



Research Article

Study on a Parameterized Scalar Field Model

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Abstract

In the theory of Brans-Dicke (BD) scalar-tensor gravity, different models have been proposed for different potential functional types, usually by setting the scalar field to the power function form of the cosmic distance factor (power exponent is constant). However, in previous studies, the observational limitation in the solar system range of the coupling parameter (BD coupling parameter) between the BD scalar field and gravity is obtained from negative values to several thousand. In particular, previous studies investigating holographic scalar field models using cosmological holographic principle, in which the BD scalar field is placed in the form of a power function of the cosmic distance factor, have considered the problem of accelerated expansion of the late universe and cosmological coincidence problem, but none of the models have been fully explained. However, the use of a model with the coupling strength of dark matter and dark energy as a non-constant variable requires parameterization, provided that previous studies have been conducted to address the problem of Hubble constant tension and cosmological coincidence. Therefore, in this study, we construct a scalar field model that generalizes the exponent in the expression of the scalar field to a function of the redshift using well-known Jassal-Bagla-Padmanabhan (JBP) parameterization in cosmology, and obtain a cosmological constraint on the BD coupling parameter, in the context of the theory of BD scalar-tensor gravity, not the Einstein's theory of tensor gravity, and show that the accelerating expansion of the late Universe and cosmological coincidence problem are well explained by the proposed model. First, we solve the continuity equation under the assumption that there is no interaction between matter and the scalar field, and obtain an analytical expression for the energy density of the mass and the BD scalar field. Then, using 580 supernova observations, the best-fitting values of the proposed model parameters and the range of values of the parameters at different confidence levels are obtained by minimizing the chi-square function. We have obtained the result that the scalar field decreases, the BD coupling parameter increases, and the current value of BD coupling parameter is around -1.746, from past to present. Also, the coincidence parameter now converges to 1, so that the problem of cosmic coincidence problem is alleviated by the proposed model, and the rate of change of the gravitational constant is considered.

Keywords

Scalar, Coupling, (JBP) Parameterization

1. Introduction

The so-called dark energy that contributes to the cosmological observations including supernovae, cosmic microwave background radiation, baryon acoustic oscillation,

and Hubble parameters, suggest the accelerated expansion of the late universe. [1-7] Within the scope of Einstein's tensor theory of gravity, several dark energy models have

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been proposed, including the cosmological constant model, which often encounter difficulties such as fine-tuning problem and cosmological coincidence problem. [8-11] The Brans-Dicke (BD) scalar theory of gravity is the first historical attempt to extend Einstein's theory of gravity. [12] In this theory, the Newton's gravitational constant is assumed to vary with time, and instead of the gravitational constant, a scalar field is introduced in the action term, where the coupling parameter characterizing the coupling with gravity plays an important role and is called the BD parameter. Because of the importance of this BD parameter, several studies have been made to constraints the use of different observations with different models. Observations of Nordtvedt effect using the Lunar Laser Ranging experiment yielded a slightly smaller value $\omega > 1000$. Observations of orbital period derivative of the airicular white dwarf-neutron star binary system PSR J1012+5307 yielded $\omega > 1250$. [13] On the other hand, the recent estimations of the parameter yield ever negative values and $-407.0 < \omega < 175.89$ at 95% confidence level and a best fit value $\omega = -1.477$ by using SNIa data. [13, 14] Where their explanation of such a wide range of estimation of the parameter ω from minus a few hundreds to a few tens of thousand should be found? On the other hand, parameterization of cosmological quantities such as the parameters of the state equation, the holographic constant in the holographic dark energy model, and the dark matter-dark energy coupling, has shown in several studies the possibility of solving the unsolved problem such as the problem of cosmological coincidence. [15-18] For the universal parameterization in cosmology, we can include Jassal-Bagla-Padmanabhan (JBP) parameterization, Ma-Zhang parameterization, and Wetterich parameterizations. [18-21] Therefore, we will parameterize the BD parameter using JBP parameterization, find the optimal values and ranges of the model parameters using supernova observations, and investigate the parameterized BD scalar gravity theory with cosmic evolution by considering the variation of the scalar field with the redshift. The structure of the paper is as follows. In Section 2, model validation using the basic equations and observational data of the parameterized BD scalar gravity theory is presented, and in Section 3, the relationship between scalar fields and cosmic evolution is investigated.

2. Basic Equation

The action for the BD theory is given by

$$S = \int d^4x \sqrt{-g} \left[\frac{1}{2} (-\phi R + \frac{\omega}{\phi} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi) + L_m \right], \quad (1)$$

where ϕ is a BD scalar field which couples with gravity, R denotes the Ricci scalar curvature, ω is a coupling parameter between gravity and scalar field called BD parameter and L_m represents the matter lagrangian density. [12].

