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Research Article

Dynamic Evolution of Quantum Correlations with PSO Algorithm in the Three-qubit Heisenberg XYZ Model Considering DM and KSEA Interactions

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Abstract

In this paper, we consider the three-qubit Heisenberg XYZ model considering DM interaction and KSEA interaction. We use the concurrence, geometric quantum discord, and local quantum uncertainty between the first and third spin as quantum correlation measures. In the absence of a magnetic field and in the presence of a constant magnetic field, quantum correlations appear sudden death and birth, and exhibit a random behavior. In summary, the Heisenberg XYZ model, no adjustment of the exchange interaction strength, DM interaction strength, KSEA interaction strength, and external magnetic field can eliminate the sudden death and births in the evolution of quantum correlations. Thus, we use the particle swarm optimization algorithm (PSO) to determine the magnetic field strength at each time step to increase the quantum correlation. The numerical simulation results show that the quantum correlation increases and reaches a maximum value by our method, and the quantum correlation remains constant at a maximum even after the external magnetic field is cancelled. This result shows that, for any exchange interaction strength, DM interaction strength, and KSEA interaction strength, the quantum correlations reach a certain value and remain at their maximum after the magnetic field is quenched, once the time-varying magnetic field is properly designed. This result provides sufficient possibilities for the use of Heisenberg spin chains as quantum channel.

Keywords

Concurrence, Geometric Quantum Discord, Local Quantum Uncertainty, Particle Swarm Optimization (PSO), Heisenberg XYZ Model

1. Introduction

Quantum entanglement plays an important role in quantum information theory as a special type of quantum correlation and has been widely applied in many quantum information tasks [5-8], such as quantum computing [1], quantum communication [2, 3], quantum key distribution [4]. However, recent studies show that there are some separable quantum states with quantum correlation even in the absence of quantum entanglement [9-12]. Thus, in [12] quantum discord was introduced by quantum correlation, and in [13] quantum mismatch was shown to be more robust than quantum entanglement in dispersive systems.

In addition, entropy quantum discord [14] and geometric

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quantum discord [15] have been proposed as measures of quantum correlation. Here, entropy quantum discord is defined as the difference between quantum interaction information and classical information, and geometric quantum discord (GQD) is defined as the minimum distance between a given state and zero discord state. Although geometric quantum discord is computationally easier and intuitive than entropy quantum discord, it cannot be a suitable measure to show non-classical correlations.

To solve such problems, the local quantum uncertainty (LQU) was introduced in [16]. Quantifying the local quantum uncertainty is easier than computing other measures such as quantum discord, and it was shown that the quantum correlation measured by LQU is consistent with the quantum correlation measured by quantum mismatch and exceeds the entanglement in Ref. [17]. Therefore, we use the concurrence, geometric quantum discord and local quantum uncertainty as measures of quantum correlation.

To use the Heisenberg spin chain as a quantum channel, many studies have been carried out on the dynamic evolution of quantum correlations such as geometric discord [12, 18-21], local quantum uncertainty [22, 23] and concurrence [21, 24-27] in this model. In addition, many studies have been carried out to further incorporate the DM interaction and KSEA interaction considering spin-orbit coupling into the Heisenberg model [28-36]. Also, the dynamic evolution of quantum correlations under homogeneous or inhomogeneous magnetic fields is widely studied. For example, in Ref. [28], investigating the quantum correlation of the Heisenberg XXZ spin chain with DM interaction under sinusoidal magnetic fields, it was shown that the quantum correlation can be strongly modulated by changing the magnetic field strength and magnetic field inhomogeneity of the external magnetic field.

However, all previous studies have considered the application of homogeneous and inhomogeneous fields to space. In addition, previous work has considered the evolution of quantum correlations under the action of a changing magnetic field, which is represented by a simple cosine function, rather than a field to increase quantum correlation even for a time-varying field. However, no study has been carried out to consider both DM and KSEA interactions in the three-qubit Heisenberg XYZ model, especially under the influence of time-varying magnetic fields, and the dynamic evolution of quantum correlations has not been considered. Therefore, in this paper, we consider the three-qubit Heisenberg XYZ model considering the DM interaction (D_x, D_y, D_z) and KSEA interaction ($\Gamma_x, \Gamma_y, \Gamma_z$), and in particular the dynamic evolution of quantum correlation when applying a varying magnetic field to increase quantum correlations.

