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Research Article

Effect of Parabolically Varying Non-Homogeneity on Thermally Induced Vibration of Orthotropic Trapezoidal Plate with Thickness Varies Linearly in One Direction and Parabolically in Other Direction

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Abstract

The present paper deals with the effect of parabolically varying non-homogeneity on thermally induced vibration of orthotropic trapezoidal plate with thickness varies linearly in one direction and parabolically in other direction. The two term deflection function corresponding to clamped-simply supported clamped-simply supported (C-S-C-S) boundary condition is defined by the product of the equation of the prescribed continuous piecewise boundary shape. The non-homogeneity of the plate varies parabolically. Rayleigh-Ritz method is used to solve the governing differential equation for maximum strain energy and maximum kinetic energy for orthotropic trapezoidal plate. The effect of frequencies for first and second mode investegated with the variations in structural parameters such as taper constant, non-homogeneity constant, aspect ratio and thermal gradient respectively. Results are calculated with great accuracy and compare the present model with the other in literature with the help of tables and graphs. All the results presented here are new and are not found elsewhere.

Keywords

Thermal Gradient, Non-Homogeneous, Vibration, Density, Thickness

1. Introduction

Recently the plates of variable thickness attracts the mind of researcher. Due to the change in density and thermal gradient two modes of vibration occurs in orthotropic trapezoidal plate. In this research paper we are analyzing two modes of vibration and compare them graphically. Plates of variable thickness are used in nuclear reactor structures, naval structures and aeronautical fields, electromechanical transducers for electronic telephones, mirrors and lenses in optical systems. Leissa [1] purposes a new model to design the aircraft and provide the plateform to researcher towards vibration of plates. Leissa [2] studied the vibration of plates for complicating effects. Leissa [3] studied the vibration of plates for clasical theory. Sharma *et al*. [4] studied the vibration of orthotropic trapezoidal plate with thickness varies linearly in both directions. Sharma *et al*. [5] studied the vibration of orthotropic trapezoidal plate with thickness varies paraboli-

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cally in both directions. Gupta *et al.* [6] studied the vibration of non-homogeneous trapezoidal plate with thickness varies linearly in one and parabolically in other direction with linearly varying density. Sharma *et al*. [7] studied the mathematical modelling of orthotropic trapezoidal plate with linearly varying density. Khanna and Sharma [8] studied the visco-elastic square plate of variable thickness with thermal gradient. Sobotka [9] studied the free vibration of visco-elastic orthotropic rectangular plate. Sobotka [10] studied the rheology of visco-elastic orthotropic plates. Warade and Deshmukh [11] studied the thermal deflection of a thin clamped circular plate. Srinivas et al. [12] analyzed the vibration of simply supported homogeneous and laminated thick rectangular plate. Saliba [13] studied the transverse vibration of trapezoidal plate. Kavita et al. [14] studied the vibration of bi-parabolic trapezoidal plate with parabolically varying density. Rana and Robin [15] measured the damping effect of orthotropic rectangular plate with variable thickness. Sharma and Gupta [16] studied the free transverse vibration for thin orthotropic plate. Sharma *et al.* [17] studied the effect of linearly varying non-homogeneity for orthotropic trapezoidal plate with thickness varies linearly in one direction and parabolically in other.

The main aim of the research paper is to study the effect of parabolically varying non-homogeneity on thermally induced vibration of orthotropic trapezoidal plate with thickness varies linearly in one and parabolically in other direction. Using Rayleigh-Ritz method governing differential equation has been attained by taking two term deflection function corresponding to clamped-simply supported clamped-simply supported (C-S-C-S) boundary condition. The effects of structural parameters such as non-homogeneity constant, aspect ratio and thermal gradient have also been studied. Results are calculated with great accuracy and compare the present model with the other in literature with the help of tables and graphs.

2. Analysis

2.1. Model Description

A thin, symmetric, non-homogeneous orthotropic trapezoidal plate is considered as shown in Figure 1.