In this paper, we consider a flat Friedmann-Robertson-

Walker (FRW) metric given by the line element.

$$ds^2 = dt^2 - a^2(t)(dx^2 + dy^2 + dz^2), \quad (2)$$

where a denotes the scale factor of the Universe. [12].

The variation of the action (1) with respect to the metric tensor $g^{\mu\nu}$ for the line element (2) with the energy-tensor yield the following Friedman equation.

$$3\phi H^2 = \rho_m + \rho_\phi, \quad (3)$$

where ϕ is a time-dependent scalar field called BD scalar field which coupled with gravity, H denotes the Hubble parameter, ρ_m and ρ_ϕ are the energy density of the matter and scalar field. In the original BD theory of gravity without potential, scalar field and energy density for the scalar field evolves according to relation. [12].

$$\phi = (1+z)^{-\frac{1}{1+\omega}}, \quad (4)$$

$$\rho_\phi = \frac{1}{2} \frac{\omega}{\phi} \dot{\phi}^2 - 3H\dot{\phi}, \quad (5)$$

where z represents the redshift and the over dot denotes the derivative with respect to the cosmic time t . We generalize the Eq (4) and the scalar field as follows:

$$\phi(z) = (1+z)^{\alpha(z)}, \quad (6)$$

where $\alpha(z)$ is generally a function of redshift to be determined as following

$$\alpha(z) = \alpha_0 + \alpha_1 \frac{z}{(1+z)^2}, \quad (7)$$

in which α_0 and α_1 are a dimensionless constants. It is well-known Jassal-Bagla-Padmanabhan (JBP) parameterization. [18] It is obvious that the $\alpha_1 = 0$ case is reduced to the origin scalar field with constant BD coupling parameter. The continuity equations for matter and scalar field, without interaction between them can be written as follows

$$\dot{\rho}_m + 3H\rho_m = 0, \quad (8)$$

$$\dot{\rho}_\phi + 3H(1+w_\phi)\rho_\phi = 0, \quad (9)$$

where $w_\phi = \frac{P_\phi}{\rho_\phi}$ is the equation of state parameter of scalar field. [18] From Eq (8) and Eq (9) the energy density for the matter and BD scalar field is given by

$$\rho_m = \rho_{m0}(1+z)^3, \quad (10)$$

where ρ_{m0} and $\rho_{\phi0}$ are constants. Using Eq (3), (10), (11), we find that

$$\rho_\phi = \rho_{\phi0}(1+z)^{3(1+w_\phi)}, \quad (11)$$

$$\frac{H^2}{H_0^2} \equiv E^2(z) = \Omega_{m0}(1+z)^{3-\alpha_0-\alpha_1\frac{z}{(1+z)^2}} + \Omega_{\phi0}(1+z)^{3(1+w_\phi)-\alpha_0-\alpha_1\frac{z}{(1+z)^2}}, \quad (12)$$

where $\Omega_{m0} \equiv \frac{\rho_{m0}}{3\phi_0 H_0^2}$, $\Omega_{\phi0} \equiv \frac{\rho_{\phi0}}{3\phi_0 H_0^2}$.

In this paper, we constraint observationally the model, using by 580 supernovae observations. [7] In this case the distance module of supernova is defined as

$$\mu_{th}(z) = 5 \log D_L(z) + 42.38 - 5 \log h - \frac{15}{4} \log \phi(z), \quad (13)$$

where $D_L(z) = (1+z) \int_0^z \frac{dz'}{E(z')}$ is a dimensionless luminosity

distance of the supernova, h is the Hubble parameter in units of $100 \text{ km s}^{-1} \text{ Mpc}^{-1}$. We construct the χ^2 - function as follows

$$\chi^2_{SN} = \sum_{i=1}^{580} \frac{(\mu_{th}(z_i) - \mu_{ob}(z_i))^2}{\sigma_i^2}, \quad (14)$$

where $\mu_{th}(z_i)$, $\mu_{ob}(z_i)$ and σ_i are the theoretical distance module, the observational distance module and error of the observation for the $\mu_{ob}(z_i)$ of the supernova, respectively. Minimizing the χ^2 - function parameters of the model are determined.

3. Conclusion

The obtained best fitting value of parameters and the contours of the parameters at the different confidence levels are shown in Figure 1.

