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# Robust $H_\infty$ Control Approach for Neutral Time-delay Systems Under Uncertainty Consideration: A Case Study on Permanent Magnet Synchronous Generators

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## Abstract

Time-delay in many systems and applications is often a source of instability and reduced system performance where among them Neutral time delay systems are a class of functional differential equations where the derivatives of the state not only depend on the current and past states but also explicitly on delay derivatives. These systems arise naturally in various engineering, biological and economic models where delays affect both the state and its rate of change. Also, the presence of uncertainties such as parameter variations, unmodeled dynamics and external disturbances further complicates the analysis and control of such systems. so, this paper deals with the robust $H_\infty$ state-feedback control design for a class of uncertain systems subject to neutral Time-delay varying with multiple state and state derivatives delays is investigated in this paper. The parametric uncertainties are time varying and unknown but norm bounded. The main motivation of the paper is to develop a robust $H_\infty$ controller, which ensures robust asymptotic stability of the system as well as a desired $H_\infty$ performance and satisfy a prescribed $\gamma$ performance level for all admissible uncertainties. By constructing a Lyapunov-Krasovskii functional, some sufficient conditions for the existence of the $H_\infty$ state-feedback controller is derived in terms of linear matrix inequalities (LMI). Finally Numerical examples are provided to illustrate the effectiveness of the proposed method.

## Keywords

Robust $H_\infty$, Control, Neutral Time-delay Systems, Lyapunov-Krasovskii Function, Stability Analysis, LMI

## 1. Introduction

$H_\infty$ control theory is one of the fundamental aspects in control theories, and in recent decades, it has emerged as a prominent and efficient research area [1]. Modern Robust control has been developed in two areas which one area is Robustness through quadratic optimization and the other Robustness under parametric uncertainty [2]. As is well-known, from a practical point of view, most process models suffer from unpredictable behaviors. Thus, both of system uncertainties such as parametric [3] or unstructured uncertainties [4] should always be taken into account when a control system is designed for both performance and stability.

Time-delays are commonly encountered in numerous industrial systems that need to be controlled, such as distributed networks system [5], telecommunication [6], robotics [7], Wind Turbine [8], etc. Neglecting the impact of time delays can significantly degrade system performance and may even lead to instability. Thus, Time-delay controllers have practical

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significance [9, 10]. Stability assessment methods for Time-Delay Systems (TDS) are broadly divided into two categories: delay independent type [11] and delay-dependent [12], where the former category is delay-independent and often yields conservative results, particularly when the delay is small. In contrast, delay-dependent criteria have primarily been investigated under the assumption of identical delays in both neutral and discrete-time systems [13]. In general, there are two types of time-delay systems, i.e., retarded type and neutral type. The retarded type contains delays only in its states, while the neutral type contains delays in both its states and its derivatives of the states [14]. likewise, Stability of these systems proves to be a more complex issue because the system involves the derivative of the delayed state and the main feature of neutral-type time-delay systems is that they incorporate retarded derivatives of state variables, which makes the analysis and design of this class of systems very challenging in many engineering systems including power systems, robotic manipulators and etc. [15].

In recent literature on neutral time-delay systems with parametric uncertainties, the problem of robust $H_\infty$ asymptotic stabilization has been investigated for a class of uncertain neutral-type systems with time-varying delays. A sufficient condition has been derived to ensure that the considered system achieves robust asymptotic stability with a prescribed disturbance attenuation level γ [16]. Furthermore, [17] investigates the stability analysis of neutral systems subjected to mixed time-varying delays and nonlinear disturbances.

Lyapunov-based methods are extensively utilized in the analysis and synthesis of control strategies for neutral delay systems, owing to their effectiveness and general applicability, and also, the linear matrix inequality (LMI) provides an efficient and powerful tool for solving problems such as stability analysis, stabilization $H_\infty$ control problems [18].

Motivated by the above discussions, this paper investigates the problem of delay-dependent robust $H_\infty$ control for a class of uncertain systems with neutral time-delay which the nonlinear uncertainties are supposed to be time-varying and norm bounded. A sufficient condition for the $H_\infty$ control problem is proposed in terms of the LMI approach by using the Lyapunov functional in order to get better control performance. At last, to validate the proposed results, numerical simulations are provided, including a case study on a wind energy conversion system modeled using a permanent magnet synchronous generator (PMSG).

