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Research Article

Investigation on Optimal Ultrasonic Frequency for Effective Particle Aggregation in Suspension

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Abstract

An analytical investigation is conducted to obtain optimal ultrasonic frequency for effective particle aggregation in suspension. In this investigation, we are discussed the proper mode of action of ultrasound to produce the ultrasonic standing wave field, due to the fact that the aggregation of particles in suspension occurs in the ultrasonic standing wave field. The mechanism of ultrasonic aggregation is also investigated qualitatively. Equation for calculating optimal ultrasonic frequency is derived based on the consideration of the forces acting on the particle in ultrasonic standing wave. According to current results, the optimal ultrasonic frequency is proportional to −3/2 power of the particle size. According to the experimental results, when the size of the particles to aggregate is micrometer, the effective ultrasonic frequency becomes the MHz band. The order calculations using this relationship are in good agreement with the experimental results that the effective frequency of ultrasonic cohesion is in the MHz range. In addition, a comparison with previous studies is also carried out to validate the relationship. According to previous study, the most effective frequency for pulverized coal recovery is 100kHz, among 80kHz, 100 kHz and 120 kHz. The theoretical calculations of this using the relations we derived are 101.806 kHz, which is in close agreement with Wang's results. This equation can be used effectively in several fields such as mineral dressing, particle control and separation of blood cells.

Keywords

Ultrasonic Standing Wave, Aggregation, Separation, Optimal Frequency

1. Introduction

Ultrasonic aggregation is used efficiently in mineral dressing process, fine coal recovery, bubble control, heterologous particle separation and so on [1-3].

There are two kinds of ultrasonic aggregation: one in aerosol and in suspension. Aerosol is a mixed medium of solid or liquid particles in a gas and these particles are aggregated by applied ultrasonic wave. According to a number of preceded theoretical and experimental studies, ultrasonic standing wave and travelling wave both cause aggregation in aerosol and its optimal frequency is in audible sound range: 1~10 kHz [4]. In suspension, solid particles are mixed in liquid, ultrasonic aggregation is caused by only standing wave and its optimal frequency is about from a few hundred kHz to several MHz, much higher than in aerosol [2, 3, 8-10]. When the size of particles is smaller, it shows a tendency that the optimal frequency gets higher. Frequency, intensity and acting time of applied ultrasonic wave are main factors for the aggregation and the critical one is frequency [1, 3].

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In this investigation, we dealt with ultrasonic aggregation for suspension and it is mainly caused by force of radiation pressure [8].

The force acting on particles in ultrasonic field already has been studied a lot [1-4]. Typically, there are King's research in 1934, Yoshioka and Gawasima in 1955, Korkov in 1961 and Henrik's research in 2012. In 1934, King derived the force acting on incompressible particle in acoustic field and Yoshioka and Gawasima derived for compressible one in 1955 [8, 10]. Korkov summarized and generalized their studies in 1961 [11]. In 2012, Henrik Brus studied acoustic radiation forces acting on particles in ultrasonic standing wave field [8]. The forces acting on particles in ultrasonic standing wave field had been studied a lot. However, relationship between optimal frequency and radiation force has been scarcely studied [5-7]. The radiation force on particles depends on radius and density of particles in ultrasonic standing wave field [12-14]. Whereas main factor of ultrasonic aggregation is frequency, the relationship between optimal frequency and particle size can be derived from forces acting on particles in ultrasonic standing wave field.

Therefore, forces acting on particles were considered and the relation between frequency of ultrasonic standing wave and radius of particles was derived in this paper.

This investigation is organized to four sections. In Section 2, we investigate the mechanism of aggregation based on the consideration of the forces acting on the particles in suspension. In Section 3, we derive the relationship between the optimal ultrasonic frequency and particle size. In Section 4, calculations are carried out to verify the results of the relationship. Section 5 gives the conclusions of this investigation.

