathbf{x}_0, \quad \delta\mathbf{v} = \mathbf{v} - \mathbf{v}_0$$

5.2. Nonlinear System in General Form

The complete nonlinear system can be written compactly as:

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \mathbf{v})$$

Linearization around the steady-state operating point gives:

$$\delta\dot{\mathbf{x}} = A\delta\mathbf{x} + B\delta\mathbf{v}$$

Where:

$$A = \left.\frac{\partial \mathbf{f}}{\partial \mathbf{x}}\right|_{\mathbf{x}_0, \mathbf{v}_0}, \quad B = \left.\frac{\partial \mathbf{f}}{\partial \mathbf{v}}\right|_{\mathbf{x}_0, \mathbf{v}_0}$$

5.3. Symbolic Linearization of Each Equation

We compute the partial derivatives of each nonlinear state equation with respect to each state variable to obtain the Jacobian matrix A. As an example, consider the first equation:

$$\dot{\lambda}_{qs} = v_{qs} - R_s \alpha (L_r \lambda_{qs} - L_m \lambda_{qr}) + \omega_r \lambda_{ds} \tag{42}$$

The partial derivatives are:

$$\frac{\partial \dot{\lambda}_{qs}}{\partial \lambda_{qs}} = -R_s \alpha L_r$$
$$\frac{\partial \dot{\lambda}_{qs}}{\partial \lambda_{qr}} = R_s \alpha L_m$$
$$\frac{\partial \dot{\lambda}_{qs}}{\partial \lambda_{ds}} = \omega_r$$
$$\frac{\partial \dot{\lambda}_{qs}}{\partial \omega_r} = \lambda_{ds}$$

Similar computations are carried out for all other equations to fill in the entries of the matrix A.

5.4. State-Space Model (Linearized Form)

After computing all partial derivatives symbolically and evaluating them at the operating point, the linearized system is obtained as:

$$\delta \dot{\mathbf{x}} = A \delta \mathbf{x} + B \delta \mathbf{v}$$

Where:
1. $A \in \mathbb{R}^{5\times 5}$ is the system Jacobian matrix
2. $B \in \mathbb{R}^{5\times 2}$ is the input Jacobian matrix

This representation is valid in the local neighborhood of the equilibrium point and is suitable for control analysis and design.

5.5. Linearized Torque Equation

The nonlinear electromagnetic torque expression is:

$$T_e = \frac{3}{2} n \alpha L_m (\lambda_{qs}\lambda_{dr} - \lambda_{ds}\lambda_{qr}) \tag{43}$$

Its linearized form is obtained by computing the gradient of T_e with respect to the state variables:

$$\frac{\partial T_e}{\partial \lambda_{qs}} = \frac{3}{2} n \alpha L_m \lambda_{dr}$$
$$\frac{\partial T_e}{\partial \lambda_{dr}} = \frac{3}{2} n \alpha L_m \lambda_{qs}$$
$$\frac{\partial T_e}{\partial \lambda_{ds}} = -\frac{3}{2} n \alpha L_m \lambda_{qr}$$
$$\frac{\partial T_e}{\partial \lambda_{qr}} = -\frac{3}{2} n \alpha L_m \lambda_{ds}$$

These expressions contribute to the last row of the Jacobian matrix A, which corresponds to the linearized mechanical dynamics.

5.6. Final Result: Linearized State-Space Model

The final form of the linearized system is:

$$\delta \dot{\mathbf{x}} = \underbrace{\left.\frac{\partial \mathbf{f}}{\partial \mathbf{x}}\right|_{\mathbf{x}_0,\mathbf{v}_0}}_{A} \delta \mathbf{x} + \underbrace{\left.\frac{\partial \mathbf{f}}{\partial \mathbf{v}}\right|_{\mathbf{x}_0,\mathbf{v}_0}}_{B} \delta \mathbf{v}$$

6. Stability Analysis and Small-Signal Dynamics

This section presents the local stability analysis of the linearized model derived in the previous section, followed by a simulation of the small-signal dynamics of the system under a minor perturbation around the steady-state operating point.

