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A Simplified Trajectory for a Radiating Charge Moving in a Uniform Magnetic Field

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Abstract: Is shown that a radiating charge moving in a constant uniform magnetic field has an approximated trajectory that copies the Euler spiral. Is made a classical study for the motion with some plausible approximations. By comparing the graph of Cornu's spiral, which is considered not from the initial point, but from an advanced point to satisfy the initial conditions, and the image obtained with the bubble chamber, a strong correspondence can be seen. This simple result is the springboard for an exact result in which starting from a formula adapted to overcome the inconsistencies of the current formulas for the radiation reaction force, one obtains again the same result presented in this article as a limiting case. This suggests that the new formula can describe the phenomenon of the radiation reaction force with more accuracy, combined with the fact that the new formula, which I hope to be able to present in a future article, provides in other cases of motion without the magnetic field, quite plausible results.

Keywords: Pair Production, Trajectory, Magnetic Field, Cornu Spiral, Bubble Chamber, Radiation Reaction Force

1. Introduction

From first times are known analytic expressions for classical trajectories of charged particles, the hyperbolic motion and the circular or helical motion. There is a third class of trajectories, the motion of a charged particle in a cloud chamber with constant magnetic field not yet expressed analytically. In this article a model for this lacking expression is found. It is established a connection with the radiation reaction force and the result obtained copies well the known pictures of the phenomenon.

2. Method

Consider an electric charge e moving with constant velocity in the x direction, entering a region with a uniform magnetic field B in the z direction. Being the radiation reaction force (Lechner - Campi elettromagnetici, pag.272)

$$\frac{d\vec{p}_{rad}}{dt} = -\frac{e^2}{6\pi}w^2\vec{v} \tag{1}$$

the newtonian equation of motion is (the speed of light c set to 1)

$$\frac{d\vec{p}}{dt} = -\lambda w^2\vec{v} + e\vec{v}\wedge\vec{B} \tag{2}$$

$$\lambda = \frac{e^2}{6\pi} \tag{3}$$

The magnetic force

$$e\vec{v}\wedge\vec{B} \tag{4}$$

is always perpendicular to the velocity and cannot change his modulo. Changes in the modulo of v arise from the component of force tangent to trajectory

$$\frac{d\vec{p}_t}{dt} = -\lambda w^2 \vec{v} \tag{5}$$

w^2 is the square of the acceleration that is compound of the tangential component proportional to λ and the normal component, the Lawrence force. Neglecting the first (very small for a particle) in comparison to the last,

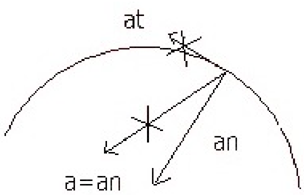

Figure 1. *The approximation.*

we have that the square of acceleration w^2 is

$$w^2 = \not{a}_t^{\,2} + a_n^2 \approx a_n^2 = (\frac{evB}{m})^2 \tag{6}$$

and the complete tangential force is

$$ma_t = -\lambda(\frac{evB}{m})^2 \cdot v = -\lambda(\frac{eB}{m})^2 \cdot v^2 \cdot v = -\frac{e^2}{6\pi}\omega^2 v^3 \tag{7}$$

then the tangential equation of motion is

$$m\frac{dv}{dt} = -\frac{e^2}{6\pi}\omega^2 v^3 \tag{8}$$

Restoring the constant c, we have

$$\frac{dv}{dt} = -\frac{e^2}{6\pi mc^3}\frac{\omega^2}{c^2}v^3 \tag{9}$$

defining

$$\alpha = \frac{e^2}{6\pi mc^3} \approx 0.6 \cdot 10^{-23} sec. \tag{10}$$

we obtain

$$\frac{dv}{dt} = -\frac{\alpha\omega^2}{c^2}v^3 \tag{11}$$

Placing

$$k = -\frac{\alpha\omega^2}{c^2} \tag{12}$$

$$\frac{dv}{dt} = -kv^3 \tag{13}$$

solving the differential equation, we have (v_0 the initial velocity)

$$-\frac{1}{2v^2} = -kt - A \tag{14}$$

$$v = \frac{1}{\sqrt{2kt + \frac{1}{{v_0}^2}}} \tag{15}$$

and solving again (choosing the spatial origin at $v = \infty$)