2. Physical Principle of Ultrasonic Aggregation

2.1. Forces Acting on the Particles

Figure 1 shows forces acting on solid particles in suspension. There is an ultrasonic source on the left and it emits plane waves in the liquid medium.

Above all, the radiation force from ultrasonic standing wave field acts on particles. The force is given as follows [4]:

$$F_{rad} = 4\pi\varphi k r^3 E_{ac} \sin(2kx) \tag{1}$$

Where, $\varphi = \frac{1}{3}(\frac{5\rho_p - 2\rho_0}{2\rho_p + \rho_0})$ is contrast factor, k is wave number, r is radius of the particle, E_{ac} is energy density of ultrasonic field and ρ_p, ρ_0 are densities of particle and medium, respectively.

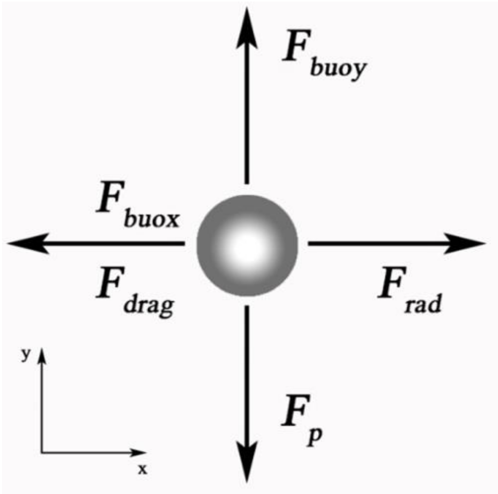

Figure 1. *Forces acting on a particle.*

Also, the viscous fluid exerts drag force on moving particle. It can be written by Stokes law as follows.

$$F_{drag} = -6\pi\eta r v_p \tag{2}$$

Where, η is viscosity, v_p is velocity of particle.

Then, gravity and buoyancy force acts on the particle by own mass.

$$P = mg = \frac{4}{3}\pi r^3 \rho g \tag{3}$$

$$F_{buo} = \frac{4}{3}\pi r^3 (\rho_0 - \rho_p) g \tag{4}$$

Here, meanings of parameters and suffixes are same as above.

2.2. Ultrasonic Standing Wave

In the suspension, the particles are aggregated when the ultrasonic wave is standing wave field.

There are various methods to form the ultrasonic standing wave field in a suspension. In ultrasonic aggregation of a suspension, the direction of applied ultrasonic waves is an important factor. Thus, there are three methods: from top to bottom, from bottom to top and from side to side for the vessel [15].

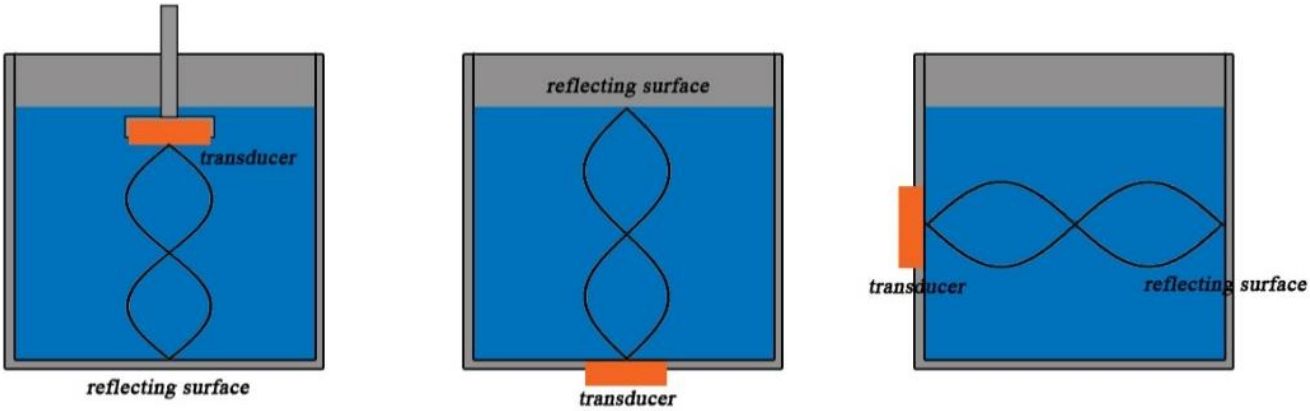

Figure 2. Action mode of ultrasound for ultrasonic standing wave field.

