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Second Order Properties of Nearest Neighbor Regression with Uniform Weighting Function for Strongly Mixing Processes

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Abstract: We examine the statistical properties of nearest neighbor regression function estimation beyond the classical i.i.d assumption. Second-order properties of this estimator for uniformly mixing processes have been derived in previous studies. Nevertheless, uniform mixing is a very strong form of dependence that is difficult to achieve. In contrast, strong mixing conditions are satisfied by a broad class of stochastic processes commonly encountered in theoretical and applied time series modeling. This paper focuses on the analysis of the second-order properties of the nearest neighbor regression estimator under strong mixing dependence measure. Under appropriate regularity conditions, including strong mixing assumptions and smoothness of the underlying density functions, we derive expressions for both the bias and the quadratic mean squared error (QMSE) of the nearest neighbor regression estimator with a uniform weighting scheme for estimating the unknown conditional mean function. Our results demonstrate that the QMSE of the nearest neighbor estimator attains the minimax-optimal rate for estimating a p-smooth regression function in a d-dimensional embedding space. The theoretical analysis integrates the dependence structure specific to strongly mixing processes with the geometric characteristics of k-nearest neighborhoods. This combination enables the identification of an optimal choice for the number of nearest neighbors that effectively balances the trade-off between bias and variance. Overall, these findings provide a rigorous theoretical foundation for the application of nearest neighbor regression methods to short-range dependent data. Furthermore, the explicit bias and variance characterizations lay the groundwork for establishing asymptotic normality, thereby enabling the construction of valid confidence intervals and supporting reliable statistical inference in dependent data settings.

Keywords: Nearest Neighbor Regression; α-mixing Processes, Bias, Variance, Quadratic Mean Square Error

1. Introduction

The k nearest neighbor (k-NN) regression estimate is a nonparametric based regression method. It has been claimed more flexible compared to the foundational nonparametric kernel regression estimate [1] and has been showed great interest in applications [11, 13, 19, 22, 24]. Various theoretical results on this regression estimate have been established.

Properties of the k-NN regression estimate depend on the dependence structure of the process. For independent and identically distributed ($i.i.d$) processes, many results have been obtained about the properties of the k-NN regression function estimate. Not to be exhaustive, we can mention the work on k-NN regression estimate with uniform weighting function. Reference [26] studied the MSE, the MISE and the asymptotic normality of such k-NN regression. Later, [17] extended this result for non-uniformly weighted k-NN regression estimate. He investigated the bias and the asymptotic normality of the k-NN regression estimate with a more general weighting function, again under the $i.i.d$ assumption. Other types of convergence such as L^2 consistent and uniform convergence are discussed in [10, 27]. Moreover, [28] has weakened the assumption on finiteness of the third moment of the underlying process in [17] and has obtained the asymptotic normality of the univariate k-NN regression estimate.

An interesting extension would be on relaxing the $i.i.d$ assumption, may we still establish similar statistical properties of the k-NN regression estimate even the processes are dependent. That would ensure the use of k-NN estimate to time series processes.

In contrast to independent framework, fewer results on k-NN regression estimate have been obtained for dependent processes. They are mainly obtained in univariate setting. Among them, reference [5] has provided the piecewise convergence for univariate dependent processes. This work has been extended by [29] on the quadratic mean square error of such k-NN regression estimate. Similar result has been obtained for multivariate strongly dependent processes as in [14]. Fewer categories of theoretical known processes met the strong dependence condition. However, many studies have found that the processes involved in time series modelling exhibit weaker dependence [4, 6, 12, 18]. We will relax the strongly-dependent constraint on processes to some weak dependence structure. Clearly, we will analyze the bias, the variance and the quadratic mean square error of the k-NN estimate for weakly-dependent processes. This work is an amended part of the pre-print version in [23].

We organize the paper as follows. In section 2, we provide an overview the k-NN nonparametric regression function estimate. In section 3, we study the properties of this nonparametric regression function estimate for dependent processes. In last section, we conclude.