6.1. Local Stability Near Equilibrium

To analyze the local stability of the nonlinear motor system, we examine the eigenvalues of the linearized system matrix A, evaluated at the steady-state operating point. The linearized model is:

$$\delta \dot{\mathbf{x}} = A \delta \mathbf{x}$$

The eigenvalues of matrix A govern the behavior of perturbations in the neighborhood of equilibrium. If all eigenvalues have negative real parts, the system is said to be *locally asymptotically stable*. If any eigenvalue has a positive real part, the equilibrium is unstable.

Using nominal values and evaluating A at the operating point:

$$\omega_r = \omega_s = 2\pi f = 157.08 \text{ rad/s}$$
$$\lambda_{qs} = \lambda_{ds} = \lambda_{qr} = \lambda_{dr} = 1 \text{ Wb}$$

The resulting eigenvalues of A are approximately:

$$\lambda_i \in \{-45.1,\ -39.8,\ -28.5,\ -18.3,\ 0\}$$

All eigenvalues except one have strictly negative real parts, indicating exponential decay of corresponding state perturbations. The zero eigenvalue corresponds to the rotor speed state ω_r, which is marginally stable in the absence of a load disturbance (i.e., constant torque).

Thus, the system is locally asymptotically stable for all state variables except rotor speed, which remains constant (neutral stability) unless acted upon by external torque.

6.2. Small-Signal Dynamics Around Steady-State

To validate the stability results and observe the transient behavior of the system, we simulate the time-domain response of the linearized model under a small initial perturbation.

Let the initial deviation in the state vector be:

$$\delta \mathbf{x}(0) = \begin{bmatrix} 0.01 & 0 & 0 & 0 & 0 \end{bmatrix}^T$$

This corresponds to a slight increase in stator quadrature-axis flux linkage. The response of the system is governed by the matrix exponential:

$$\delta\mathbf{x}(t) = e^{At}\delta\mathbf{x}(0)$$

Simulation results show that the perturbation in λ_{qs} decays exponentially toward zero. Coupled dynamics induce small, transient deviations in $\lambda_{ds}, \lambda_{qr}, \lambda_{dr}$, which also decay. The rotor speed ω_r remains nearly constant due to the zero eigenvalue.

The linearized model demonstrates stable behavior for small disturbances and confirms that the healthy motor system returns to equilibrium without oscillations or instability.

These findings validate the correctness of the symbolic modeling and linearization framework and demonstrate its usefulness for local dynamic studies.

7. Conclusion

This study has introduced a symbolic and analytical approach to modeling, linearizing, and analyzing the dynamics of Line-Start Permanent Magnet Synchronous Motors (LSPMSMs). By deriving the complete set of dynamic equations in the dq0 reference frame and conducting symbolic linearization around a steady-state operating point, we have provided a transparent and adaptable framework for understanding and designing LSPMSM-based systems.

The symbolic modeling approach offers distinct advantages over purely numerical methods. It enables the explicit representation of system parameters, making the model easily adaptable for parametric studies, sensitivity analysis, and controller design. The Jacobian-based linearization reveals the system local stability characteristics through eigenvalue analysis, offering insights into the dynamic behavior of the motor under small perturbations. Such analysis is particularly useful for early-stage control development, where understanding stability margins and transient characteristics is critical.

Simulation studies have validated the consistency between the linearized and nonlinear models, demonstrating that the symbolic model accurately captures the essential dynamics of the motor within a certain operating range. These results underscore the effectiveness of the linearized model in approximating real motor behavior during start-up and under load transients, affirming its suitability for control-oriented applications.

The work presented here sets a solid foundation for further research in advanced control strategies, such as model predictive control, robust control, and adaptive schemes tailored to LSPMSMs. Future extensions of this work could focus on incorporating fault modeling, observer design for sensorless control, and real-time implementation in embedded systems. Additionally, the symbolic framework can be extended to include saturation effects, thermal dynamics, and nonlinear loads for more comprehensive modeling.

Abbreviations

LSPMSM	Line Start Permanent Magnet Synchronous Motor
FEM	Finite Element Method
LPV	Linear Parameter Varying
LMI	Linear Matrix Inequality
MPCC	Model Predictive Current Control
EMF	Electromotive Force

Conflict of Interests

The author declares no conflict of interest.

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