Among them, it is most effective to apply ultrasonic waves on the side. This is due to the fact that the most stable standing wave is formed by this direction. Hence, we considered that ultrasonic waves were emitted from the side to derive the relational equation.

2.3. Mechanism of Particle Aggregation

As shown in Eq. (1), the radiation force acting on a particle in ultrasonic standing wave field is spatially periodic. This radiation force causes the particles to be aggregated in nodes and anti-nodes of the ultrasonic standing wave. The direction of this force depends on densities of the medium and the particles. This is because the radiation force, as shown in Eq. (1), contains a contrast factor (shown in Figure 3), related to the densities of the medium and the particle. Figure 3 shows contrast factors of various solid particles in several mediums. As shown, both of magnitudes and signs of the contrast factors vary with materials.

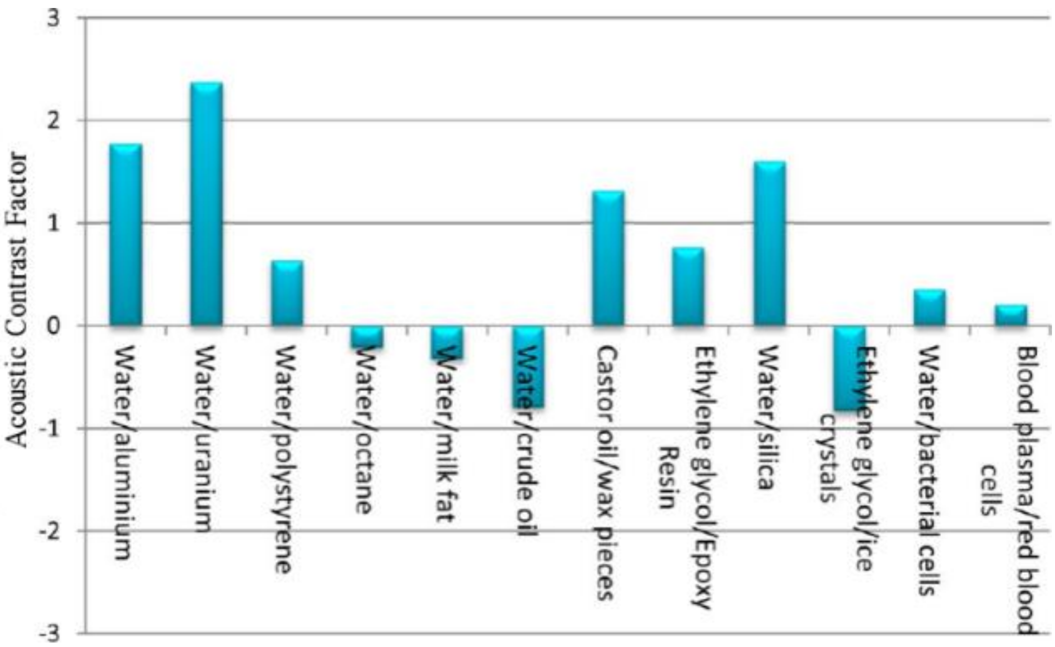

Figure 3. Acoustic contrast factors of materials in different liquid medium. The values of the factors vary from substance to substance. Hence, the radiation force will also change [3].

Particles are aggregated in nodes and anti-nodes of the ultrasonic standing wave by the radiation force, where Bernoulli forces promote the aggregation of particles. That is, the forces of attraction between particles moving with relative velocities in the medium act to aggregate.