Notation. The notation in this paper is quite standard. The superscript '$T$' stands for the transpose of a matrix; $R^n$ and $R^{n*n}$denote an $n$-dimensional Euclidean space and the set of all $n \times n$ real matrices, respectively; $I$ is the identity matrix of appropriate dimension; is the Euclidean vector norm, and the symmetric terms in a symmetric matrix are denoted by $*$.

## 2. Problem Formulation and Preliminaries

Consider the following systems with neutral time-delay and uncertainty parameters as follows:

$$\begin{cases} \dot{x}(t) = Ax(t) + Bx\big(t-\tau(t)\big) + C\dot{x}\big(t-d(t)\big) + Hh(x(t),t) \\ \quad +Gg(x(t-\tau(t)),t) + Nn(x(t-d(t)),t) + E_w w(t) \\ \quad x(t) = \phi(t) \quad t \in [0, \max\{\mathrm{d}, \tau\}] \\ \quad Z(t) = C_1 x(t) \end{cases} \tag{1}$$

Where $x(t) = [x_1(t), \dots, x_n(t)]^T \in R^n$ and $A, B, C, H, G, N, E_w$ are constant matrices with appropriate dimensions; and $\tau(t), d(t)$ are the time-varying continuous function; the uncertainties $h(x(t),t)$, $g(x(t-\tau(t)),t), n(x(t-d(t)),t)$ represent nonlinear perturbations. $\phi(t)$ is a real-valued continuous initial function and $w(t)$ is the disturbance of finite energy and belong to $L_2[0,\infty)$.

Definition 2.1: The uncertain TDS of the form (1) is said to be robustly asymptotically stable in Lyapunov sense with an $H_\infty$ disturbance attenuation $\gamma > 0$ if system (1) with $w(t)$ =0 is robustly stable and moreover, under zero initial condition, there is

$$\int_0^\infty z^T(t)z(t)dt \le \gamma^2 \int_0^\infty w^T(t)w(t)dt \tag{2}$$

Definition 2.2: The delay $\tau(t)$ and neutral delay $d(t)$ are time-varying continuous function that satisfies:

$$0 \le \tau(t) \le \tau, \dot{\tau}(t) \le \mu_1,$$

$$0 \le d(t) \le d, \dot{d}(t) \le \mu_2.$$

where $\mu_1$, $\mu_2$, $\tau$ and $d$ are given real constants.

Assumption 2.1: we assume that the nonlinear uncertainties of the system $h(x(t),t), g(x(t-\tau),t), n(x(t-d),t)$ are bounded, i.e.

$$\|h(x(t),t)\| \le \alpha\|x(t)\| \tag{3}$$

$$\|g(x(t-\tau(t)),t)\| \le \beta\|x(t-\tau(t)\| \tag{4}$$

$$\|n(x(t-d(t)),t)\| \le \sigma\|x(t-d(t))\| \tag{5}$$

Where $\alpha, \beta, \sigma$ are positive scalers.

Lemma 2.1: (Schur complement). Let $M, P$, $Q$ be given matrices such that $Q > 0$, then

$$\begin{bmatrix} P & M^T \\ M & -Q \end{bmatrix} < 0 \iff P + M^T Q^{-1} M < 0 \tag{6}$$

Lemma 2.2: Let $D$, $E$ be real matrices of appropriate dimensions, and $F(t)$ satisfying $F^T(t)F(t) \leq I$. Then, the following statement holds for any scaler and vector:

$$DF(t)E + E^T F^T(t) D^T \leq \varepsilon DD^T + \varepsilon^{-1} E^T E \tag{7}$$

## 3. Main Result

The focus of the problem in this section will be on examining asymptotic stability and $H_\infty$ performance analysis.

A. $H_\infty$ Performance Analysis

Theorem 1. Under Definitions 2.1, uncertain time-delay system (1) is asymptotically stable and fulfills $H_\infty$ performance condition (2) if there exist positive-definite matrices $P, Q_1, Q_2, Q_3, R_1, R_2$ and scalers $\mu_1, \mu_2, d, \tau > 0$ satisfying the following LMI:

$$\Sigma = \begin{bmatrix} \Theta & P^T\bar{B} + P^T\bar{G}\beta^2 & P^T\bar{N}\sigma^2 & P^T\bar{C} & P^T\bar{E}w & P^T\bar{H} \\ * & -(1-\mu_1)R_1 & 0 & 0 & 0 & 0 \\ * & 0 & -(1-\mu_2)Q_1 & 0 & 0 & 0 \\ * & 0 & 0 & -(1-\mu_2)Q_2 & 0 & 0 \\ * & 0 & 0 & 0 & -\gamma^2 I & 0 \\ * & 0 & 0 & 0 & 0 & -I \end{bmatrix} < 0 \tag{8}$$