First, in Section 2, we introduce the three-qubit Heisenberg XYZ model with DM interaction, KSEA interaction and X-axis magnetic field and calculate the Hamiltonian of the model. Then, three measures for estimating quantum correlation and PSO algorithm used in control magnetic field design are introduced in this paper. Then, in Section 3, we consider the influence of several parameters on the dynamic evolution of quantum correlations under constant magnetic field (including zero magnetic field). We also consider the dynamic evolution of quantum correlations under the action of the designed control field and the external control field design by the particle swarm optimization algorithm. Finally, we give the results of this paper.

2. Model and Measure of Quantum Correlation

2.1. Model

In this paper, we consider a three-qubit Heisenberg XYZ model with DM interaction and KSEA interaction in the presence of a time-varying external magnetic field, which can be expressed as:

$$
\begin{aligned}
&H=\sum_{i=1}^{N}\left[J_i(\sigma_i^x\sigma_{i+1}^x+\sigma_i^y\sigma_{i+1}^y+\gamma\sigma_i^z\sigma_{i+1}^z)+B_x(t)\sigma_i^x+B_z(t)\sigma_i^z+\mathrm{D}(\sigma_i\times\sigma_{i+1})+\sigma_i\Gamma\sigma_{i+1}\right]\\
&\mathrm{D}(\sigma_i\times\sigma_{i+1})=D_x(\sigma_i^y\sigma_{i+1}^z-\sigma_i^z\sigma_{i+1}^y)+D_y(\sigma_i^x\sigma_{i+1}^z-\sigma_i^z\sigma_{i+1}^x)+D_z\left(\sigma_i^x\sigma_{i+1}^y-\sigma_i^y\sigma_{i+1}^x\right)\\
&\sigma_i\Gamma\sigma_{i+1}=\Gamma_x(\sigma_i^y\sigma_{i+1}^z+\sigma_i^z\sigma_{i+1}^y)+\Gamma_y(\sigma_i^x\sigma_{i+1}^z+\sigma_i^z\sigma_{i+1}^x)+\Gamma_z(\sigma_i^x\sigma_{i+1}^y+\sigma_i^y\sigma_{i+1}^x)
\end{aligned}
\tag{1}
$$

where $\sigma_i^\alpha\left(\alpha=x,y,z\right)$ is the Pauli operator acting on the i th spin, and J_x, J_y, J_z represents the exchange interaction in the x, y and z directions. D_x, D_y, D_z and $\Gamma_x, \Gamma_y, \Gamma_z$ are the DM interaction and KSEA interaction factors acting in the X, Y, Z direction, respectively, and $B_x(t)$ is the time-dependent magnetic fields directed in the x direction. Positive J_x, J_y, J_z describes antiferromagnetic and negative J_x, J_y, J_z describes ferromagnetic. We divide the Hamiltonian of the system into a magnetic field-free part, i.e., an internal Hamiltonian and a controlled Hamiltonian, for control. Then, when the internal Hamiltonian of the system is expanded to a basis $\{|000\rangle,|001\rangle,|010\rangle,|011\rangle,|100\rangle,|101\rangle,|110\rangle,|111\rangle\}$, the internal Hamiltonian of the system and the control Hamiltonian of the x-axis can be written in matrix form as:

$$H_0 = \begin{pmatrix} 3J_z & 2\Gamma_y - 2i\Gamma_x & 2\Gamma_y - 2i\Gamma_x & J_x - J_y - 2i\Gamma_z & J_x - J_y - 2i\Gamma_x & J_x - J_y - 2i\Gamma_z & J_x - J_y - 2i\Gamma_z & 0 \\ 2\Gamma_y + 2i\Gamma_x & -J_z & J_x + J_y + 2iD_z & -2D_y + 2iD_x & J_x + J_y - 2iD_z & 2D_y - 2iD_x & 0 & J_x - J_y - 2i\Gamma_z \\ 2\Gamma_y + 2i\Gamma_x & J_x - J_y - 2iD_z & -J_z & 2D_y - 2iD_x & J_x + J_y + 2iD_z & 0 & -2D_y + 2iD_x & J_x - J_y - 2i\Gamma_z \\ J_x - J_y + 2i\Gamma_z & -2D_y - 2iD_x & 2D_y + 2iD_x & -J_z & 0 & J_x - J_y + 2iD_z & J_x - J_y - 2iD_z & -2\Gamma_y + 2i\Gamma_x \\ 2\Gamma_y + 2i\Gamma_x & J_x + J_y + 2iD_z & J_x - J_y - 2iD_z & 0 & -J_z & -2D_y + 2iD_x & 2D_y - 2iD_x & J_x - J_y - 2i\Gamma_z \\ J_x - J_y + 2i\Gamma_z & 2D_y + 2iD_x & 0 & J_x + J_y - 2iD_z & -2D_y - 2iD_x & -J_z & J_x + J_y + 2iD_z & -2\Gamma_y + 2i\Gamma_x \\ J_x - J_y + 2i\Gamma_z & 0 & -2D_y - 2iD_x & J_x + J_y + 2iD_z & 2D_y + 2iD_x & J_x + J_y - 2iD_z & -J_z & -2\Gamma_y + 2i\Gamma_x \\ 0 & J_x - J_y + 2i\Gamma_z & J_x - J_y + 2i\Gamma_z & -2\Gamma_y - 2i\Gamma_x & J_x - J_y + 2i\Gamma_z & -2\Gamma_y - 2i\Gamma_x & -2\Gamma_y - 2i\Gamma_x & 3J_z \end{pmatrix} \quad (2)$$