2. Nearest Neighbor Regression Estimate

This section presents the general problem of the conditional mean's estimation by the nearest neighbor regression nonparametric method and some dependent structures.

Given a probability space $(\Omega, \mathcal{F}, P)$ where $(\Omega, \mathcal{F})$ is a measurable space and P is a probability measure on $(\Omega, \mathcal{F})$. Consider two real-valued stochastic processes $\{X_i, \quad i = 1, 2, ...\}$ and $\{Y_i, \quad i = 1, 2, ...\}$ on the probability space $(\Omega, \mathcal{F}, P)$ that is X_i and Y_i are $\mathcal{F}_i$-measurable with $\mathcal{F}_i$ a filtration of $\mathcal{F}$. We focus on regression analysis which concerns the estimation of the relationship between the two processes $Y = \{Y_i, i = 1, 2, ...\}$ (the dependent variable) and $X = \{X_i, i = 1, 2, ...\}$ (the explanatory variable). This can be summarized by the expression $Y = m(X)$, where $m(.)$ is the link function. In general, the link function $m(.)$ is unknown. Therefore, this function has to be estimated. We can reformulate this problem by relating it with the conditional mean. The underlying principle that theoretical laws usually do not hold in every individual case but merely on average will be considered and $m(x)$ appears as conditional expectation of Y given x:

$$m(x) = E[Y|X = x]. \tag{1}$$

Under mild conditions, equation (1) says that the equation holds in average, i.e. the expectation of Y conditional on $X = x$ is given by $m(x)$. When we observe (x_i, y_i), $i = 1, .., n$ of (X,Y), such that:

$$y_i = m(x_i) + \varepsilon_i, \;\; i = 1, \cdots, n, \tag{2}$$

where $(\varepsilon_i)_i$ is a white noise process satisfying $E[\varepsilon_i|X_i = x] = 0$ and $Var[\varepsilon_i|X_i = x] = \sigma^2(x)$, the equation (2) says that the relationship $Y = m(X)$ does not hold exactly for the i-th observation but is disturbed by the random variable ε_i. Considering the set of previous observations, we can estimate $m(.)$ by m_n in the following way:

$$m_n(x) = \sum_{i=1}^{n} \omega_{i,n}(x) y_i, \tag{3}$$

where $\omega_{i,n}$ are weights to be specify or to be estimated. A general form of regression function estimate by nonparametric methods lies on relation (3).

Let provide a brief presentation of a k-NN regression estimate of the unknown conditional mean. Consider a more general process $(Y, \underline{X})$ with valued in $\mathbb{R}$x$\mathbb{R}^d$. We assume the existence of the joint density function f for $(Y, \underline{X})$, the conditional density $f(y \mid \underline{x})$ for $Y \mid (\underline{X} = \underline{x})$ and the marginal density h for $\underline{X}$. For $\underline{x} \in \mathbb{R}^d$, we are interested in estimation of the conditional mean $m(\underline{x}) = E[Y \mid \underline{X} = \underline{x}]$.

Given a sample $(Y_1, \underline{X}_1), ..., (Y_n, \underline{X}_n)$, with size n, for the process $(Y, \underline{X})$, the k-NN regression estimate of the conditional mean $m(\underline{x})$ can be rewritten as follows:

$$m_n(\underline{x}) = \sum_{i=1}^{n} w(\underline{x} - \underline{X}_i) Y_i, \tag{4}$$

where w is a weighting function associated to neighbors. A general expression of the weighting function w is:

$$w(\underline{x} - \underline{X}_i) = \frac{\frac{1}{nR_n^d} K(\frac{\underline{x} - \underline{X}_i}{R_n})}{\frac{1}{nR_n^d} \sum_{i=1}^{n} K(\frac{\underline{x} - \underline{X}_i}{R_n})}, \tag{5}$$

where R_n will be defined as a distance, according to the Euclidean norm in $\mathbb{R}^d$, from $\underline{x}$ to its k-th neighbors, and $K(u)$ is a bounded, non negative function satisfying

$$\int K(u) du = 1 \quad and \quad K(u) = 0 \quad for \quad |u| \geq 1. \tag{6}$$

The uniform weighting function satisfies these conditions. An assumption on the weighting function will not be needed.