Where

$$\Theta = P^T\bar{A} + \bar{A}^T P + \begin{bmatrix} Q_1 + R_1 + C_1^T C_1 & 0 \\ 0 & Q_2 + \tau R_2 + dQ_3 \end{bmatrix}.$$

Proof. Represent (1) in the equivalent descriptor form:

$$\dot{x}(t) = \eta(t)$$

$$0 = -\eta(t) + Ax(t) + Bx(t-\tau(t)) + C\eta(t-d(t)) + Hh(x(t),t) + Gg(x(t-\tau(t)),t) + Nn(x(t-d(t)),t) + E_w w(t) \tag{9}$$

Consider the following Lyapunov- Krasovskii functional candidate of the form,

$$V(t) = \sum_{i=1}^{4} V_i(t) \tag{10}$$

where

$$V_1(t) = x^T(t)P_1x(t) = \zeta^T(t)TP\zeta(t)$$

with

$$T = \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix}, P = \begin{bmatrix} P_1 & 0 \\ P_2 & P_3 \end{bmatrix}, \zeta(t) = \begin{bmatrix} x(t) \\ \eta(t) \end{bmatrix}, P_1 = P^T > 0$$

And

$$V_2(t) = \int_{t-d(t)}^{t} x^T(s)\, Q_1 x(s) ds + \int_{t-d(t)}^{t} \eta^T(s) Q_2 \eta(s) ds + \int_{t-\tau(t)}^{t} x^T(s) R_1 x(s) ds$$

$$V_3(t) = \int_{-\tau}^{0} \int_{t+\theta}^{t} \eta^T(s) R_2\, \eta(s) ds d\theta$$

$$V_4(t) = \int_{-d}^{0} \int_{t+\theta}^{t} \eta^T(s) Q_3\, \eta(s) ds d\theta$$

The time derivation of $V(t)$ along the trajectory of the system (1) is given by

$$\dot{V}_1(t) = \dot{\zeta}^T(t)TP\zeta(t) + \zeta^T(t)TP\dot{\zeta}(t) = 2\zeta^T(t)TP\dot{\zeta}(t) = 2\zeta^T(t)P^T \begin{bmatrix} \dot{x}(t) \\ 0 \end{bmatrix}$$

That by substituting (1) in $\dot{V}_1(t)$ we have:

$$\dot{V}_1(t) = 2\zeta^T(t)P^T\begin{bmatrix}\eta\\0\end{bmatrix}$$

$$= 2\zeta^T(t)P^T\{\bar{A}\zeta(t) + \bar{B}x(t-\tau(t)) + \bar{C}\eta(t-d(t)) + \bar{H}h(x(t),t) + \bar{G}g(x(t-\tau(t)),t) + \bar{N}n(x(t-d(t)),t) + \bar{E}_w w(t)\}$$

Where

$$\bar{A} = \begin{bmatrix}0 & I\\A & -I\end{bmatrix}, \bar{B} = \begin{bmatrix}0\\B\end{bmatrix}, \bar{C} = \begin{bmatrix}0\\C\end{bmatrix}, \bar{H} = \begin{bmatrix}0\\H\end{bmatrix}, \bar{G} = \begin{bmatrix}0\\G\end{bmatrix}, \bar{N} = \begin{bmatrix}0\\N\end{bmatrix}, \bar{E} = \begin{bmatrix}0\\E\end{bmatrix}$$

And

$$\dot{V}_2(t) = x^T(t)Q_1x(t) + \eta^T(t)Q_2\eta(t) - (1-\dot{d}(t))x^T(t-d(t))Q_1x(t-d(t)) - (1-\dot{d}(t))\eta^T(t-d(t))Q_2\eta(t-d(t)) + x^T(t)R_1x(t) - (1-\dot{\tau}(t))x^T(t-\tau(t))R_1x(t-\tau(t))$$

$$\dot{V}_2(t) \le x^T(t)(Q_1+R_1)x(t) + \eta^T(t)Q_2\eta(t) - \left(1-\dot{d}(t)\right)x^T\left(t-d(t)\right)Q_1x\left(t-d(t)\right) - \left(1-\dot{d}(t)\right)\eta^T\left(t-d(t)\right)Q_2\eta\left(t-d(t)\right) - (1-\dot{\tau}(t))x^T(t-\tau(t))R_1x(t-\tau(t))$$

