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Research Article

A New Discussion on Banach-Type Fixed Point Result in Double-Composed Metric Spaces

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Abstract

One of the important research directions of fixed point theory is the generalization of metric spaces. An interesting generalization of metric space is the modification of triangular inequality. In the last few decades, many generalizations of metric space have been introduced in the field of fixed point theory by changing triangle inequality using multiplication of constants or functions. Recently, a new generalization of metric space with changing triangle inequality using the composition of two functions, namely, a double-composed metric space, has been introduced. A double-composed metric space is a new concept using the composition of functions, unlike the previous generalizations of metric space that modify the triangular inequality using the multiplication of functions. And, Banach-type fixed point result and Kanan-type fixed point result are established under certain assumptions in the setting of double-composed metric spaces. In this paper, we reconsider the Banach-type fixed point result in the setting of double-composed metric spaces under new and simple conditions. We have proved the fixed point theorem by using a new proof method and, consequently, we have demonstrated that Banach's contractions in double-composed metric spaces have a unique fixed point under different assumptions from the previous one. We also present an example showing the validity of our fixed point result. Finally, we apply our fixed point result to show the existence of solution of Fredholm integral equations.

Keywords

Fixed Point, Banach Contraction, Double-Composed Metric Space, Fredholm Integral Equation

1. Introduction and Preliminaries

Fixed point theory has wide applications in various fields including nonlinear analysis, engineering, computer science, mathematical modeling, etc., [1]. Banach's contraction principle [2] plays a key role in fixed point theory. Many researchers have been engaged in research to extend and improve Banach's contraction principle, where recent attention is focused on generalization of metric spaces. Several abstract spaces that generalize triangular inequalities such as b-metric space [3], extended b-metric space [4], controlled metric space [5], and double controlled metric space [6] have been introduced in the field of fixed point theory. In 2023, Ayoob et al. [7] defined a new extension of metric space by

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using the composition of functions, namely, double-composed metric space, as follows.

Definition 1.1 ([7]). Let $\mathcal{F},\mathcal{G}:[0,\infty)\to[0,\infty)$ be two non-constant functions. A double-composed metric on $K\neq\emptyset$ is a function $\mathcal{D}_c:K\times K\to[0,\infty)$ such that $\forall\kappa,\iota,\nu\in K$:

1) $\kappa=\iota\Leftrightarrow\mathcal{D}_c(\kappa,\iota)=0$;
2) $\mathcal{D}_c(\kappa,\iota)=\mathcal{D}_c(\iota,\kappa)$;
3) $\mathcal{D}_c(\kappa,\iota)\le\mathcal{F}(\mathcal{D}_c(\kappa,\nu))+\mathcal{G}(\mathcal{D}_c(\nu,\iota))$.

The pair $(K,\mathcal{D}_c)$ is called a double-composed metric space. Obviously, each metric space is a double-composed metric space via $\mathcal{F}(x)=\mathcal{G}(x)=x$ for all $x\ge0$.

More recently, several papers have been published concerning the double-composed metric space and its generalization, see [8-15].

Example 1.1 ([8]). Let $K=\mathbb{R}$. Define $\mathcal{D}_c:K\times K\to[0,\infty)$ as $\mathcal{D}_c(\kappa,\iota)=e^{|\kappa-\iota|}-1$, for all $\kappa,\iota\in K$. Then $(K,\mathcal{D}_c)$ is a double-composed metric space via $\mathcal{F},\mathcal{G}:[0,\infty)\to[0,\infty)$ with $\mathcal{F}(x)=\mathcal{G}(x)=\frac{x^2+2x}{2}$ for all $x\ge0$, which is not a standard metric space.

The authors of [7] presented some concepts concerning double-composed metric spaces.

Definition 1.2 ([7]). Let $(K,\mathcal{D}_c)$ be a double-composed metric space.

1) $\{\kappa_i\}\subset K$ converges to κ if $lim_{i\to\infty}\mathcal{D}_c(\kappa_i,\kappa)=0$ for some $\kappa\in K$.
2) $\{\kappa_i\}\subset K$ is said to Cauchy if $lim_{i,j\to\infty}\mathcal{D}_c(\kappa_i,\kappa_j)=0$;
3) $(K,\mathcal{D}_c)$ is said to complete if every Cauchy sequence on K converges to some point of K.

Proposition 1.1 ([7]). Let $(K,\mathcal{D}_c)$ be a double-composed metric space via two non-constant functions $\mathcal{F},\mathcal{G}:[0,\infty)\to[0,\infty)$. If $\mathcal{F}$ and $\mathcal{G}$ are continuous and $\mathcal{F}(0)=\mathcal{G}(0)=0$, then every convergent sequence has a unique limit.