$$H_x = \begin{pmatrix} 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 \end{pmatrix} \quad (3)$$

$$H = H_0 + B_x(t)H_x \quad (4)$$

The density matrix of the system at time t can be delineated as follows:

$$\rho(t) = -i[H_0 + H_x(t), \rho(t)] \quad (5)$$

For the quantum system in Eq. (5), its dynamics can also be described by a unitary operator $U(t)$ where $U(t)U^+(t) = U^+(t)U(t) = I$ as

$$\dot{U}(t) = -i(H_0 + B_x(t)H_x)U(t) = -iH(t)U(t)$$

with $U(0) = I$. Given an initial state $\rho(0) = \rho_0$, the quantum state at time t, $\rho(t)$, can always be written as

$$\rho(t) = U(t)\rho_0 U^+(t), \quad t \geq 0$$

2.2. The Measures of Quantum Correlation

We use the concurrence [28], geometric quantum discord [37] and local quantum uncertainty [38] as a measure to evaluate quantum correlation, and these quantities are defined as follows.

$$C_{ij} = \max\left\{0, \sqrt{\lambda_1} - \sqrt{\lambda_2} - \sqrt{\lambda_3} - \sqrt{\lambda_4}\right\} \quad (6)$$

where λ_k, $k = 1,2,3,4$ denotes the eigenvalues of the matrix

$$R_{lm} = \rho_{lm}\left(\sigma_1^y \otimes \sigma_2^y\right)\rho_{lm}^*\left(\sigma_1^y \otimes \sigma_2^y\right) \quad (7)$$

and is in decreasing order.

The 2-norm geometric quantum discord used in Ref. [37] is defined as:

$$D_2(\rho) = \min_{\chi \in \Omega_0} \|\rho - \chi\|^2 \quad (8)$$

where Ω_0 is the set of zero-discord states $\chi = \Sigma_j p_j |\psi_j\rangle\langle\psi_j| \otimes \rho_j$. Also, $|\psi_j\rangle (j = 1,2)$ is two orthogonal bases of subsystem A, and ρ_j is two-dimensional density matrices of subsystem B. And $\|\rho - \chi\|^2 = Tr(\rho - \chi)^2$ is a 2-norm in Hilbert-Schmidt space.

Another measure of quantum correlation used in this paper is the local quantum uncertainty, which is defined as follows [38].

$$U(\rho) = 1 - \max[\lambda_1, \lambda_2, \lambda_3] \quad (9)$$

where $\lambda_i (i = 1,2,3)$ is the eigenvalues of the matrix M, and the elements of this matrix are

$$m_{ij} = tr\left\{\sqrt{\rho}(\sigma^i \otimes I)\sqrt{\rho}(\sigma^j \otimes I)\right\} \quad (10)$$

where $\sigma^i (i = 1,2,3)$ is Pauli matrices.

$$\sigma^1 = \sigma^x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \sigma^2 = \sigma^y = \begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix}, \sigma^3 = \sigma^z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad (11)$$

2.3. Particle Swarm Optimization Algorithm

In 1995, scientists proposed a particle swarm optimization (PSO) algorithm from the foraging process of a bird swarm. The algorithm uses the concept of population intelligence evolution, and each particle adjusts its own flight speed and direction based on its own self-optimality (*pbest*) and global optimality (*gbest*). When birds visit their habitat, all birds are in a blind flight until one bird finds its habitat. If birds find safer and more comfortable habitats, they immediately leave the site and fly to better habitats to form new swarm. Birds accelerate the speed of search through sharing information about the location between members of the flock.