Where by applying Definition 2.2 we can rewrite $\dot{V}_2(t)$ as follows:

$$\dot{V}_2(t) \le x^T(t)(Q_1+R_1)x(t) + \eta^T(t)Q_2\eta(t) - (1-\mu_2)x^T\left(t-d(t)\right)Q_1x\left(t-d(t)\right) - (1-\mu_2)\eta^T\left(t-d(t)\right)Q_2\eta\left(t-d(t)\right) - (1-\mu_1)x^T\left(t-\tau(t)\right)R_1x\left(t-\tau(t)\right)$$

And also,

$$\dot{V}_3(t) = \tau\ \eta^T(t)R_2\eta(t) - \int_{t-\tau}^{t}\eta^T(s)R_2\eta(s)ds$$

$$\le \tau\ \eta^T(t)R_2\eta(t)$$

And

$$\dot{V}_4(t) = d\eta^T(t)Q_3\eta(t) - \int_{t-d}^{t}\eta^T(s)Q_3\eta(s)ds$$

$$\le d\eta^T(t)Q_3\eta(t)$$

Therefore,

$$\dot{V}(t) = \sum_{i=1}^{4}\dot{V}_i(t)$$

$$\le 2\zeta^T(t)P^T\{\bar{A}\zeta(t) + \bar{B}x\left(t-\tau(t)\right) + \bar{C}\eta(t-d(t0) + \bar{H}h(x(t),t) + \bar{G}g\left(x\left(t-\tau(t)\right),t\right) + \bar{N}n\left(x\left(t-d(t)\right),t\right) + \bar{E}_w w(t)\} + x^T(t)(Q_1+R_1)x(t) + \eta^T(t)(Q_2+\tau R_2+dQ_3)\eta(t) - (1-\mu_2)x^T\left(t-d(t)\right)Q_1x\left(t-d(t)\right) - (1-\mu_2)\eta^T(t-d(t))Q_2\eta(t-d(t)) - (1-\mu_1)x^T(t-\tau(t))R_1x(t-\tau(t)) \quad (10)$$

According to definition 2.1, the $H_\infty$ disturbance attenuation express as:

$$J(t) = \int_0^\infty[z^T(t)z(t) - w^T(t)w(t)]dt \quad (11)$$

Considering (1) where $z(t) = C_1x(t)$, we can obtain

$$x^T(t)C_1^TC_1x(t) \le \gamma^2\omega^T(t)\omega(t).$$

From (10) and (11) it can be shown that

$$\dot{v}(t) + x^T(t)C_1^TC_1x(t) - \gamma^2\omega^T(t)\omega(t) < X_{avg}^T\sum X_{avg}$$

Or

$$2\zeta^T(t)P^T\{\bar{A}\zeta(t)+\bar{B}x(t-\tau(t))+\bar{C}\eta(t-d(t))+\bar{H}h(x(t),t)+\bar{G}g(x(t-\tau(t)),t)+\bar{N}n(x(t-d(t)),t)+\bar{E}_w w(t)\}+x^T(t)(Q_1+R_1)x(t)+\eta^T(t)(Q_2+\tau R_2+dQ_3)\eta(t)-(1-\mu_2)x^T(t-d(t))Q_1x(t-d(t))-(1-\mu_2)\eta^T(t-d(t))Q_2\eta(t-d(t))-(1-\mu_1)x^T(t-\tau(t))R_1x(t-\tau(t))+x^T(t)c_1^Tc_1x(t)-\gamma^2w^T(t)w(t)<X_{avg}^T\sum X_{avg}$$

Where $X_{avg}=[\zeta^T(t) \; x^T(t-\tau(t)) \; x^T(t-d(t)) \; \eta^T(t-d(t)) \; w^T(t)]$. By Schur complement lemma in (6), our proof is complete and one has:

$$\Sigma=\begin{bmatrix}\Theta & P^T\bar{B}+P^T\bar{G}\beta^2 & P^T\bar{N}\sigma^2 & P^T\bar{C} & P^T\bar{E}w & P^T\bar{H}\\ * & -(1-\mu_1)R_1 & 0 & 0 & 0 & 0\\ * & 0 & -(1-\mu_2)Q_1 & 0 & 0 & 0\\ * & 0 & 0 & -(1-\mu_2)Q_2 & 0 & 0\\ * & 0 & 0 & 0 & -\gamma^2I & 0\\ * & 0 & 0 & 0 & 0 & -I\end{bmatrix}<0 \tag{12}$$

With

$$\Theta=P^T\bar{A}+\bar{A}^TP+\begin{bmatrix}Q_1+R_1+C_1^TC_1 & 0\\ 0 & Q_2+\tau R_2+dQ_3\end{bmatrix}.$$

B. Control Design synthesis

This section examines the robust $H_\infty$ control design for the TDS in (1) in the presence of neutral delay.