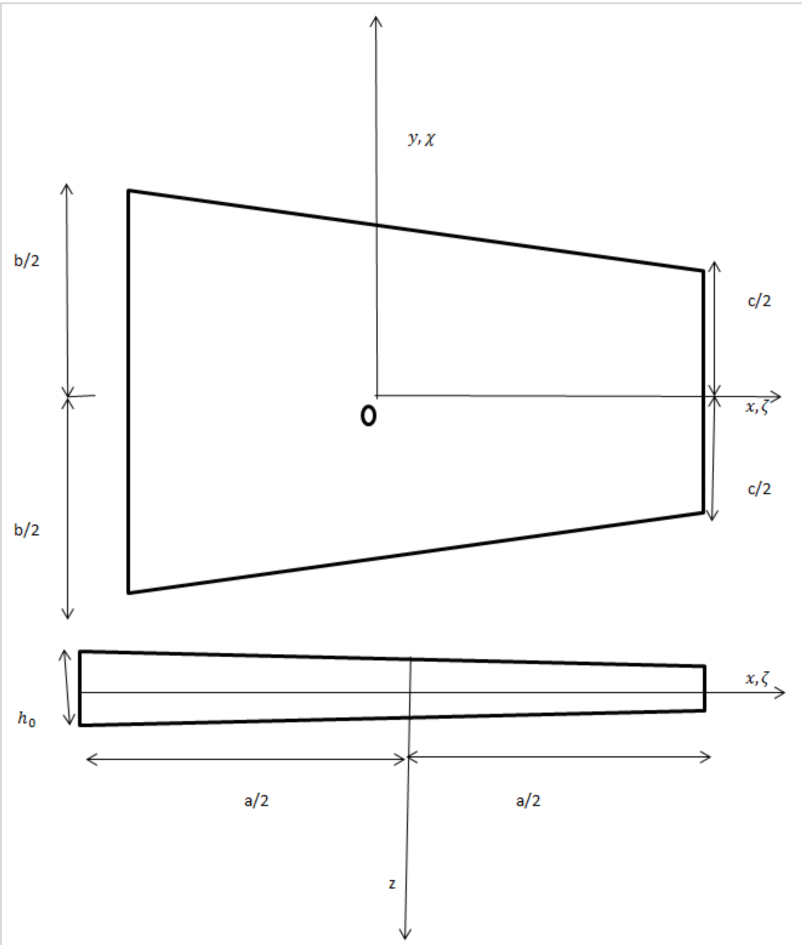

Figure 1. *Geometry of orthotropic trapezoidal plate.*

2.2. Thickness

For tapering in plate thickness has important role as compared to uniform thickness. The general equation of thickness with parabolic variation in both directions is

$$h(\zeta) = h_0\left[1-(1-\delta_1)\left(\zeta+\tfrac{1}{2}\right)\right]\left[1-(1-\delta_2)\left(\chi+\tfrac{1}{2}\right)^2\right] \quad (1)$$

where h_o is the maximum plate thickness occurring at the left edge, δh_0 is the minimum plate thickness occurring at the right edge and δ_1, δ_2 are the taper constants.

2.3. Density

For non-homogeneity of the plate density is assumed linear in x-direction. The general equation of density with linear variation in x-direction is

$$\rho = \rho_0[1-(1-ß)\left(\zeta+\tfrac{1}{2}\right)^2] \quad (2)$$

where ß is the non-homogeneity constant of the plate, ρ_0 is the mass density at $\zeta = -\frac{1}{2}$.

2.4. Temperature

The general equation of temperature with linear temperature distribution in x-direction is

$$\theta = \theta_0(\tfrac{1}{2}-\zeta) \quad (3)$$

where θ and θ_0 denote the temperature excess above the reference temperature on the plate at any point and at the origin, respectively.

For most orthotropic materials, modulus of elasticity is described as a function of temperature as

$$\check{E}_\zeta(\theta) = \check{E}_1(1-\gamma\theta),$$

$$\check{E}_\chi(\theta) = \check{E}_2(1-\gamma\theta),$$

$$\mathcal{G}_{\zeta\chi}(\theta) = \mathcal{G}_0(1-\gamma\theta), \quad (4)$$

where $\check{E}_\zeta$ and $\check{E}_\chi$ are Young's moduli in x-direction and y-direction, respectively, and $\mathcal{G}_{\zeta\chi}$ is the shear modulus, γ is slope of vibration of moduli with temperature, and $\check{E}_1, \check{E}_2$ and $\mathcal{G}_0$ are the values of moduli at some refrence temperature; that is, $\theta = 0$.

Using (3), (4) becomes

$$\check{E}_\zeta(\theta) = \check{E}_1\left(1-\delta\left(\tfrac{1}{2}-\zeta\right)\right),$$

$$\check{E}_\chi(\theta) = \check{E}_2\left(1-\delta\left(\tfrac{1}{2}-\zeta\right)\right),$$

$$\mathcal{G}_{\zeta\chi}(\theta) = \mathcal{G}_0\left(1-\delta\left(\tfrac{1}{2}-\zeta\right)\right). \quad (5)$$

where $\delta = \gamma\theta_0$ $(0 \le \delta < 1)$ known as thermal gradient.

2.5. Deflection Function and Corresponding Boundary Condition

Two term deflection function with boundary condition clamped simply supported- clamped simply supported (C-S-C-S) for vibrational analysis has been considered and can be written as

$$\psi = P_1\left\{\left(\zeta+\tfrac{1}{2}\right)\left(\zeta-\tfrac{1}{2}\right)\right\}^2\left\{\chi-\left(\tfrac{b-c}{2}\right)\zeta+\left(\tfrac{b+c}{4}\right)\right\}.\left\{\chi+\left(\tfrac{b-c}{2}\right)\zeta-\left(\tfrac{b+c}{4}\right)\right\}+P_2\left\{\left(\zeta+\tfrac{1}{2}\right)\left(\zeta-\tfrac{1}{2}\right)\right\}^3.\left\{\chi-\left(\tfrac{b-c}{2}\right)\zeta+\left(\tfrac{b+c}{4}\right)\right\}^2\left\{\chi+\left(\tfrac{b-c}{2}\right)\zeta-\left(\tfrac{b+c}{4}\right)\right\}^2, \quad (6)$$

P_1 and P_2 are two unknown constants to be determined.