$$\frac{ds}{dt} = \frac{1}{\sqrt{2kt + \frac{1}{{v_0}^2}}} \tag{16}$$

$$s = \frac{1}{k}\sqrt{2kt + \frac{1}{{v_0}^2}} \tag{17}$$

Then

$$v = \frac{1}{ks} \tag{18}$$

but

$$\omega = \frac{v}{r} \tag{19}$$

constant, and then the curvature 1/r is proportional to 1/v

$$\frac{1}{r} = \frac{\omega}{v} = \omega ks \tag{20}$$

then the curvature must be proportional to the distance s from the beginning of curve and the trajectory is a Cornu spiral [8]

$$rs = \frac{1}{\omega k} = const. = \frac{1}{2a^2} \tag{21}$$

with the parameter a

$$a = \sqrt{\frac{k\omega}{2}} = \sqrt{\frac{\alpha\omega^3}{2c^2}} \tag{22}$$

The parametric representation of trajectory is

$$\begin{cases} x = \frac{1}{a}C(s) \\ x = \frac{1}{a}S(s) \end{cases} \tag{23}$$

where

$$C(s) = \int_0^s cos(u^2)\,\mathrm{d}u \tag{24}$$

$$S(s) = \int_0^s sin(u^2)\,\mathrm{d}u \tag{25}$$

The polar angle ϑ equals s^2 (normalized spiral).

In fact we have

$$\omega t = \frac{k\omega}{2}s^2 - \frac{1}{{v_0}^2} \cdot \frac{\omega}{2k} \tag{26}$$

$$x = \int_0^t v cos(\omega t' + \varphi)\mathrm{d}t' = \int_{s_1}^{s_2} \frac{1}{ks} cos\left[\frac{k\omega s^2}{2} - \frac{\omega}{2kv_0^2} + \varphi\right] ks\mathrm{d}s \tag{27}$$

choosing

$$\varphi = \frac{\omega}{2kv_0^2} \tag{28}$$

we have

$$x = \int_{s_1}^{s_2} cos\left(\frac{k\omega s^2}{2}\right)\mathrm{d}s \tag{29}$$

$$s_1 = \frac{1}{kv_0} \tag{30}$$

$$s_2 = \frac{1}{k}\sqrt{2kt + \frac{1}{{v_0}^2}} \tag{31}$$

$$x = \int_{s_1}^{s_2} cos(a^2 s^2)\mathrm{d}s = \frac{1}{a}\int_{as_1}^{as_2} cos(s^2)\mathrm{d}s \tag{32}$$

In the same way

$$y = \frac{1}{a}\int_{as_1}^{as_2} sin(s^2)\mathrm{d}s \tag{33}$$

and for the length of trajectory crossed

$$x = \int_{s_1}^{s_2} \sqrt{dx^2 + dy^2} = \int_{s_1}^{s_2} \sqrt{\frac{2}{k\omega}}\mathrm{d}s = \sqrt{\frac{2}{k\omega}}(s_2 - s_1) \tag{34}$$

As an application find the time for the velocity to became the half,

$$v = \frac{v_0}{2} \tag{35}$$

$$\frac{v_0}{2} = \frac{1}{\sqrt{2kt + \frac{1}{{v_0}^2}}} \tag{36}$$

follows

$$t = \frac{3}{2k{v_0}^2} = \frac{3c^2}{2\alpha\omega^2{v_0}^2} \tag{37}$$

For example suppose that an electron of mass $m = 9.1 \cdot 10^{-28}g$ is accelerated by a voltage of 1000 Volt and suddenly the electric field is extinguished and a magnetic constant and uniform field B=10^3 gauss acts on in a large enough region around the particle. We have the final velocity of accelerated particle

$$v = \sqrt{\frac{2e\Delta V}{m}} = 1.8 \cdot 10^9 \frac{cm}{sec} = 0.06c \tag{38}$$

The angular velocity is

$$\omega = \frac{eB}{mc} = 1.7 \cdot 10^{10} sec^{-1} \tag{39}$$

Applying the formula

$$t_{\frac{1}{2}} = \frac{3c^2}{2\alpha\omega^2{v_0}^2} = 0.24 \cdot 10^6 sec \cong 2.8 days \tag{40}$$

The following image shows well the result. A charged particle, coming from left, at the moment of activation of the magnetic field, enters in the spiral at a point (s_1) after the origin (s=0) with velocity tangent to the spiral itself. The initial curvature R^{-1} equals the inverse radius of the circle of the magnetic field trajectory $\left(R = \dfrac{mv}{eB}\right)$.