Theorem 2. Consider TDS (1) with neutral Time-delay. The system (1) with a control input $u(t)=kx(t)$ is asymptotically stable and satisfies disturbance attenuation level $\gamma>0$, if there exist positive-definite matrices $P, Q_1, Q_2, Q_3, R_1, R_2$ and matrix, $\bar{K}$ and and scalers $\mu_1,\mu_2,d,\tau>0$ such that the following LMI holds:

$$\Sigma=\begin{bmatrix}\Theta & P^T\bar{B}+P^T\bar{G}\beta^2 & P^T\bar{N}\sigma^2 & P^T\bar{C} & P^T\bar{E}w & P^T\bar{H}\\ * & -(1-\mu_1)R_1 & 0 & 0 & 0 & 0\\ * & 0 & -(1-\mu_2)Q_1 & 0 & 0 & 0\\ * & 0 & 0 & -(1-\mu_2)Q_2 & 0 & 0\\ * & 0 & 0 & 0 & -\gamma^2I & 0\\ * & 0 & 0 & 0 & 0 & -I\end{bmatrix}<0 \tag{13}$$

With $\Theta=P^T\bar{A}+\bar{K}[I \;\; 0]+\bar{A}^TP+[I \;\; 0]^T\bar{K}^T+\begin{bmatrix}Q_1+R_1+\alpha^2I+C_1^TC_1 & 0\\ 0 & Q_2+\tau R_2+dQ_3\end{bmatrix}$. Then the control state-feedback gain $K$ is obtained by

$$K=\bar{B}_1{}^{+}(P^T)^{-1}\;\bar{K}$$

Where $\bar{B}_1{}^{+}$ is the pseudo-inverse matrix.

Proof. By substituting the control signal $u(t)=kx(t)$ in (1), we have

$$\dot{x}(t)=(A+B_1K)x(t)+Bx(t-\tau(t))+C\dot{x}(t-d(t))+Hh(x(t),t)+Gg(x(t-\tau(t)),t)+Nn(x(t-d(t)),t)+E_w w(t)$$

According to Theorem 1, by replacing $\bar{A}$ with $\bar{A}+\bar{B}_1K[I \;\; 0]$, we can obtain

$$\Sigma=\begin{bmatrix}\Theta & P^T\bar{B}+P^T\bar{G}\beta^2 & P^T\bar{N}\sigma^2 & P^T\bar{C} & P^T\bar{E}w & P^T\bar{H}\\ * & -(1-\mu_1)R_1 & 0 & 0 & 0 & 0\\ * & 0 & -(1-\mu_2)Q_1 & 0 & 0 & 0\\ * & 0 & 0 & -(1-\mu_2)Q_2 & 0 & 0\\ * & 0 & 0 & 0 & -\gamma^2I & 0\\ * & 0 & 0 & 0 & 0 & -I\end{bmatrix}<0 \tag{14}$$

With

$$\Theta = P^T((\bar{A}+\bar{B}_1K[I\ \ 0]) + (\bar{A}+\bar{B}_1K[I\ \ 0])^TP + \begin{bmatrix} Q_1+R_1+\alpha^2I+C_1^TC_1 & 0 \\ 0 & Q_2+\tau R_2+dQ_3 \end{bmatrix}$$

Due to the nonlinear condition in (14), it must be transformed into a linear matrix inequality form by supposing $P^T\bar{B}_1K=\bar{K}$ where $\bar{K}$ is matrix with appropriate dimension that it can be obtained by solving a linear matrices inequality form of follow:

$$\Sigma = \begin{bmatrix} \Theta & P^T\bar{B}+P^T\bar{G}\beta^2 & P^T\bar{N}\sigma^2 & P^T\bar{C} & P^T\bar{E}w & P^T\bar{H} \\ * & -(1-\mu_1)R_1 & 0 & 0 & 0 & 0 \\ * & 0 & -(1-\mu_2)Q_1 & 0 & 0 & 0 \\ * & 0 & 0 & -(1-\mu_2)Q_2 & 0 & 0 \\ * & 0 & 0 & 0 & -\gamma^2I & 0 \\ * & 0 & 0 & 0 & 0 & -I \end{bmatrix} < 0 \tag{15}$$

With

$$\Theta = P^T\bar{A}+\bar{K}[I\ \ 0]+\bar{A}^TP+[I\ \ 0]^T\bar{K}^T+\begin{bmatrix} Q_1+R_1+\alpha^2I+C_1^TC_1 & 0 \\ 0 & Q_2+\tau R_2+dQ_3 \end{bmatrix}$$

By Schur complement lemma in (6) and solving linear matric inequality and using pseudo inverse matric we can obtain control feedback gain $K$ as follow:

$$K = \bar{B}_1{}^{+}(P^T)^{-1}\ \bar{K}$$

## 4. Illustrative Example

In this section, the applicability of the proposed method is validated by Three examples in the sequel.