Thus, for all the non-homogeneous trapezoidal plate equation (6) satisfied the following conditions clamped simply supported- clamped simply supported (C-S-C-S) for vibrational analysis such as

$$\chi = \frac{c}{4b}-\frac{\zeta}{2}+\frac{1}{4}+\frac{c\zeta}{2b},$$

$$\chi = -\frac{c}{4b}+\frac{\zeta}{2}-\frac{1}{4}-\frac{c\zeta}{2b},$$

$$\zeta = -\frac{1}{2},$$

$$\zeta = \frac{1}{2}, \quad (7)$$

introducing the following non-dimensional variables as $\zeta = \frac{x}{a}$ and $\chi = \frac{y}{b}$.

Expression for maximum strain energy and maximum kinetic energy for orthotropic trapezoidal plate are [3] as

$$P = \frac{ab}{2}\iint\left[Đ_\zeta\left(\frac{\partial^2\psi}{\partial\zeta^2}\right)^2 + Đ_\chi\left(\frac{\partial^2\psi}{\partial\chi^2}\right)^2 + 2Đ_1\left(\frac{\partial^2\psi}{\partial\zeta^2}\right)\left(\frac{\partial^2\psi}{\partial\chi^2}\right) + 4Đ_{\zeta\chi}\left(\frac{\partial^2\psi}{\partial\zeta\partial\chi}\right)^2\right]d\zeta d\chi, \quad (8)$$

and

$$K = \frac{ab}{2} \omega^2 h_0 \iint \rho h(\zeta)\psi^2 \, d\zeta d\chi, \quad (9)$$

where ω is the angular frequency of vibration.

Also flexural and torsion rigidity is given by [4] as

$$Đ_\zeta = \frac{Ĕ_\zeta h^3}{12(1-\nu_\chi \nu_\zeta)},$$

$$Đ_\chi = \frac{Ĕ_\chi h^3}{12(1-\nu_\chi \nu_\zeta)},$$

$$Đ_{\zeta\chi} = \frac{\mathcal{G}_{\zeta\chi} h^3}{12}. \quad (10)$$

Using (5), (10) becomes

$$Đ_\zeta = \frac{Ĕ_1 h^3\left(1-\delta\left(\frac{1}{2}-\zeta\right)\right)}{12(1-\nu_\chi \nu_\zeta)},$$

$$Đ_\chi = \frac{Ĕ_2 \; h^3\left(1-\delta\left(\frac{1}{2}-\zeta\right)\right)}{12(1-\nu_\chi \nu_\zeta)},$$

$$Đ_{\zeta\chi} = \frac{\mathcal{G}_0 h^3\left(1-\delta\left(\frac{1}{2}-\zeta\right)\right)}{12}. \quad (11)$$

Also

$$Đ_1 = \nu_\chi Đ_\zeta = \nu_\zeta \; Đ_\chi, \quad (12)$$

where h is the thickness of the plate.

After applying boundary conditions (7), (8) and (9) becomes

$$P = \frac{ab}{2} \int_{-\frac{1}{2}}^{\frac{1}{2}} \int_{-\frac{c}{4b}+\frac{\zeta}{2}-\frac{1}{4}-\frac{c\zeta}{2b}}^{\frac{c}{4b}-\frac{\zeta}{2}+\frac{1}{4}+\frac{c\zeta}{2b}} \left[Đ_\zeta \left(\frac{\partial^2\psi}{\partial\zeta^2}\right)^2 + Đ_\chi \left(\frac{\partial^2\psi}{\partial\chi^2}\right)^2 + 2Đ_1 \left(\frac{\partial^2\psi}{\partial\zeta^2}\right)\left(\frac{\partial^2\psi}{\partial\chi^2}\right) + 4Đ_{\zeta\chi}\left(\frac{\partial^2\psi}{\partial\zeta\partial\chi}\right)^2\right] \; d\zeta d\chi \quad (13)$$

$$K = \frac{ab}{2} \omega^2 h_0 \int_{-\frac{1}{2}}^{\frac{1}{2}} \int_{-\frac{c}{4b}+\frac{\zeta}{2}-\frac{1}{4}-\frac{c\zeta}{2b}}^{\frac{c}{4b}-\frac{\zeta}{2}+\frac{1}{4}+\frac{c\zeta}{2b}} [\rho h(\zeta)\psi^2] \; d\zeta d\chi \quad (14)$$

2.6. Rayleigh Ritz Technique

To obtain equation of frequency and vibrational frequency Rayleigh Ritz technique is used according to which $\delta(P-K) = 0$ implies

$$\delta(P_1 - \mu^2 K_1) = 0$$

$$P_1 = \int_{-\frac{1}{2}}^{\frac{1}{2}} \int_{-\frac{c}{4b}+\frac{\zeta}{2}-\frac{1}{4}-\frac{c\zeta}{2b}}^{\frac{c}{4b}-\frac{\zeta}{2}+\frac{1}{4}+\frac{c\zeta}{2b}} \left[1-(1-\delta_1)\left(\zeta+\frac{1}{2}\right)^2\right]\left[1-(1-\delta_2)\left(\chi+\frac{1}{2}\right)^2\right]^3 \left[\left(\frac{\partial^2\psi}{\partial\zeta^2}\right)^2 + \frac{E_2}{E_1}\left(\frac{\partial^2\psi}{\partial\chi^2}\right)^2 + 2\nu_\zeta\left(\frac{\partial^2\psi}{\partial\zeta^2}\right)\left(\frac{\partial^2\psi}{\partial\chi^2}\right) + 4\frac{\mathcal{G}_0(1-\nu_\chi\nu_\zeta)}{E_1}\left(\frac{\partial^2\psi}{\partial\zeta\partial\chi}\right)^2\right] d\zeta d\chi$$