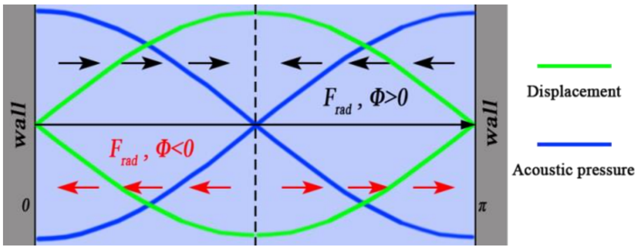

Figure 4. Direction of radiation force with contrast factors in ultrasonic standing wave.

As shown in the Figure 4, the direction of radiation force varies with the sign of the contrast factor. Then, particles with positive values of the contrast factor are aggregated in nodes of acoustic pressure while others are aggregated in anti-nodes.

3. Relational Equation Between Optimal Frequency and Particle Size

First of all, let's consider what forces should be taken into calculation to derive the relational equation between particle size and the optimal frequency which is effective for ultrasonic aggregation in the ultrasonic standing wave field. The geometrical shape of particle is assumed to be spherical. The forces acting on a particle are described in the previous section, where gravity and buoyancy are negligible compared to acoustic radiation and drag forces. In fact, the gravity on a particle, which is expressed as Eq. (3), is proportional to the third power of the radius. Considering that the particle size is on the micrometer scale, the value is very small and negligible. Also, the buoyancy force is proportional to third power of the particle size, but it is also negligible when the particle density is larger than the medium. Even if a particle is gravitationally downward, it will experience a Stokes drag force that corresponds to the velocity due to the movement, which is proportional to the radius and is much larger than the force of gravity and buoyancy. Hence, neither gravity nor buoyancy can be neglected [16].

The forces considered in the derivation are the acoustic radiation force and Stokes drag force that act on the particles in the direction of aggregation to nodes and anti-nodes of the ultrasonic standing wave. Acoustic radiation force is expressed as Eq. (1) and it can be rewritten as follows.

$$F_{rad} = 4\pi\varphi k r^3 E_{ac} \sin(2kx) \tag{5}$$

This is the time-averaged force. The acoustic radiation force acting on the particle during the aggregation process is balanced against the Stokes drag force.

$$\vec{F}_{rad} + \vec{F}_{drag} = 0 \tag{6}$$

Where, F_{drag} is Stokes drag force.

However, the drag force expressed in Eq. (2) is not the time-averaged force, but the instantaneous force.

Hence, the Stokes force should be time-averaged to derive the relationship between the optimal frequency and particle size. If the viscosity of the medium is a constant and the particle vibrates harmonically in the ultrasonic field, the time-averaged of the Stokes force will be zero. However, the viscosity depends on the temperature and the temperature within the ultrasonic field changes periodically. Assuming that the thermodynamic processes are adiabatic in the vessel, the relationship between the pressure and temperature variations is:

$$\Delta T = T_0 \alpha_T \frac{\Delta P}{\rho_0 c_p} \tag{7}$$

Where, α_T is the thermal expansion coefficient, T_0 is the equilibrium temperature, ρ_0 is the equilibrium density of the medium, and c_p is the static pressure specific heat.

The thermal expansion coefficient is written as:

$$\alpha_T = \frac{1}{V_0}\frac{dV}{dT} \tag{8}$$

Also, the pressure variation value ΔP is given by $\Delta P \equiv P = \rho_0 c_0 v$, applied a first order approximation.