We wish to extend results for independent processes to dependent processes. Let us remind the notion of dependence.

The concept of dependence may occur in different ways according to the structure of the process to be studied and could be ranged from the weakest to the strongest. Here, we consider some mixing conditions which measure the dependence structure on the process. They are the strong, the uniform and the regular mixings. Consider a process $\{X_i, i \in \mathbb{N}\}$ which is assumed to be stationary and denote by $\mathcal{F}_r^s = \sigma(X_\ell, r \leq \ell \leq s)$ the σ-algebra generated by

$\{X_\ell, r \leq \ell \leq s\}$. We define the following mixing coefficients:

$$\alpha(n) = \underset{A\in\mathcal{F}_1^t, B\in\mathcal{F}_{n+t}^\infty}{Sup} |P(B\cap A) - P(A)P(B)|. \quad (7)$$

$$\beta(n) = \underset{B\in\mathcal{F}_{n+t}^\infty}{Sup} |P(B) - P(B \mid \mathcal{F}_1^t)|. \quad (8)$$

$$\phi(n) = \underset{A\in\mathcal{F}_1^t, B\in\mathcal{F}_{n+t}^\infty, P(A)\neq 0}{Sup} |P(B \mid A) - P(B)|. \quad (9)$$

Then the process $\{X_i, i \in \mathbb{N}\}$ is said to be α-mixing (or strong mixing) if $\alpha(n) \to 0$ as $n \to \infty$, β-mixing if $\beta(n) \to 0$ as $n \to \infty$ and ϕ-mixing (or uniform mixing) if $\phi(n) \to 0$ as $n \to \infty$. Moreover, such process is said to be geometrically strongly mixing if there exist $c_0 > 0$ and $\rho \in [0, 1[$ such that $\alpha(n) \leq c_0\rho^n$. For more development on the concepts of dependent processes [2, 3, 7, 15, 20, 25]. Example of dependent processes are the linear and the nonlinear parametric models, since processes like the ARMA models, related GARCH processes and Markov switching processes are known to be mixing [4, 6, 12, 18]. For other direction on dependent structure studies [8, 9].

3. Second Order Properties

Our result concerns the bias and the quadratic mean square error of the k-NN regression estimate of $m(.)$. For strongly mixing processes with suitable conditions (on the density functions, on the weighting function and on the regression function) the quadratic mean square error of the k-NN regression estimate is of order $O(n^{-Q})$ with $Q = \frac{2p}{2p+d}$ and p the degree of smoothness.

Before all, we introduce regularity conditions in the following assumptions. The first assumption characterizes the dependence on the process.

Assumption 3.1. $(X_n)_n$ is α-mixing process. Moreover, for a certain $\delta > 0$, $|X_n|^{2+\delta}$ is uniformly integrable, $\sum_n n^{2/\delta}\alpha(n) < \infty$ and $inf_k Var(X_k) > 0$.

The next assumption concerns regularity condition on the joint, on the marginal and on the conditional density functions of the process.

Assumption 3.2. $h(\underline{x}), f(y \mid \underline{x})$ and $f(y, \underline{x})$ are p continuously differentiable and $f(y \mid \underline{x})$ is bounded.

The regularity condition on the marginal and conditional distribution of the time series in assumption 3.2 can be overcome when the function $m(\underline{x})$ satisfies Lipschitz's condition with order $1 \leq \delta < 2$.

We now present our theoretical result, which characterises the order of the bias, variance and quadratic mean square error of the nearest neighbor regression estimate with a uniform weighting function $m_n(\cdot)$ of the unknown conditional mean $m(\cdot)$.