Example 4.1. considering the simulation of a wind energy conversion system model based on a permanent magnet synchronous generator model. It can be shown that by utilization of Park's transformation to the (*abc*) coordinate frame PMSG model and linearizing the nonlinear model about an operating point, the following linear model in the d-q coordinate frame model can be obtained (Mittal, R., Sandhu, K., and Jain, D. (2012)),

$$\begin{bmatrix} \dot{i}_d(t) \\ \dot{i}_q(t) \\ \dot{\omega}_r(t) \end{bmatrix} = \begin{bmatrix} g_1 & g_2 & 0 \\ -g_2 & g_1 & 0 \\ 0 & g_3 & 0 \end{bmatrix}\begin{bmatrix} i_d(t) \\ i_q(t) \\ \omega_r(t) \end{bmatrix} + \begin{bmatrix} 0 & 0 & \frac{Pi_q^*}{2} \\ 0 & 0 & g_4 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} i_d(t-\tau(t)) \\ i_q(t-\tau(t)) \\ \omega_r(t-\tau(t)) \end{bmatrix} + \begin{bmatrix} \frac{-1}{L_s} & 0 \\ 0 & \frac{-1}{L_s} \\ 0 & 0 \end{bmatrix}\begin{bmatrix} u_d(t) \\ u_q(t) \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} w(t) \tag{16}$$

with $g_1=\frac{-R_s+\Delta R_s}{L_s+\Delta L_s}, g_2=\frac{P\omega_r^*}{2}$ and $g_3=\frac{-\Psi_m P}{4J}, g_4=\frac{4P}{2L_s+\Delta L_s}-\frac{Pi_d^*}{2}$, where $u_d$ and $u_q$ are the d- and q-axis stator voltage components; $i_d$ and $i_q$ are the d- and q-axis stator current components, respectively; and $L_s$, $R_s$ are the stator inductance and resistance, respectively. $\Psi_m$ is the flux, $\omega_r$ is the rotor electrical angular speed and P the number of the poles.

Assuming small deviation of resistance and inductance values from nominal corresponding values, we can represent the system (16) in the following form,

$$\dot{x}(t) = \begin{bmatrix} \frac{-R_s}{L_s} & \frac{P\omega_r^*}{2} & 0 \\ -\frac{P\omega_r^*}{2} & \frac{-R_s}{L_s} & 0 \\ 0 & \frac{-\Psi_m P}{4J} & 0 \end{bmatrix} x(t) + \begin{bmatrix} 0 & 0 & \frac{Pi_q^*}{2} \\ 0 & 0 & \frac{2P}{L_s}-\frac{Pi_d^*}{2} \\ 0 & 0 & 0 \end{bmatrix} x\big(t-\tau(t)\big) + \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}\Delta_1\big(x(t)\big)$$

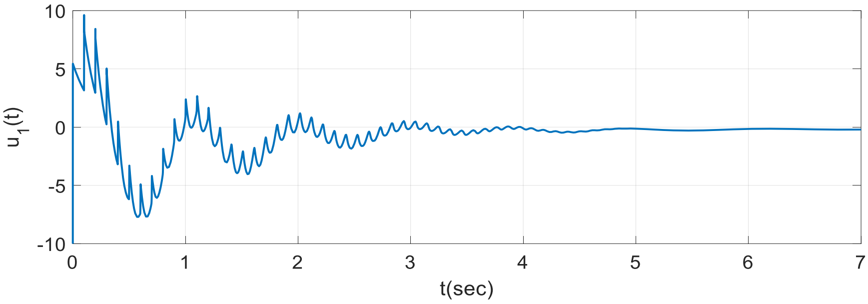
$$+\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}\ \Delta_2\big(x(t-\tau(t))\big) + \begin{bmatrix} \frac{-1}{L_s} & 0 \\ 0 & \frac{-1}{L_s} \\ 0 & 0 \end{bmatrix}\begin{bmatrix} u_d(t) \\ u_q(t) \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} w(t) \tag{17}$$