Hence, the relationship between the temperature variation and the vibration velocity is

$$\Delta T = \frac{\alpha_T T_0}{c_p} \rho_0 c_0 v \frac{1}{\rho_0 c_p} = \frac{c_0 \alpha_T T_0}{c_p} v \tag{9}$$

On the other hand, expand the viscosity depended on temperature to the first term.

$$\eta(T) = \eta_0 + \frac{d\eta}{dT}\Delta T \tag{10}$$

The time-averaged of Stokes drag force from Eq. (9) and Eq. (10) is

$$\bar{F}_{drag} = 6\pi r \eta_0 \frac{V_m{}^2}{2c_0}\left(1 - 2\frac{d\eta}{dT}\frac{\alpha_T c_0{}^2 T_0}{c_p}\right) \tag{11}$$

Where, V_m is the amplitude of the vibration velocity of the medium [5].

$$V_m = 2\sqrt{\frac{E_{ac}}{\rho_0}}\cos(kx) \tag{12}$$

Eq. (11) gives the drag force. Its sign is negative and <-> must be attached to the front.

From the time-averaged Stokes force, the balanced equation is

$$F_{drag} = F_{rad} \tag{13}$$

Rewriting Eq. (13) gives

$$12\pi r\eta_0 \frac{E_{ac}}{\rho_0 c_0}\cos^2(kx)\left(1-2\frac{d\eta}{dT}\frac{\alpha_T {c_0}^2 T_0}{c_p}\right) = 4\pi\varphi k r^3 E_{ac}\sin(2kx) \tag{14}$$

Considering that $\sin(2kx) = 2\sin(kx)\cos(kx)$ is

$$3\eta_0 \frac{\cos(kx)}{\rho_0 c_0}\left(1-2\frac{d\eta}{dT}\frac{\alpha_T {c_0}^2 T_0}{c_p}\right) = 2\varphi k r^2 \sin(kx) \tag{15}$$

Hence,

$$\frac{1}{\tan(kx)} = \frac{\cos(kx)}{\sin(kx)} = \frac{2\varphi\rho_0 c_0 k r^2}{3\eta_0\left(1-2\frac{d\eta}{dT}\frac{\alpha_T {c_0}^2 T_0}{c_p}\right)} \tag{16}$$

Where, kx represents the position of the particle and covers from 0 to $\frac{\pi}{2}$. Let x_0 be the initial position of the particle and x be the final position. Then, x is $\frac{\pi}{2}n$, nodes or anti-nodes of the ultrasonic standing wave. N is an integer, which is $n=1,2,3,\cdots$. Considering that initial position is $kx_0 \approx 0$. $\tan(kx_0)$ is orderly kx_0

Therefore,

$$0 \le \frac{2\varphi\rho_0 c_0 k r^2}{3\eta_0\left(1-2\frac{d\eta}{dT}\frac{\alpha_T {c_0}^2 T_0}{c_p}\right)} \le \frac{1}{kx_0} \tag{17}$$

Here, x_0 mustn't be zero and its minimum value is r.

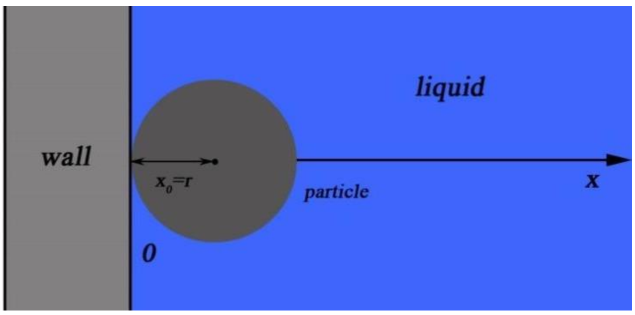

Figure 5. *Minimum of particle's initial position.*

Thus, the relational equation from initial position of particle is given by.

$$0 \le \frac{2\varphi\rho_0 c_0 k r^2}{3\eta_0\left(1-2\frac{d\eta}{dT}\frac{\alpha_T {c_0}^2 T_0}{c_p}\right)} \le \frac{1}{kr} \tag{18}$$

Here, all particles can be aggregated efficiently only when the particles farthest from the aggregation point are aggregated. To aggregate all particles, it should be:

$$\frac{2\varphi\rho_0 c_0 k r^2}{3\eta_0\left(1-2\frac{d\eta}{dT}\frac{\alpha_T {c_0}^2 T_0}{c_p}\right)} = \frac{1}{kr} \tag{19}$$