Theorem 3.1. In a kNN regression model with dependent variables and assuming that (X_n) is an α-mixing process as in assumption 3.1, the probability density function meets assumption 3.2, then we have :

(i) The k-NN regression estimate $m_n(\underline{x})$ in relation 4 is asymptotically unbiased

$$E[m_n(\underline{x})] = m(\underline{x}) + O(n^{-\beta}). \quad (10)$$

where $\beta = \frac{(1-Q)p}{d}$ and $0 \leq Q < 1$;

(ii) The variance of the k-NN regression estimate $m_n(\underline{x})$ is given by:

$$Var[(m_n(\underline{x})] = \frac{v(\underline{x})}{k(n)} \quad (11)$$

where $v(\underline{x}) = Var(Y_n/\underline{X}_n = \underline{x})$;

(iii) The quadratic mean square error of the k-NN regression estimate $m_n(\underline{x})$ satisfies

$$E[(m_n(\underline{x}) - m(\underline{x}))^2] = O(n^{-Q}), \quad (12)$$

where $Q = \frac{2p}{2p+d}$.

The basic idea of the proof is as follows. In general, the number of neighbors k depends on the sample size n, denoted $k(n)$. This integer number $k(n)$ is the key unknown parameter of the k-NN regression estimate. Large-sample properties should account appropriate value for this unknown number k. The choice $k = [n^Q]$, with $0 \leq Q < 1$ and [.] the integer part symbol, is frequent for matching the usual conditions ($k = o(n)$, $log(n) = o(k)$ and $k \to \infty$ as $n \to \infty$) for large-sample properties such as for derivation of convergence rate or asymptotic distribution of the k-NN regression estimate. Then, calibration of the number k is equivalent to the characterization of Q. We calibrate the parameter k based on the bias-variance dilemma. Therefore, for dependent processes, we derive in that manner the second order properties of the k-NN regression method.

Proof of Theorem 3.1. We start from providing the proof of (i). When needed ordering of the sample according to its distance from $\underline{x}$ will be considered. We denote $B(\underline{x}, r) = \{z \in \mathbb{R}^d, \|\underline{x} - z\| \leq r\}$ the ball centered at $\underline{x}$ with radius r. If necessary, the samples will be ordered according to their distance from $\underline{x}$. From the definition of $m_n(\underline{x})$, we have:

$$\begin{aligned} E[m_n(\underline{x})] &= E[\sum_{i=1}^{k(n)} \frac{1}{k(n)} Y_i] \\ &= \frac{1}{k(n)} \sum_{i=1}^{k(n)} E[Y_i] \\ &= \frac{1}{k(n)} \sum_{i=1}^{k(n)} E[E(Y_i/\underline{X}_i)] \\ &= \frac{1}{k(n)} \sum_{i=1}^{k(n)} E[m(\underline{X}_i)] \end{aligned} \quad (13)$$

As $\underline{X}_i \in B(\underline{x}, r)$, then expansion around $\underline{x}$ of the $p-$continuously differentiable $m(.)$ yields:

$$E[m_n(\underline{x})] = m(\underline{x}) + O(r^p) \quad (14)$$

The last step is to find r such that $k(n)$ observations fall within the ball $B(\underline{x}, r)$. The idea comes from whether or not a given observation is in the ball. So by denoting $N(r, n)$ the number of observations falling in the ball $B(\underline{x}, r)$, we have $E[N(r, n)] = qn$ with q the probability that they are in the ball $B(\underline{x}, r)$ (a rare situation). If $E[N(r, n)] = k(n)$ then one expects $k(n)$ observations falling in the ball $B(\underline{x}, r)$.

$$q = \frac{k(n)}{n} \tag{15}$$

The probability q is obtained as follows:

$$\begin{aligned} q =& P(\underline{x}_i \in B(\underline{x}, r)) \\ =& \int_{B(\underline{x},r)} f(\underline{x}_i) d\underline{x}_i \\ =& f(\underline{x}) \int_{B(\underline{x},r)} d\underline{x}_i + \int_{B(\underline{x},r)} (f(\underline{x}_i) - f(\underline{x})) d\underline{x}_i \\ =& f(\underline{x}) c r^d + o(r^d), \end{aligned} \tag{16}$$