Where $x(t)=[i_d(t)\quad i_q(t)\quad \omega_r(t)]^T$ and $\Delta_1\big(x(t)\big)=\begin{bmatrix} \hat{\Delta}_1\big(x(t)\big) \\ \bar{\Delta}_1\big(x(t)\big) \end{bmatrix}$, considering the system (1) with parameters of WECS model in table 1. And $\mu_1,\mu_2=0.1, \tau=0.1\ \ d=0.1$, $\gamma=10$; and according to theorem 2.1, the corresponding LMI is solved by using LMI Toolbox, then the state-feedback control gain $K$ is obtained. Therefore, we can show behavior of input control signal in Figure 1. And state responses of closed-loop systems in figure 2.

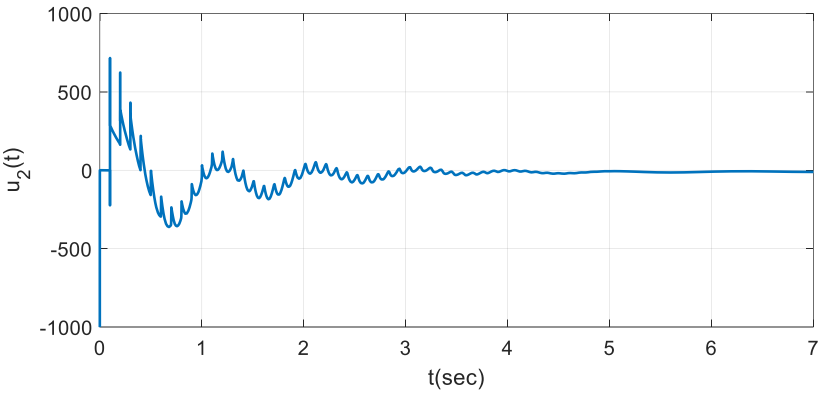
$$K = 1.0e + 03 * \begin{bmatrix} 0.0015 & 0.0022 & -0.0055 \\ -1.0991 & -1.0188 & 0.0031 \\ -0.0169 & -0.0175 & 0.0000 \end{bmatrix}$$

***Table 1.*** *Parameters of the WECS Mode.*

| | |
|---|---|
| 3.3 Ω | $R_s$ |
| $41.56*10^{-3}$ H | $L_s$ |
| 6 | P |
| $4832*10^{-4}$ V.s. | $\Psi_m$ |




(i)




(ii)

***Figure 1.*** *(i)- (ii) the Behavior of Input control signals.*

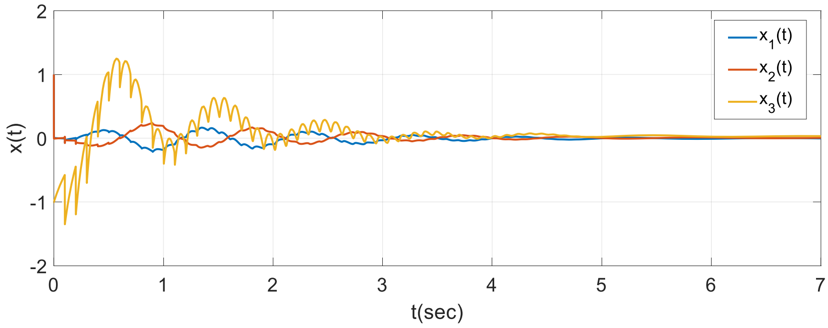



***Figure 2.*** *State Trajectories of d-axis Current, q-axis Current and rotational speed.*

Example 4.2. Considering the system (1) with the following matrices,

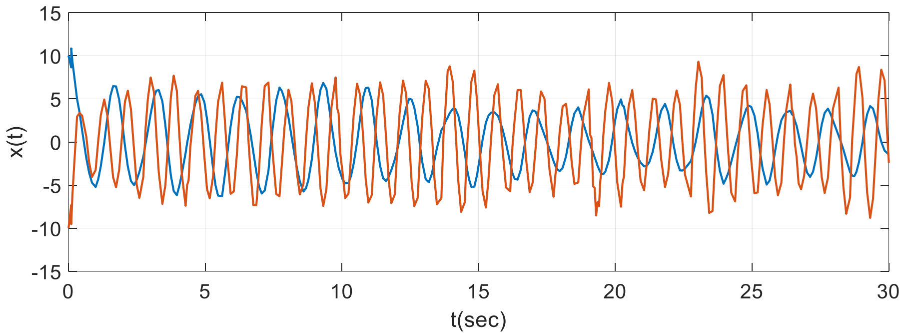
$$A = \begin{bmatrix} -1 & 0.5 \\ 0.1 & -3 \end{bmatrix} \; B = \begin{bmatrix} -0.8 & 0 \\ -0.1 & -3 \end{bmatrix} \; G = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \; C = \begin{bmatrix} -0.1 \\ -0.3 \end{bmatrix} \; E = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \; H = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$N = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix} \; E_w = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \; B_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