The following theorem is Banach-type fixed point result in the framework of double-composed metric space proposed in [7].

Theorem 1.1 ([7]). Let $(K,\mathcal{D}_c)$ be a complete double-composed metric space via continuous and non-decreasing functions $\mathcal{F},\mathcal{G}:[0,\infty)\to[0,\infty)$ with $\mathcal{F}(0)=\mathcal{G}(0)=0$, and let $\mathcal{T}:K\to K$ be a mapping such that

$$\mathcal{D}_c(\mathcal{T}\kappa,\mathcal{T}\iota)\le\lambda\mathcal{D}_c(\kappa,\iota),\ \forall\kappa,\iota\in K,$$

where $\lambda\in(0,1)$. Define a sequence $\{\kappa_i\}$ as $\kappa_i=\mathcal{T}^i\kappa_0$ for κ_0. Assume that:

1) $lim_{i,j\to\infty}\left(\sum_{n=j}^{i-2}\mathcal{G}^{n-j}\mathcal{F}(\lambda^n\mathcal{D}_c(\kappa_0,\kappa_1))+\mathcal{G}^{i-j-1}\left(\lambda^{i-1}\mathcal{D}_c(\kappa_0,\kappa_1)\right)\right)=0$, where $\mathcal{G}^{n-j}\mathcal{F}(\lambda^n\mathcal{D}_c(\kappa_0,\kappa_1))$ and $\mathcal{G}^{i-j-1}\left(\lambda^{i-1}\mathcal{D}_c(\kappa_0,\kappa_1)\right)$ denotes the composite function.
2) $\mathcal{G}$ is sub-additive.

Then $\mathcal{T}$ has a unique fixed point.

In this paper, we prove Banach-type fixed point result in double-composed metric space without the assumptions 1) and 2) of Theorem 1.1. We also present an example showing the significance of the obtained result. Moreover, we show the existence of a solution of Fredholm integral equations by applying obtained fixed point result.

2. Main Results

We reconsider the Banach-type fixed point theorem in double-composed metric spaces without the assumptions 1) and 2) of Theorem 1.1, while we use the assumption that $\mathcal{F}$ and $\mathcal{G}$ are strictly increasing instead of the assumption that they are non-decreasing.

Theorem 2.1. Let $(K,\mathcal{D}_c)$ be a complete double composed metric space via two continuous and strictly increasing functions $\mathcal{F},\mathcal{G}:[0,\infty)\to[0,\infty)$ with $\mathcal{F}(0)=\mathcal{G}(0)=0$, and let $\mathcal{T}:K\to K$ be a mapping such that

$$\mathcal{D}_c(\mathcal{T}\kappa,\mathcal{T}\iota)\le\lambda\mathcal{D}_c(\kappa,\iota),\ \forall\kappa,\iota\in K, \tag{1}$$

where $\lambda\in[0,1)$. Then $\mathcal{T}$ has a unique fixed point.

Proof. Since $\lambda\in[0,1)$, for given $0<\eta<1$, we can take $n\in\mathbb{N}$ such that

$$\lambda^n<min\left\{\mathcal{F}^{-1}\left(\frac{\eta}{4}\right),\mathcal{G}^{-1}\left(\frac{\eta}{4}\right)\right\}. \tag{2}$$

Let us take $\kappa_0\in K$ arbitrarily. We define $\Psi:=\mathcal{T}^n$ and construct a sequence $\{\kappa_i\}$ by $\kappa_i=\Psi^i\kappa_0$.

Firstly, we will see that $\{\kappa_i\}$ is a Cauchy sequence.

From (1), we obtain

$$\mathcal{D}_c(\Psi\kappa,\Psi\iota)=\mathcal{D}_c(\mathcal{T}^n\kappa,\mathcal{T}^n\iota)\le\lambda\mathcal{D}_c(\mathcal{T}^{n-1}\kappa,\mathcal{T}^{n-1}\iota)\cdots\le\lambda^n\mathcal{D}_c(\kappa,\iota) \tag{3}$$

By (3), we get for all $i\in\mathbb{N}$,

$$\mathcal{D}_c(\kappa_i,\kappa_{i+1})=\mathcal{D}_c(\Psi\kappa_{i-1},\Psi\kappa_i)\le\lambda^n\mathcal{D}_c(\kappa_{i-1},\kappa_i)\le\lambda^{ni}\mathcal{D}_c(\kappa_0,\kappa_1).$$