$$K_1 = \int_{-\frac{1}{2}}^{\frac{1}{2}} \int_{-\frac{c}{4b}+\frac{\zeta}{2}-\frac{1}{4}-\frac{c\zeta}{2b}}^{\frac{c}{4b}-\frac{\zeta}{2}+\frac{1}{4}+\frac{c\zeta}{2b}} \left[\rho\left[1-(1-\delta_1)\left(\zeta+\frac{1}{2}\right)^2\right]\left[1-(1-\delta_2)\left(\chi+\frac{1}{2}\right)^2\right]\psi^2\right] \; d\zeta d\chi \quad (15)$$

Where

$$\mu^2 = \frac{12 \;\; \rho_0 \omega^2 a^5 (1-\nu_\chi\nu_\zeta)}{Ĕ_1 {h_0}^2} \quad (16)$$

Equation (15) involves two constants C_1 and C_2 to be evaluated as follows

$$\frac{\partial(\Omega_1 - \mu^2 \Theta_1)}{\partial C_m} = 0\text{: m=1,2} \quad (17)$$

On simplifying (16) We get

$$\Theta_{m1} C_1 + \Theta_{m2} C_2 = 0\text{: m=1,2} \quad (18)$$

In this way the frequency equation can be obtained as

$$\begin{vmatrix} \Theta_{11} & \Theta_{12} \\ \Theta_{21} & \Theta_{22} \end{vmatrix} = 0 \quad (19)$$

and results validated with preexisting literature.

3. Results and Discussion

With the help of Mathematica Software two values of frequency parameter are calculated for different values of thermal gradient, aspect ratio and non-homogeneity constant.

Table 1. *In the following table first and second mode values of frequency parameter μ for a orthotropic trapezoidal plate for different values of thermal gradient δ are calculated by fixing other parameters such as taper constant $\delta_1 = \delta_2$=0.0, 0.2, 0.6, non-homogeneity constant ß=0.4, 1.0 and aspect ratio $\frac{a}{b} = 1.0, \frac{c}{b} = 0.5$ respectively.*

	ß=0.4				ß=1.0			
δ	$\delta_1 = \delta_2 = 0.0$		$\delta_1 = \delta_2 = 0.6$		$\delta_1 = \delta_2 = 0.0$		$\delta_1 = 0.2,\ \delta_2 = 0.6$	
	First Mode	Second Mode	First Mode	Second Mode	First Mode	Second Mode	First Mode	Second Mode
0.0	30.0362	121.386	29.5453	148.464	30.9674	125.188	27.7872	122.075
0.2	27.1463	111.905	27.0057	138.276	27.9885	115.408	25.1968	113.000
0.4	23.9051	101.543	24.1994	127.276	24.6475	104.718	22.3043	103.131
0.6	20.1395	89.9979	21.0188	115.23	20.6475	92.808	18.969	92.2123
0.8	15.4576	76.7407	17.2551	101.77	15.9397	79.1304	14.8882	79.8172
1.0	8.37256	60.6697	12.377	86.2374	8.63517	62.5482	9.06927	65.1131

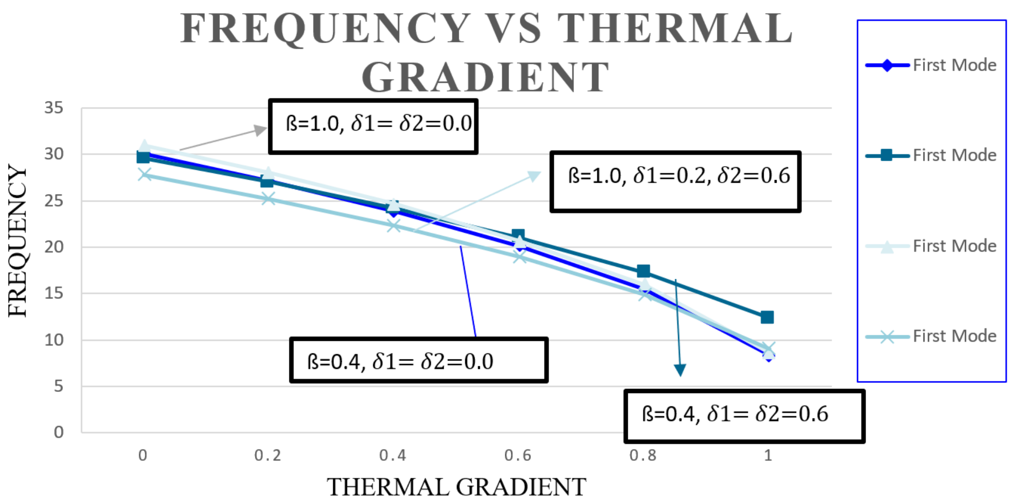

Figure 2. *Represents the first mode of frequency parameter μ for a orthotropic trapezoidal plate for different values of thermal gradient δ and other parameters such as taper constant $\delta_1 = \delta_2$=0.0, 0.6, non-homogeneity constant ß=0.4, 1.0 and aspect ratio $\frac{a}{b} = 1.0, \frac{c}{b} = 0.5$.*