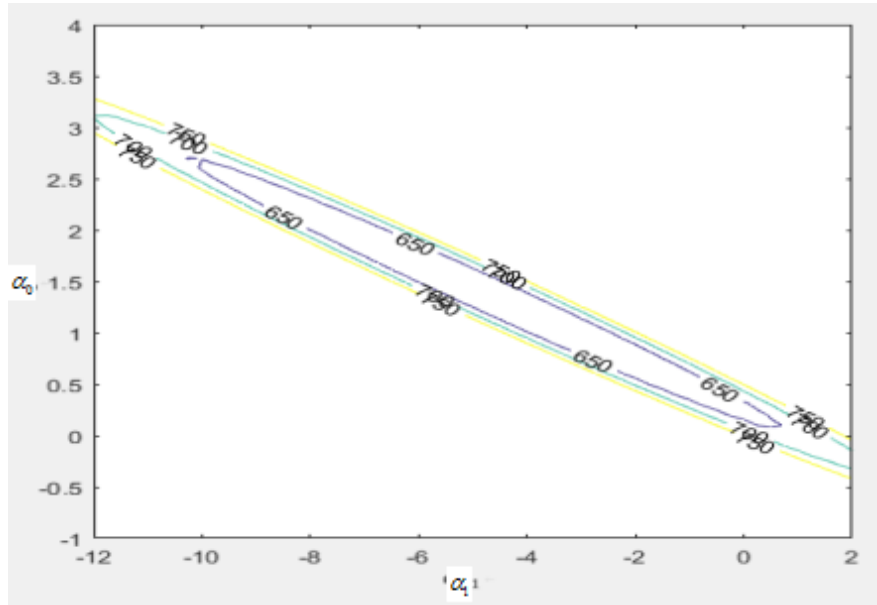


Figure 1. The contours of the parameters (α_0, α_1) at the different confidence levels ($\Omega_{m0} = 0.267$, $\Omega_{\phi0} = 0.723$).

From Eq (3) and Eq (7), we get

$$\omega = -1 - \frac{(1+z)^2}{\alpha_0(1+z)^2 + \alpha_1 z}. \quad (15)$$

The variation of the BD parameter ω for the best fitting value of the parameters (α_0, α_1) with redshift z is shown in Figure 2.

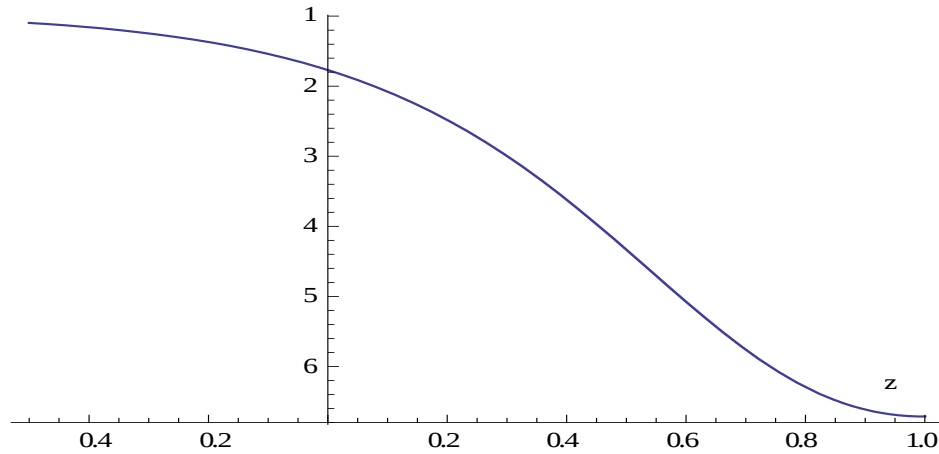


Figure 2. The variation of the BD parameter with redshift ($\alpha_0 = 1.3$, $\alpha_1 = -4.5$).

It can be seen from Figure 2 that the BD parameter shows an increasing trend from past to present. The present time, i.e. $z = 0$ correspond to the BD parameter $\omega = -1.746$. A variation of the

scalar field ϕ for the best fitting value of the parameters (α_0, α_1) with redshift z is shown in Figure 3.