Thus,

$$lim_{i\to\infty}\mathcal{D}_c(\kappa_i,\kappa_{i+1})=0. \tag{4}$$

So we can take $h\in\mathbb{N}$ satisfying

$$\mathcal{D}_c(\kappa_h,\kappa_{h+1})<min\left\{\mathcal{F}^{-1}\left(\frac{\eta}{4}\right),\mathcal{G}^{-1}\left(\frac{\eta}{4}\right)\right\}. \tag{5}$$

Denote by $B_{dc}[\kappa_h,\eta/2]:=\{\iota\in K:\mathcal{D}_c(\kappa_h,\iota)\le\eta/2\}$. Since it is clear that $\kappa_h\in B_{dc}[\kappa_h,\eta/2]$, it holds that $B_{dc}[\kappa_h,\eta/2]\neq\emptyset$. Let us take an arbitrary $\nu\in B_{dc}[\kappa_h,\eta/2]$. Then, $\mathcal{D}_c(\kappa_h,\nu)\le\eta/2$.

By (2) and (3), we get

$$\mathcal{D}_c(\Psi\kappa_h,\Psi\nu)\le\lambda^n\mathcal{D}_c(\kappa_h,\nu)\le min\left\{\mathcal{F}^{-1}\left(\frac{\eta}{4}\right),\mathcal{G}^{-1}\left(\frac{\eta}{4}\right)\right\}.$$
$$\frac{\eta}{2}\le min\left\{\mathcal{F}^{-1}\left(\frac{\eta}{4}\right),\mathcal{G}^{-1}\left(\frac{\eta}{4}\right)\right\}. \tag{6}$$

By triangle inequality (iii), (5), (6) and the fact that $\mathcal{F}$ and $\mathcal{G}$ are strictly increasing, we get

$$\mathcal{D}_c(\kappa_h,\Psi\nu)\le\mathcal{F}\big(\mathcal{D}_c(\kappa_h,\kappa_{h+1})\big)+\mathcal{G}\big(\mathcal{D}_c(\kappa_{h+1},\Psi\nu)\big)$$
$$=\mathcal{F}\big(\mathcal{D}_c(\kappa_h,\kappa_{h+1})\big)+\mathcal{G}\big(\mathcal{D}_c(\Psi\kappa_h,\Psi\nu)\big)$$
$$\le$$
$$\mathcal{F}\left(min\left\{\mathcal{F}^{-1}\left(\frac{\eta}{4}\right),\mathcal{G}^{-1}\left(\frac{\eta}{4}\right)\right\}\right)+\mathcal{G}\left(min\left\{\mathcal{F}^{-1}\left(\frac{\eta}{4}\right),\mathcal{G}^{-1}\left(\frac{\eta}{4}\right)\right\}\right)$$
$$\le\mathcal{F}\left(\mathcal{F}^{-1}\left(\frac{\eta}{4}\right)\right)+\mathcal{G}\left(\mathcal{G}^{-1}\left(\frac{\eta}{4}\right)\right)$$
$$\le\frac{\eta}{2}.$$

Hence, $\Psi\nu\in B_{dc}[\kappa_h,\eta/2]$. This means that Ψ maps $B_{dc}[\kappa_h,\eta/2]$ into itself. Thus we have $\Psi\kappa_h\in B_{dc}[\kappa_h,\eta/2]$ for $\kappa_h\in B_{dc}[\kappa_h,\eta/2]$. Repeating this process, we get $\kappa_j\in B_{dc}[\kappa_h,\eta/2]$ for all $j>h$.

For all $j>i\ge h$ (where $i=h+l$ for some $l\ge 0$), by (3) and $\kappa_{j-l}\in B_{dc}[\kappa_h,\eta/2]$, we have

$$\mathcal{D}_c(\kappa_i,\kappa_j)=\mathcal{D}_c(\Psi\kappa_{i-1},\Psi\kappa_{j-1})\le\lambda^n\mathcal{D}_c(\kappa_{i-1},\kappa_{j-1})$$
$$\le\lambda\mathcal{D}_c(\kappa_{i-1},\kappa_{j-1})$$
$$\le\lambda^2\mathcal{D}_c(\kappa_{i-2},\kappa_{j-2})$$
$$\cdots$$
$$\le\lambda^l\cdot\mathcal{D}_c(\kappa_h,\kappa_{j-l})$$
$$\le\mathcal{D}_c(\kappa_h,\kappa_{j-l})$$
$$\le\frac{\eta}{2}<\eta.$$

Thus, we have $lim_{i,j\to\infty}\mathcal{D}_c(\kappa_i,\kappa_j)=0$ and $\{\kappa_i\}$ is Cauchy. By the completeness of K, there exists $\kappa^*\in K$ such that $lim_{i\to\infty}\mathcal{D}_c(\kappa_i,\kappa^*)=0$.