Therefore, wave number *k* is:

$$k = \sqrt{\frac{3\eta_0\left(1-2\frac{d\eta}{dT}\frac{\alpha_T {c_0}^2 T_0}{c_p}\right)}{2\varphi\rho_0 c_0}} \cdot r^{-\frac{3}{2}} \tag{20}$$

Where, $k = 2\pi f / c_0$, Then the optimal frequency is

$$f = \sqrt{\frac{3\eta_0 c_0\left(1-2\frac{d\eta}{dT}\frac{\alpha_T {c_0}^2 T_0}{c_p}\right)}{8\pi^2\varphi\rho_0}} \cdot r^{-\frac{3}{2}} \tag{21}$$

4. Results and Discussion

4.1. Order Comparison with Frequency Range of Ultrasonic Aggregation

As mentioned above, the frequency range of ultrasonic aggregation covers from a few hundred kHz to several MHz.

The order size of each factor in Eq. (22) is as follows.

Table 1. *Order magnitude of the factors involved in the relational equation.*

Factor	order	Factor	order
η_0	10^{-3}	ρ_0	10^3
c_0	10^3	φ	10^0
T_0	10	r	10^{-6}
α_T	10^{-4}	c_p	10^3

Using the values of Table 1, the order magnitude of the frequency is about 10^6Hz. Thus, Eq. (22) satisfies the experimental data for the frequency range of ultrasonic aggregation in suspension [6-10].

4.2. Comparison Calculation with the Experiment Data of Fine Coal Recovery

To verify the derived relationship, we carried out a comparison calculation with experimental data for fine coal recovery of L. Jin et al [6]. The liquid medium in the suspension is water, which is mixed with fine coal particles. The density of fine coal particles is about $1300 kg/m^3$, and the particle radius is $45\times10^{-6} m$ by considering the experimental data. This is because the particles had these size are the most abundant with 73.87%. The values of the factors for the calculation were defined as follows.

Table 2. *Magnitude of factors for calculation of optimal frequency.*

Factor	Value	Factor	Value
η_0	$1.025\times10^{-3} Pa\cdot s$	ρ_0	$1000 kg/m^3$
c_0	$1418 m/s$	φ	0.416
T_0	20℃	r	$45\times10^{-6} m$
α_T	2.1×10^{-4} ℃$^{-1}$	c_p	$4200 J/(kg\cdot$℃$)$

The viscosity gradient $\frac{d\eta}{dT}$ has not been determined yet. This gradient is determined using numerical differentiation from the following values:

Table 3. *Viscosities variation with temperatures.*

Temperature/℃	Viscosity/Pa s	Temperature/℃	Viscosity/Pa s
0	1.792×10^{-3}	60	0.469×10^{-3}
10	1.308×10^{-3}	70	0.406×10^{-3}
20	1.025×10^{-3}	80	0.357×10^{-3}
30	0.801×10^{-3}	90	0.317×10^{-3}
40	0.656×10^{-3}	100	0.284×10^{-3}
50	0.549×10^{-3}		

Hence, the optimal frequency is 101806.42 Hz.

The experimental data showed that the highest yield was obtained at 100 kHz, among the values of 80 kHz, 100 kHz and 120 kHz. This shows good agreement between the experimental and calculated values.

The variation of the optimal frequency with particle size in fine coal recovery can be visualized from the relationship graph. As shown in Eq. (22), the optimal frequency is proportional to the -3/2 power of the particle radius. Hence, the relationship graph between them is as follows.

Actually, particles size has not certain value and lie within a given range in a suspension.