We get an expression of r from 15 and 16

$$r = \left(\frac{k(n)}{n}\right)^{\frac{1}{d}} D(\underline{x}), \tag{17}$$

with $D(\underline{x}) = \left(\frac{1}{f(\underline{x})c}\right)^{\frac{1}{d}}$. The finiteness of r requires the choice of $k(n)$ as in integer part of n^Q with $0 \leq Q < 1$. Replacing $k(n)$ in relation (17) by its value, we get:

$$\begin{aligned} r =& \left(\frac{n^Q}{n}\right)^{\frac{1}{d}} D(\underline{x}) \\ =& n^{-\frac{(1-Q)}{d}} D(\underline{x}) \end{aligned} \tag{18}$$

Plugging this expression of r in equation (14), we have

$$E[m_n(\underline{x})] = m(\underline{x}) + O(n^{-\frac{(1-Q)p}{d}}) \tag{19}$$

We get the desired result for this special case. For the general situation $q_i = P(\underline{x}_i \in B(\underline{x}, r))$, relation (19) has followed from Lemma 1 in [14].

Next we prove point (ii) of Theorem 3.1 which concerns the variance of the k-NN regression estimate $m_n(\underline{x})$. The variance of $m_n(\underline{x})$ can be written as follows:

$$\begin{aligned} Var(m_n(\underline{x})) =& \sum_{i=1}^{k(n)} Var(\frac{1}{k(n)} Y_i) + \sum_{i=1}^{k(n)} \sum_{j \neq i}^{k(n)} cov(\frac{1}{k(n)} Y_i, \frac{1}{k(n)} Y_j) \\ =& \frac{1}{k(n)^2} \sum_{i=1}^{k(n)} Var(Y_i) + \frac{1}{k(n)^2} \sum_{i=1}^{k(n)} \sum_{j \neq i}^{k(n)} cov(Y_i, Y_j) \end{aligned} \tag{20}$$

By rewriting the equation (20), it becomes:

$$|Var(m_n(\underline{x})) - \frac{1}{k(n)^2} \sum_{i=1}^{k(n)} Var(Y_i)| = |\frac{1}{k(n)} \sum_{i=1}^{k(n)} \sum_{j \neq i}^{k(n)} cov(Y_i, Y_j)| \leq \frac{1}{k(n)^2} \sum_{i=1}^{k(n)} \sum_{j \neq i}^{k(n)} |cov(Y_i, Y_j)| \tag{21}$$

Using the inequality on the covariance bound for α-mixing processes in [16], we have:

$$\begin{aligned} \frac{1}{k(n)} \sum_{i=1}^{k(n)} \sum_{j \neq i}^{k(n)} |cov(Y_i, Y_j)| \leq& \frac{1}{k(n)^2} \sum_{i=1}^{k(n)} \sum_{j \neq i}^{k(n)} \alpha(|i-j|)(E[Y_i^2])^{1/2}(E[Y_j^2])^{1/2} \\ \leq& \frac{1}{k(n)^2} \sum_{i=1}^{k(n)} \sum_{j \neq i}^{k(n)} \alpha(|i-j|)(E[Y_i^2])^{1/2}(E[Y_j^2])^{1/2}) \\ \leq& \frac{1}{k(n)^2} \sum_{i=1}^{k(n)} \sum_{j \neq i}^{k(n)} \alpha(|i-j|) E[Y_i^2] \leq \frac{1}{k(n)^2} \sum_{i=1}^{k(n)} \sum_{j \neq i}^{k(n)} \alpha(|i-j|) Var(Y_i)^{1/2} Var(Y_j)^{1/2} \end{aligned} \tag{22}$$

Combining relations (21) and (22), we get:

$$|Var(m_n(\underline{x})) - \frac{1}{k(n)^2} \sum_{i=1}^{k(n)} Var(Y_i)| \leq \frac{1}{k(n)^2} \sum_{i=1}^{k(n)} \sum_{j \neq i}^{k(n)} \alpha(|i-j|) Var(Y_i)^{1/2} Var(Y_j)^{1/2} \tag{23}$$