Secondly, we will prove that κ^* is a unique fixed point of Ψ. By triangle inequality (iii) and (3),

$$\mathcal{D}_c(\kappa^*,\Psi\kappa^*)\le\mathcal{F}\big(\mathcal{D}_c(\kappa^*,\kappa_{i+1})\big)+\mathcal{G}\big(\mathcal{D}_c(\kappa_{i+1},\Psi\kappa^*)\big)$$
$$=\mathcal{F}\big(\mathcal{D}_c(\kappa^*,\kappa_{i+1})\big)+\mathcal{G}\big(\mathcal{D}_c(\Psi\kappa_i,\Psi\kappa^*)\big)$$
$$\le\mathcal{F}\big(\mathcal{D}_c(\kappa^*,\kappa_{i+1})\big)+\mathcal{G}\big(\lambda^n\mathcal{D}_c(\kappa_i,\kappa^*)\big),$$

for all $i\in\mathbb{N}$.

Taking the limit as $\boldsymbol{i}\to\infty$, by the continuity of $\boldsymbol{\mathcal{F}}$ and $\boldsymbol{\mathcal{G}}$ we obtain that

$$\mathcal{D}_c(\kappa^*,\Psi\kappa^*)\le$$
$$\mathcal{F}\big(lim_{i\to\infty}\mathcal{D}_c(\kappa^*,\kappa_{i+1})\big)+\mathcal{G}\big(\lambda^n lim_{i\to\infty}\mathcal{D}_c(\kappa_i,\kappa^*)\big)=$$
$$\mathcal{F}(0)+\mathcal{G}(0)=0.$$

So κ^* is a fixed point of Ψ.

Now, let Ψ has two fixed points κ^* and ι^*, i.e., $\kappa^*=\Psi\kappa^*$ and $\iota^*=\Psi\iota^*$.

By (3)

$$\mathcal{D}_c(\kappa^*,\iota^*)=\mathcal{D}_c(\Psi\kappa^*,\Psi\iota^*)\le\lambda^n\mathcal{D}_c(\kappa^*,\iota^*).$$

That is $\mathcal{D}_c(\kappa^*,\iota^*)=0$, and the fixed point of Ψ is unique.

Finally, since $\kappa^*=\Psi\kappa^*=\mathcal{T}^n\kappa^*$, we get $\mathcal{T}\kappa^*=\mathcal{T}^{n+1}\kappa^*=\Psi(\mathcal{T}\kappa^*)$. That is, $\mathcal{T}\kappa^*$ is also a fixed point of Ψ. Since the fixed point of Ψ is unique, $\kappa^*=\mathcal{T}\kappa^*$. Consequently, κ^* is a fixed point of $\mathcal{T}$. The uniqueness can be easily showed.

Example 2.1. We discuss the double composed metric space $(K,\mathcal{D}_c)$ defined in Example 1.1. The completeness of K can be easily checked, and $\mathcal{F}$ and $\mathcal{G}$ are continuous and strictly increasing with $\mathcal{F}(0)=\mathcal{G}(0)=0$.

Take $\mathcal{T}:K\to K$ by $\mathcal{T}\kappa=\frac{\kappa}{3}$.

Since $e^{x/3}-1\le\frac{1}{3}(e^{x/3}-1)$, $\forall x\in\mathbb{R}$, we have

$$\mathcal{D}_c(\mathcal{T}\kappa,\mathcal{T}\iota)=e^{|\mathcal{T}\kappa-\mathcal{T}\iota|}-1=e^{\frac{|\kappa-\iota|}{3}}-1\le\frac{1}{3}\left(e^{|\kappa-\iota|}-1\right)\le$$
$$\lambda\mathcal{D}_c(\kappa,\iota),$$

for all $\lambda\in[\frac{1}{3},1)$. Thus all the conditions of Theorem 2.1 are satisfied and $\mathcal{T}$ has a unique fixed point $\kappa^*=0$. On the other hand, since $\mathcal{G}$ is not sub-additive, so we cannot apply Theorem 1.1 (Theorem 3 in [7]).