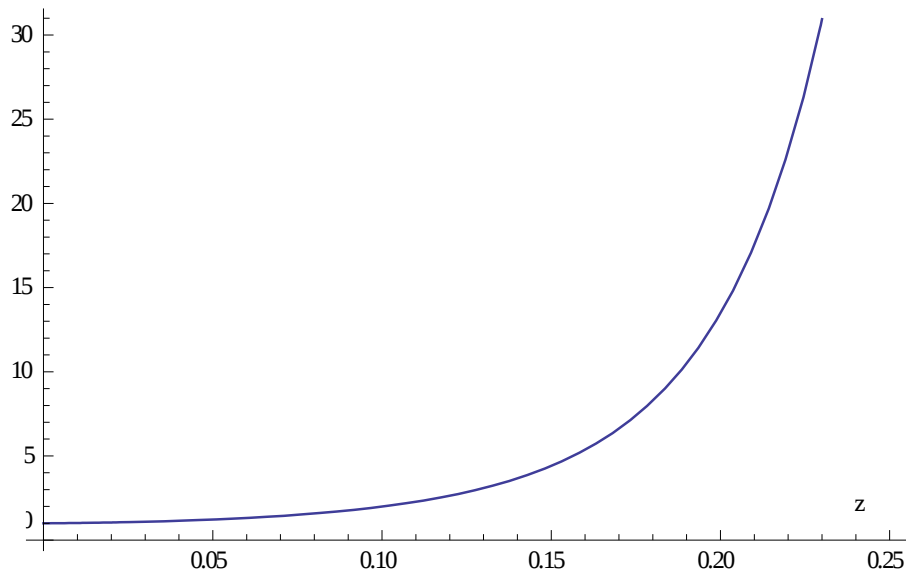


Figure 3. The variation of the scalar field with redshift ($\alpha_0 = 1.3$, $\alpha_1 = -4.5$).

From the Figure 3, we can see that the scalar field decreases from the past to the present. It is also found that the BD scalar field has decreased rapidly in the past and has become almost constant recently. This shows that the BD scalar theory of gravity is true in the past description of cosmic evolution, but now the Einstein's tensor theory is valid. This is in agreement with the previous results. [22-24] This means that the scalar

field plays a fundamental role in cosmic evolution. On the other hand, using the Eq (3)- Eq (6) we get the energy density for the BD scalar field as follows

$$\rho_\phi = H^2 \phi A, \quad (16)$$

$$A = \left(\alpha'(z) \ln(1+z) + \frac{\alpha(z)}{1+z} \right) \left[-\frac{1}{2} \left(1 + \frac{1}{\alpha(z)} \right) (1+z)^2 \left(\alpha'(z) \ln(1+z) + \frac{\alpha(z)}{1+z} \right) + 3(1+z) \right]. \quad (17)$$

Substituting Eq (3) and Eq (9) to the Eq (15), the energy

density of the BD scalar field is rewritten

$$\rho_\phi = \frac{A}{1-A} \Omega_{m0} (1+z)^3 H_0^2. \quad (18)$$

From Eq (3) and (16) the energy density ratio called the coincidence parameter r is written as follows

$$r = \frac{3}{A} - 1. \quad (19)$$

From Eqs. (17) and (19), it can be seen that when the red shift z goes to zero, i.e., the consistency parameter at the present time becomes

$$z \rightarrow 0, A \rightarrow \frac{\alpha_0}{2} (5 - \alpha_0), \quad (20)$$

$$z \rightarrow 0, r \rightarrow O(1). \quad (21)$$

This is in good agreement with previous studies [25, 26]. Therefore, we conclude that the late time accelerating universe is explained by the model with parameterized BD parameter. On the other hand, the rate of change of G obtained by our model is

$$\left| \frac{\dot{G}}{G} \right| \sim \left| \frac{\dot{\phi}}{\phi} \right| = (1+z) \left[\alpha' \ln(1+z) + \frac{\alpha}{1+z} \right] H.$$

From the above expression, we can see that the rate of change of G at the current point is only related to α_0 , which is

$$\left| \frac{\dot{G}}{G} \right| \sim \left| \frac{\dot{\phi}}{\phi} \right|_{z=0} = \alpha_0 H_0 = 9.295 \times 10^{-11} \text{ yr}^{-1},$$

where we have taken $H_0 = 70 \text{ km/s} \cdot \text{Mpc} = 7.15 \times 10^{-11} \text{ yr}^{-1}$.

Using the same observation as ranging and Doppler data from the Mars Global Surveyor, Mars Odyssey, Mars Reconnaissance Orbiter, and other spacecraft, Ref. [27] obtain a strong

observational constraint: $\frac{\dot{G}}{G} < 7.5 \times 10^{-14} \text{ yr}^{-1}$. In addition,

Ref. [28] use big bang nucleosynthesis to obtain the following restrictions: $\frac{\dot{G}}{G} \approx 10^{-13} \text{ yr}^{-1}$. Our result are much different

from this. Using other cosmological observations such as the Hubble parameter, we can obtain a more reliable constraint on

$\left| \frac{\dot{G}}{G} \right|$. In the present model, the cosmic coincidence problem

has been alleviated more effectively. Future work on varying BD parameter will be more interesting.

Abbreviations

BD	Brans-Dicke
JBP	Jassal-Bagla-Padmanabhan

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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