Focusing on $Var(Y_i)$, from the law of total variance, we have

$$Var(Y_i) = Var(E(Y_i/\underline{X}_i)) + E(Var(Y_i/\underline{X}_i)) = Var(m(\underline{X}_i)) + E(v(\underline{X}_i)) \tag{24}$$

Since $\underline{X}_i \in B(\underline{x}, r)$, then from taylor expansion around $\underline{x}$ of $m(.)$ and $v(.)$, we get $Var(Y_i) = v(\underline{x})$ within the choice of $k(n)$ as integer part of n^Q. So relation (25) is equivalent to:

$$|Var(m_n(\underline{x})) - \frac{v(\underline{x})}{k(n)}| \leq \frac{v(\underline{x})}{k(n)^2} \sum_{i=1}^{k(n)} \sum_{j \neq i}^{k(n)} \alpha(|i-j|) \tag{25}$$

From assumption 3.1, we get $\sum_{i=1}^{k(n)} \sum_{j \neq i}^{k(n)} \alpha(|i-j|) < \infty$ as n tends to infinity. So $Var(m_n(\underline{x})) - \frac{v(\underline{x})}{k(n)}$ vanishes to zero as n tends to infinity. We have just specified the variance of the estimate m_n.

We now prove (iii) of theorem 3.1. We derive the proof of (iii) from (i) and (ii) of theorem 3.1. We start by writing down the bias-variance decomposition of the quadratic mean square error.

$$E[(m_n(\underline{x}) - m(\underline{x}))^2] = Var(m_n(\underline{x})) + (E[m_n(\underline{x})] - m(\underline{x}))^2. \tag{26}$$

We reconsider the last expression in the early proof that is $Var(m_n(\underline{x})) = \frac{1}{k(n)} v(\underline{x})$ when r is small or $\frac{k(n)}{n}$ tends to zero. We achieved this with the choice of $k(n)$ as the integer part of n^Q. Therefore, it follows:

$$\begin{aligned} Var(m_n(\underline{x})) &= \frac{v(\underline{x})}{n^Q} \\ &= O(n^{-Q}) \end{aligned} \tag{27}$$

We have also the following expression of the bias from proof of point (i):

$$(E[m_n(\underline{x}) - m(\underline{x})])^2 = O(n^{-2\beta}). \tag{28}$$

Plugging equations (27) and (28) inside equation (26), we get $2\beta = Q$ or $Q = \frac{2p}{2p+d}$ and the proof is complete.

The result in theorem 3.1 provides knowledge of the bias and speed rate of the variance of the k-NN regression estimate in dependent case. Such knowledge allows for controling finite sample as well as asymptotic behavior of the estimator. Then, it ensures the use of such regression estimator to time series processes.

4. Conclusion

In summary, we have characterised the bias, variance, and quadratic mean-square error of the k-nearest neighbor (k-NN) regression estimate of the conditional mean under the strongly mixing condition, subject to suitable regularity conditions. Specifically, the quadratic mean square error of the k-NN regression estimate with a uniform weighting function is of order $O(n^{-Q})$.

One strength of our results is that the variance of the k-NN regression estimate does not depend on the true density or the integral of the squared weighting function. This is a common result in the literature, arising from the flexibility of the information set used in k-NN regression [14]. This adds value to the nearest neighbor method for regression tasks compared to popular kernel regression methods, especially for dependent processes.

Our developments have their limits. This is especially the case with regard to the mixing conditions. These make it more difficult to provide illustrations using simulations or real datasets. In addition, our attention has been focused on the uniformly weighted nearest neighbor regression.

There are several ways in which future research could be conducted. One option would be to focus on weakening the dependent conditions. Another option would be to look at the method itself, for example by thinking about a more general weighting scheme.

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Abbreviations

ARMA	Autoregressive Moving Average
GARCH	Generalized Autoregressive Conditional Heteroskedasticity
k-NN	k-Nearest Neighbor
MISE	Mean Integrated Squared Error
MSE	Mean Squared Error
QMSE	Quadratic Mean Squared Error

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Conflicts of Interest

The authors declare no conflicts of interest.

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