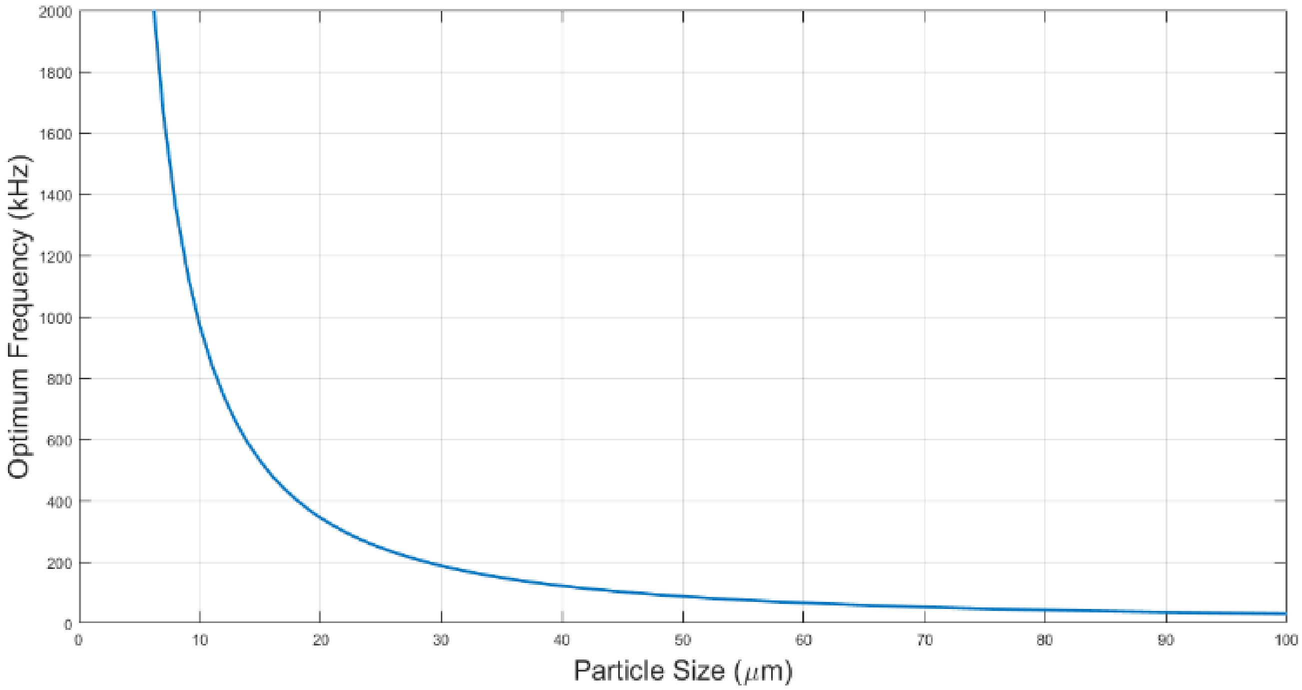

Figure 6*. Relation between particle size and optimal frequency in fine coal recovery.*

Figure 6 shows that the frequency range corresponding to a given range of particle sizes is much narrower. This allows the calculation to be convenient within a certain particle size range when the optimal frequency is calculated using this relationship.

4. Conclusion

In this investigation, we derived a relational equation to obtain the optimal frequency of the ultrasonic field according to the particle density and size in ultrasonic aggregation. The derived relational equation shows that the smaller the particle size is, the higher the optimal frequency is. To be concrete, optimal frequency was proportional to -3/2 power of the particle size. This relational equation also satisfied the frequency range of ultrasonic aggregation in the suspension. This relational equation can be used to calculate the optimal frequency of ultrasonic devices for mineral processing, heterogeneous particle separation, separation of blood cells and fine coal recovery etc. In the forthcoming investigation, we will experiment and simulate the derived relations.

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Author Contributions

Song-Guk. Kim: Conceptualization, Formal Analysis, Investigation, Methodology, Writing-original draft, Validation

Kyong-Ho Ri: Conceptualization, Data curation, Visualization

Sang-Jin Kim: Investigation, Resources, Writing-original draft

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Data Availability Statement

Not applicable.

Conflicts of Interest

The authors declare no conflict of interest.

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Biography

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