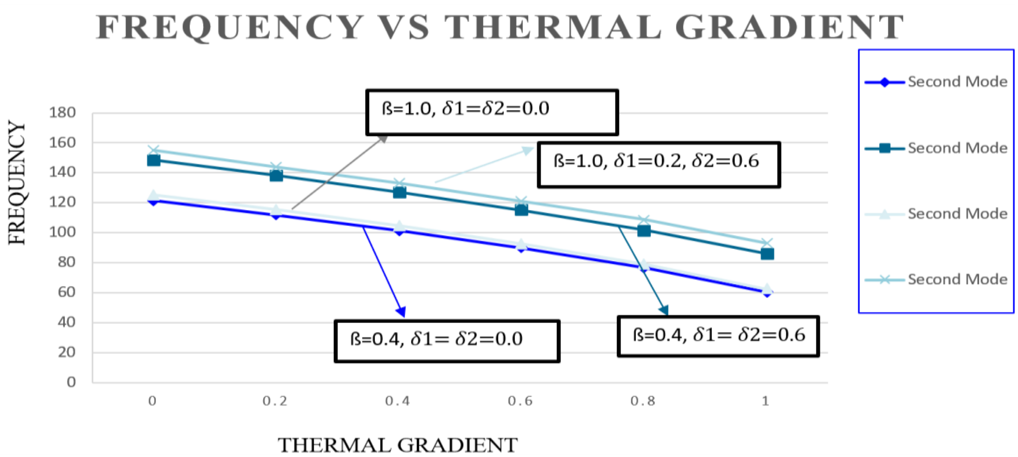

Figure 3. *Represents the second mode of frequency parameter μ for orthotropic trapezoidal plate for different values of thermal gradient δ and other parameters such as taper constant $\delta_1 = \delta_2$=0.0, 0.6, non-homogeneity constant ß=0.4, 1.0 and aspect ratio $\frac{a}{b} = 1.0, \frac{c}{b} = 0.5$.*

Table 2. *In the following table two different values of frequency parameter* μ *for a orthotropic trapezoidal plate for different values of aspect ratio* $\frac{c}{b}$ *are calculated by fixing other parameters such as aspect ratio* $\frac{a}{b} = 0.75$, *ß=0.4,* $\delta_1 = \delta_2 =$ *0.0,* 0.6 *respectively.*

	ß=0.4							
c/b	**δ1=δ2=0.0 δ=0.0**		**δ1=δ2=0.0 δ=0.4**		**δ1=δ2=0.6 δ=0.0**		**δ1=δ2=0.6, δ=0.4**	
	First Mode	**Second Mode**	**First Mode**	**Second Mode**	**First Mode**	**Second Mode**	**First Mode**	**Second Mode**
0.25	36.5464	127.094	28.8371	103.764	34.5674	138.037	27.6578	114.662
0.50	28.7149	94.5373	22.7947	77.7065	27.2593	104.782	20.0997	88.253
0.75	22.926	70.8515	18.3635	58.6103	22.4037	80.7134	18.5682	69.0722
1.0	18.8612	54.5294	15.2816	45.3861	20.9471	61.8397	16.6441	56.4667

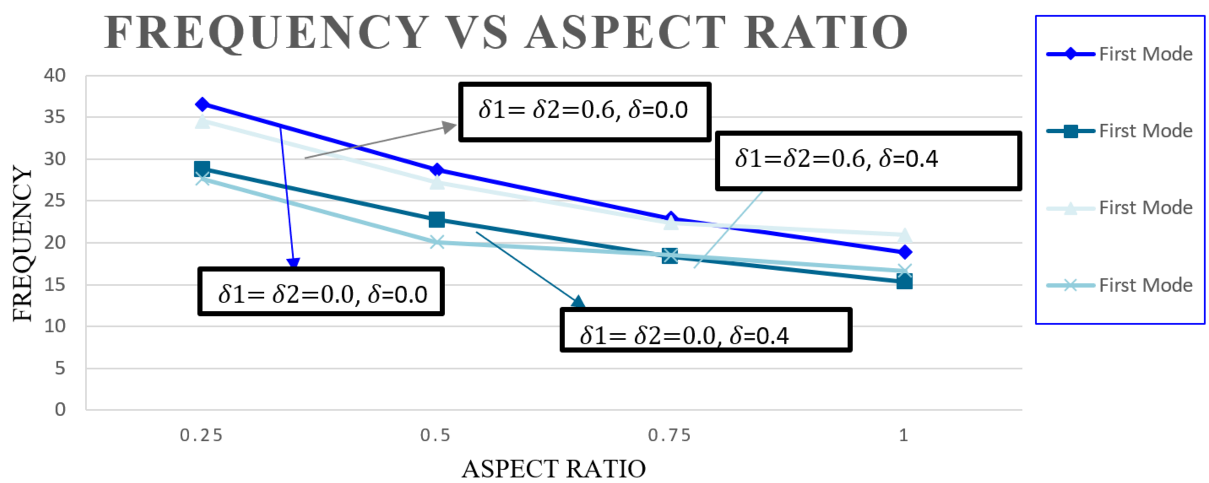

Figure 4. *Represents the first mode values of frequency parameter* μ *for a orthotropic trapezoidal plate for different values of thermal gradient* δ *and constant aspect ratio* $\frac{a}{b} = 0.75$, *non-homogeneity constant ß=0.0, Thermal gradient* $\delta = 0.0, 0.4$ *and Taper Constant* $\delta_1 = \delta_2 = 0.0, 0.6$.