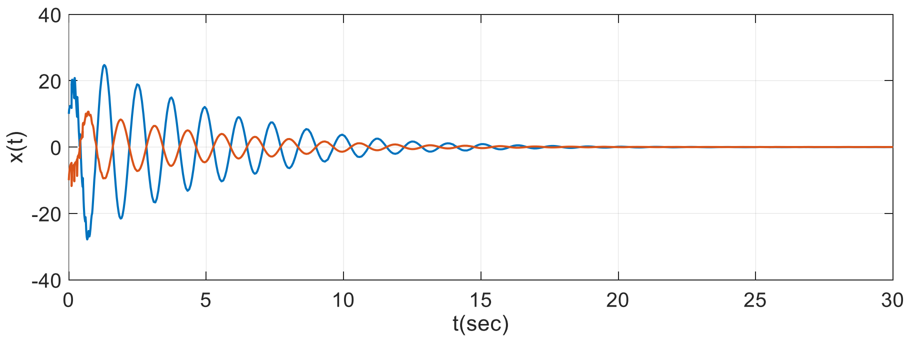
And $\mu_1, \mu_2 = 0.1, \tau = 0.1 \;\; d = 0.1, \; \gamma = 10$.
According to Theorem 2, the corresponding LMI is solved using LMI Toolbox, then the following solutions can be computed.

$$P = \begin{bmatrix} 0.9201 & -0.0225 \\ -0.0225 & 0.1607 \end{bmatrix}, Q_1 = \begin{bmatrix} 0.2860 & -0.0438 \\ -0.0438 & -0.1903 \end{bmatrix}, R_1 = \begin{bmatrix} 0.3640 & -0.0620 \\ -0.0620 & -0.0541 \end{bmatrix}$$

With the state-feedback control gain $K = [-12.0747 \quad -28.8534]$. The time behavior of state system with controller and without controller and control input are depicted and system stability effect can be shown from the figures 3 and 4.




(i)




(ii)

***Figure 3.*** *The behavior of state system without controller (i) and with controller (ii).*

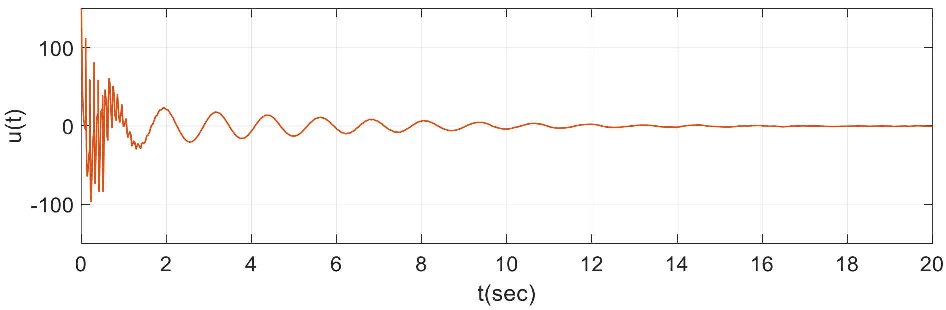



***Figure 4.*** *Input control signals.*

## 5. Conclusion

In this paper, the problem of robust $H_\infty$ state-feedback control design for a class of uncertain systems with neutral time-delay is addressed. The Lyapunov stability theory and Linear Matrix Inequalities (LMI) are employed to ensure the robust stabilization of the system under consideration. Additionally, an explicit $H_\infty$ state-feedback controller is derived. Finally, numerical examples, including a wind energy conversion system modeled using a Permanent Magnet Synchronous Generator (PMSG), are presented to demonstrate the effectiveness and validity of the proposed methods. In future work, we can focus on extending the proposed robust $H_\infty$ control design to more complex systems.

## Abbreviations

| | |
|---|---|
| LMI | Linear Matrix Inequality |
| PMSG | Permanent Magnet Synchronous Generator |
| TDS | Time-Delay Systems |
| LKF | Lyapunov-Krasovskii Function |

## Conflicts of Interest

The authors declare no conflicts of interest.

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