3. Application to Nonlinear Integral Equations

We apply Theorem 2.1 to discuss the existence of a solution of Fredholm integral equation as follows:

$$\kappa(t)=\int_0^1\mathcal{H}(t,s,\kappa(s))ds,\ \forall t,s\in[0,1], \tag{7}$$

where $\mathcal{H}:[0,1]^2\times\mathbb{R}\to\mathbb{R}$ is given continuous function.

Let $K=C([0,1],\mathbb{R})$. Define $\mathcal{D}_c:K\times K\to[0,\infty)$ by

$$\mathcal{D}_c(\kappa,\iota)=e^{max_{t\in[0,1]}(|\kappa(t)-\iota(t)|)}-1,\ \forall\kappa,\iota\in K.$$

From Example 1.1, we can easily check that $(K,\mathcal{D}_c)$ is a complete double-composed metric space via $\mathcal{F},\mathcal{G}:[0,\infty)\to[0,\infty)$ with $\mathcal{F}(x)=\mathcal{G}(x)=(x^2+2x)/2$ for all $x\ge 0$.

Theorem 3.1. Suppose that

$$e^{|\mathcal{H}(t,s,\kappa(s))-\mathcal{H}(t,s,\iota(s))|}-1 \le \lambda\left(e^{|\kappa(s)-\iota(s)|}-1\right) \quad (8)$$

for all $\kappa,\iota \in K$ and $t,s \in [0,1]$, where $\lambda \in [0,1)$. Then the integral equation (7) has a unique solution in K.

Proof. The inequality (8) is equivalent to the following inequality.

$$|\mathcal{H}(t,s,\kappa(s))-\mathcal{H}(t,s,\iota(s))| \le ln\left(\lambda\left(e^{|\kappa(s)-\iota(s)|}-1\right)+1\right) \quad (9)$$

for all $\kappa,\iota \in K$ and $t,s \in [0,1]$.

We define the integral operator $\mathcal{T}: K \to K$ as

$$\mathcal{T}\kappa(t) = \int_0^1 \mathcal{H}(t,s,\kappa(s))ds, \ \forall t,s \in [0,1].$$

By (9), we get

$$e^{|\mathcal{T}\kappa(t)-\mathcal{T}\iota(t)|}-1 = e^{\left|\int_0^1 \mathcal{H}(t,s,\kappa(s))ds-\int_0^1 \mathcal{H}(t,s,\iota(s))ds\right|}-1$$
$$\le e^{\int_0^1 |\mathcal{H}(t,s,\kappa(s))-\mathcal{H}(t,s,\iota(s))|ds}-1$$

$$\le e^{\int_0^1 ln(\lambda(e^{|\kappa(s)-\iota(s)|}-1)+1)ds}-1$$

$$\le e^{ln\left(\lambda\left(e^{max_{s\in[0,1]}|\kappa(s)-\iota(s)|}-1\right)+1\right)\int_0^1 ds}-1$$

$$= \lambda\left(e^{max_{s\in[0,1]}|\kappa(s)-\iota(s)|}-1\right)$$

$$\le \lambda\left(e^{max_{t\in[0,1]}|\kappa(t)-\iota(t)|}-1\right),$$

for all $t \in [0,1]$ and $\kappa,\iota \in K$.

So,

$$e^{max_{t\in[0,1]}|\mathcal{T}\kappa(t)-\mathcal{T}\iota(t)|}-1 \le \lambda\left(e^{max_{t\in[0,1]}|\kappa(t)-\iota(t)|}-1\right),$$

that is,

$$\mathcal{D}_c(\mathcal{T}\kappa,\mathcal{T}\iota) \le \lambda\mathcal{D}_c(\kappa,\iota).$$

By Theorem 2.1, $\mathcal{T}$ has a unique fixed point, which means that (7) has a unique solution in K.

4. Conclusions

In this paper, we newly discussed the Banach-type fixed point result in the framework of double-composed metric spaces. We have established the Banach-type fixed point result without the assumptions (1) and (2) of Theorem 1.1, instead replacing the assumption that $\mathcal{F}$ and $\mathcal{G}$ are non-decreasing with the assumption of strictly increasing. And we presented an example supporting our fixed point result. Finally, we show the existence of a solution of Fredholm integral equations by applying obtained fixed point result.

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Author Contributions

Gwang-Myong Kim: Writing - original draft, Methodology
Gum-Sik Kang: Writing - original draft, Conceptualization
Chol Jin Kil: Writing - review & editing, Supervision
Jin-Sim Kim: Formal Analysis, Data curation

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Conflicts of Interest

The authors declare no conflicts of interest.

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