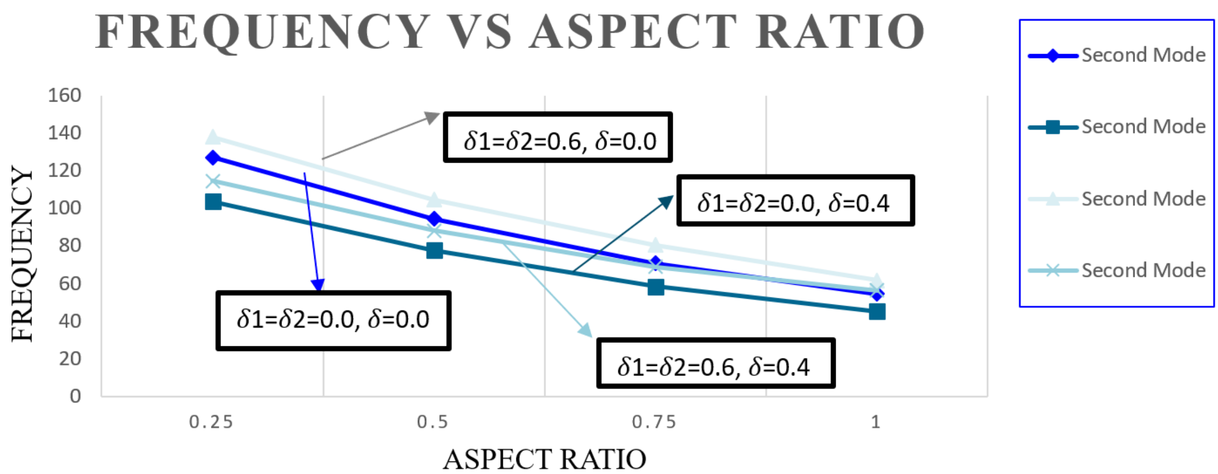

Figure 5. *Represents the second mode values of frequency parameter* μ *for a orthotropic trapezoidal plate for different of aspect ratio* $\frac{c}{b}$ *by fixing other parameters such as aspect ratio* $\frac{a}{b} = 0.75$, *ß=0.4,* $\delta_1 = \delta_2 =$ *0.0,* 0.6 *respectively.*

Table 3. *In the following table two different values of frequency parameter* μ *for a orthotropic trapezoidal plate for different values of aspect ratio* $\frac{c}{b}$ *are calculated by fixing other parameters such as aspect ratio* $\frac{a}{b} = 0.75$*, ß=0.4,* $\delta_1 = \delta_2 = 0.0, \ 0.6$ *respectively.*

	ß=0.4							
c/b	**δ1=δ2=0.0 δ=0.0**		**δ1=δ2=0.0 δ=0.4**		**δ1=δ2=0.6 δ=0.0**		**δ1=δ2=0.6, δ=0.4**	
	First Mode	**Second Mode**	**First Mode**	**Second Mode**	**First Mode**	**Second Mode**	**First Mode**	**Second Mode**
0.25	36.5611	186.296	30.0494	131.123	37.4023	187.871	30.3055	158.844
0.50	29.2556	147.332	23.9051	101.543	29.5453	148.464	24.1994	127.276
0.75	24.0939	115.569	19.4087	78.1832	24.2406	116.239	20.2154	100.966
1.0	19.492	95.6747	16.2403	60.852	20.9471	91.8397	17.9048	80.8386

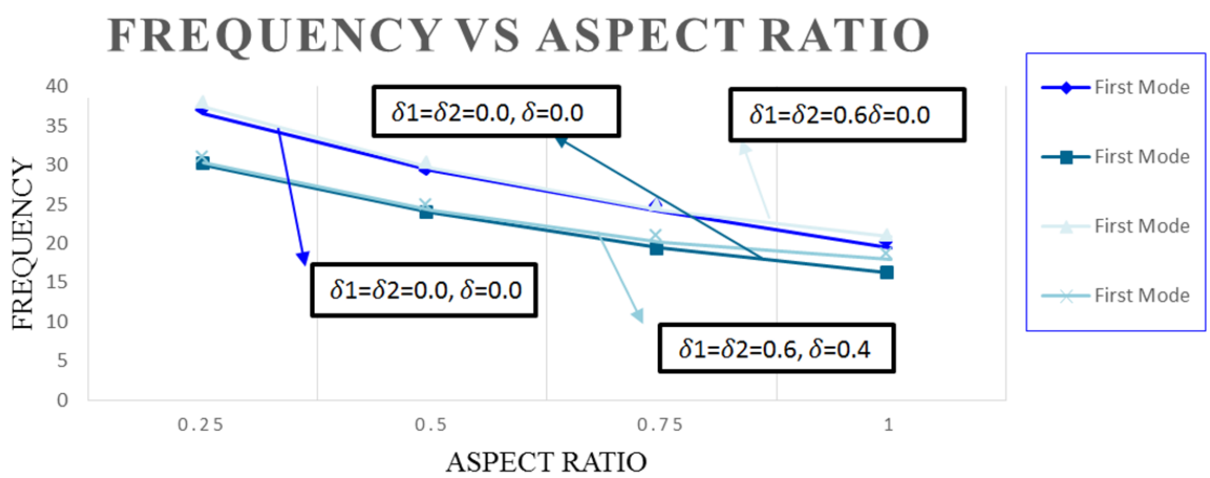

Figure 6. *Represents the first mode values of frequency parameter* μ *for a orthotropic trapezoidal plate for different values of thermal gradient* δ *and constant aspect ratio* $\frac{a}{b} = 0.75$*, non-homogeneity constant ß=0.0, Thermal gradient* $\delta = 0.0, 0.4$ *and Taper Constant* $\delta_1 = \delta_2 = 0.0, 0.6$.