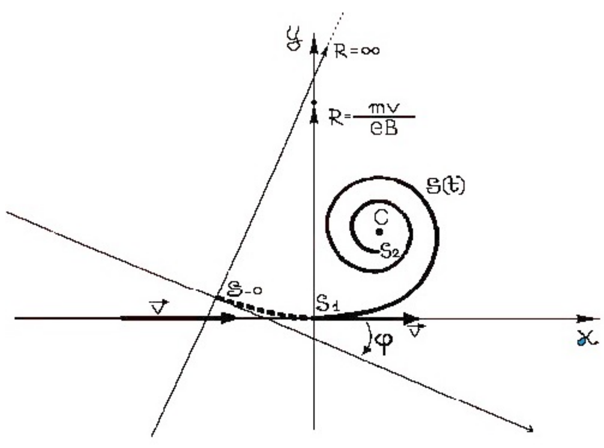

__Figure 2.__ The trajectory of the particle, coming from left.

3. Result

From the calculations performed follows that the classical trajectory of a particle charged that moves in a uniform magnetic field is a Cornu Spiral, neglecting frictional forces and relativistic effects.

4. Discussion

The particle that enters the region of the magnetic field changes trajectory, as following a circle. But the curvature increases for the decrease of velocity due to energy loss by radiation. The only effect of radiation reaction force is more weak that it is in reality because of the friction force, that accelerates the process and distorts the trajectory, that in any case adjusts itself to copy in the best possible way a Cornu spiral, as one can see in well-known images of the process (Fig. 3). Being null the angular acceleration, the reaction force compensates the Coriolis force.

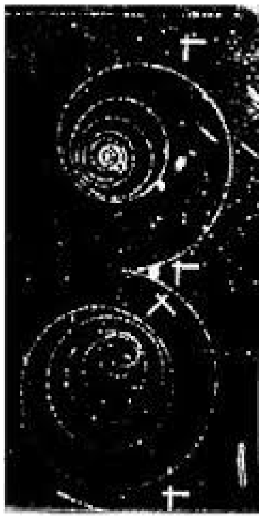

Figure 3. *A pair production (Taken from Internet).*

5. Conclusions

A particle entering with constant velocity in a uniform magnetic field begin radiating energy and his trajectory approximates well a Cornu spiral, neglecting energy loss by friction. The curvature is initially equal to the value impressed by the Lawrence force and increases indefinitely, in a way proportional to the length of the trajectory crossed. The considerations are classical, not relativistic, but the approximation appears good enough.

6. Moreover

This example that starts from the formula of the reaction force of radiation gives, after an approximation to the Newtonian case, results in agreement with intuition. The search for a formula that gives the exact reaction force comes from afar. Abraham and Lorentz began with a very ingenious method and powerful calculations that lead to a result almost in agreement with experience [1]. The mass of the electron that is obtained is 4/3 of the real one. Lorentz and Abraham [16] found for the force a appropriate analytical expression that describes the problem good enough, apart from the presence of pre-acceleration and runaway solutions not in agreement with what one expects. Others have tried to solve the problem, Landau [18], Rorlich [19], Spohn [20], to name a few, but all have obtained formulas that applied to the motion of a charge give sometimes paradoxical results and arrive at more acceptable results by force of ingenious and forced additions. The charge poses many problems both for the radiated energy and for knowing whether, for example, in the case of a uniformly accelerated rectilinear motion, the radiation is present.

Here in my opinion we are faced with the Heraclitus [21] problem, the principle cannot belong to the whole [22].

The force of a charge on itself cannot arise from itself, ”its laws”, is not included in the axioms that concern it.

We try to obtain a formula for the radiation force starting from the inside of electromagnetism, but the reaction force is at the limit of electromagnetism and the postulates of the theory may not contain such a force.

It is as if I try to lift myself by pulling myself by the hair. I do not move an inch from the ground.

A substantial modification of the reaction force formula has been obtained that also seems to agree with the Compton effect, which intervenes at the quantum level when the charge radiates.

With the new formula the motion of the charge is what one would expect and for small speeds we fall back into the Newtonian case described in this article.

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More will follow.

Conflicts of Interest

The author declares no conflicts of interest.

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