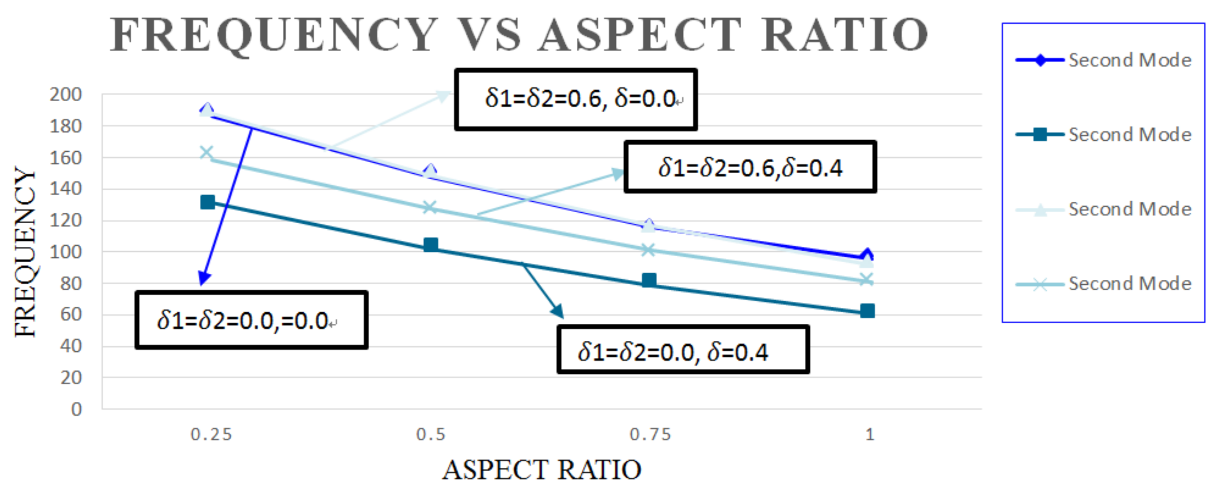

Figure 7. *Represents the second mode values of frequency parameter* μ *for a orthotropic trapezoidal plate for different of aspect ratio* $\frac{c}{b}$ *by fixing other parameters such as aspect ratio* $\frac{a}{b} = 0.75$*, ß=0.4,* $\delta_1 = \delta_2 = 0.0, \ 0.6$ *respectively.*

Table 4. *In the following table two different values of frequency parameter μ for a orthotropic trapezoidal plate for different values of non-homogeneity constant ß are calculated by fixing other parameters such as taper constant $\delta_1 = \delta_2$=0.0, 0.2, 0.6, Thermal gradient $\delta = 0.0, 0.4$ and aspect ratio $\frac{a}{b} = 1.0, \frac{c}{b} = 0.5$ respectively.*

	δ1=δ2=0.0				δ1=0.2, δ2=0.6			
ß	δ=0.0		δ=0.4		δ=0.0		δ=0.4	
	First Mode	Second Mode	First Mode	Second Mode	First Mode	Second Mode	First Mode	Second Mode
0.0	31.2976	126.54	24.9107	105.847	29.8477	131.204	23.9541	110.808
0.2	30.6475	123.88	24.3924	103.626	29.19	128.225	23.4253	108.297
0.4	30.0362	121.386	23.9051	101.543	28.5738	125.448	22.9299	105.956
0.6	29.4601	119.04	23.4458	99.5832	27.9949	122.851	22.4646	103.765
0.8	28.9159	116.827	23.012	97.7352	27.4498	120.413	22.0265	101.71
1.0	28.4007	114.736	22.6014	95.9885	26.9353	118.119	21.6131	99.7749

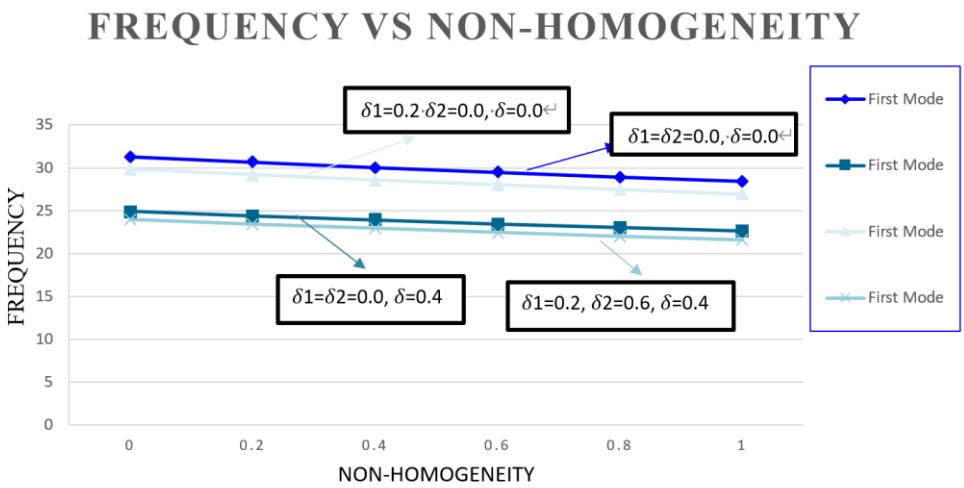

Figure 8. *Represents the first mode values of frequency parameter μ for a orthotropic trapezoidal plate for different values of non-homogeneity constant ß and aspect ratios $\frac{a}{b} = 1.0$, $\frac{c}{b} = 0.5$, $\delta = 0.0, 0.4$ $\delta_1 = \delta_2$=0.0 and $\delta_1 = 0.2, \delta_2$=0.6 respectively.*

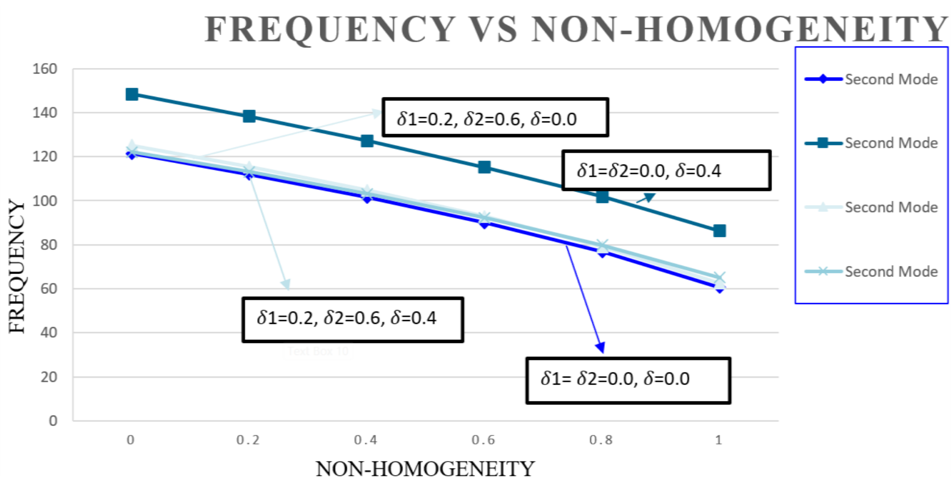

Figure 9. *Represents the second mode values of frequency parameter μ for a orthotropic trapezoidal plate for different values of non-homogeneity constant ß and aspect ratios $\frac{a}{b} = 1.0$, $\frac{c}{b} = 0.5$, $\delta = 0.0$ and $0.4, \delta_1 = \delta_2$=0.0 and $\delta_1 = 0.2, \delta_2$=0.6 respectively.*

4. Conclusion

The main aim of the research paper is to study the effect of parabolically varying non-homogeneity on thermally induced vibration of orthotropic trapezoidal plate with thickness varies linearly in one and parabolically in other direction. Using Rayleigh-Ritz method governing differential equation has been attained by taking two term deflection function corresponding to clamped-simply supported clamped-simply supported (C-S-C-S) boundary condition. The effects of structural parameters such as non-homogeneity constant, aspect ratio and thermal gradient have also been studied. Results are calculated with great accuracy and compare the present model with the other in literature with the help of tables and graphs. It is concluded by Table 1 and Figures 2 & 3 that frequency decreases in both the modes by increasing thermal gradient. Also it is concluded by Table 2 and Figures 4 & 5, Table 3 and Figures 6 & 7 frequency decreases in both the cases by taking aspect ratio $\frac{a}{b} = 0.75, 1.0$ and $\frac{c}{b}$ increasing from 0.25 to 1.0. From Table 4 and Figures 8 & 9 it is concluded that frequency decreases in both the modes as non-homogeneity increases. The results for orthotropic trapezoidal plate of parabollically varying density and thickness linear in one and parabolic in other direction are verified by the literature [6, 16].

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Author Contributions

Amit Sharma: Writing – original draft
Pragati Sharma: Supervision
Geeta: Supervision

Conflicts of Interest

The authors declare no conflicts of interest.

Appendix

ρ	Density
$\frac{a}{b}, \frac{c}{b}$	Aspect Ratio
δ	Thermal Gradient
δ_1, δ_2	Taper Constant
ß	Non-Homogeneity
μ	Frequency Parameter
$\zeta = \frac{x}{a}$ and $\chi = \frac{y}{b}$	Non-Dimensional Variables
$\check{E}_\zeta$ and $\check{E}_\chi$	Young's Moduli
$h(\zeta)$	Thickness
θ and θ_0	Temperature excess above the reference temperature and origin
γ	Slope of vibration of moduli with temperature
ω	Angular Frequency
δh_0	Minimum plate thickness
P	Maximum Strain Energy
K	Maximum Kinetic Energy
$Đ_\zeta$	Flexural Rigidity
$Đ_\chi$	Torsion Rigidity

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