The basic principles of the particle swarm optimization algorithm are as follows:

Step 1: Randomly select a particle swarm. Thus, we randomly choose the number of particles n, the initial value of each particle's position $x(i)$, and the initial value of velocity $v(i)$.

Step 2: Set a suitable estimate and calculate the estimate $f(x(i))$ of each particle.

Step 3: We compare the estimated value of each particle $f(x(i))$ with $f(p(i))$. If $f(x(i)) > f(p(i))$, update the current $f(p(i))$ to $f(x(i))$ and replace $p(i)$ by $x(i)$.

Step 4: Similarly, we compare $f(x(i))$ and $f(g(i))$ for each particle $x(i)$. If $f(x(i)) > f(p(g))$, replace the current $f(p(g))$ and $f(x(i))$ and replace $p(g)$ by $x(i)$.

Step 5: For each particle, we update the position and velocity by the following equation.

$$v_i(t+1) = wv_i(t) + c_1r_1(pbest_i(t) - x_i(t)) + c_2r_2(gbest(t) - x_i(t)) \tag{12}$$

$$x_i(t+1) = x_i(t) + v_i(t+1) \tag{13}$$

Step 6: Finally, we judge whether the final condition is satisfied. If satisfied, then the algorithm is terminated and the optimal solution is output. Otherwise, we go to step 2. Here, the optimal control condition is usually set to the fitness value or the maximum number of iterations.

3. The Dynamic Evolution of Quantum Correlations

3.1. Dynamic Evolution of Quantum Correlation in the Non-magnetic Field

The initial state of the system is W-like, as follows:

$$\begin{aligned} &\rho(0)=\frac{1-r}{8}I_8 + r|\psi\rangle\langle\psi|, \\ &|\psi\rangle = \sin\phi|001\rangle + \sin\phi|010\rangle + \sqrt{1-2\sin^2\phi}|100\rangle \quad (0 \le \phi \le \pi/4) \end{aligned} \tag{14}$$

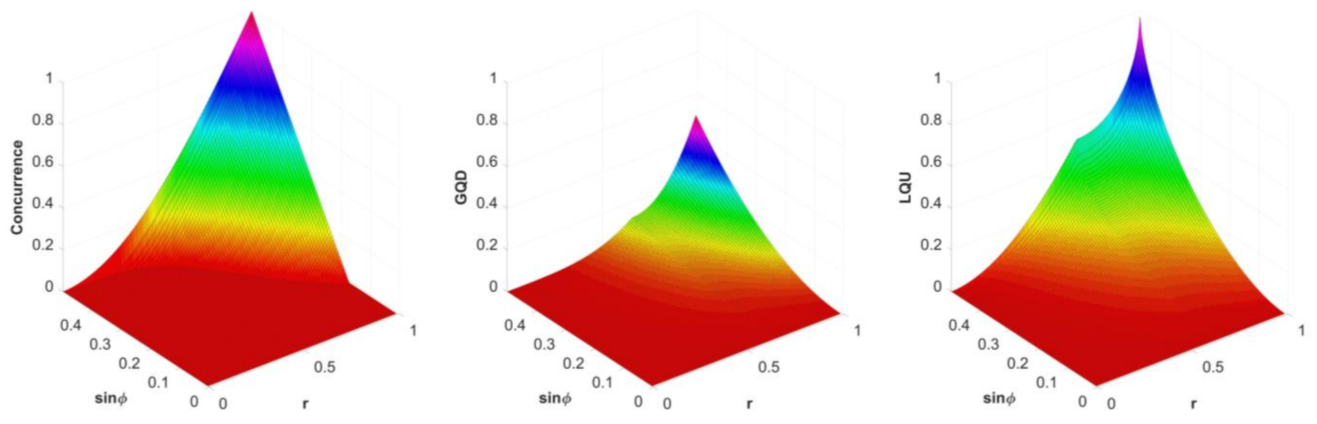

Figure 1. *The concurrence, GQD and LQU with* $(r, \sin\phi)$.

When $\sin\phi = \sqrt{1/3}$, the initial state of the system is the W state, the state of maximum entanglement in the three-spin system. First, the concurrence, GQD and LQU of initial state with $(r, \sin\phi)$ are shown in Figure 1.

As shown in Figure 1, both the concurrence and the LQU reach maximum 1 when $r = 1, \sin\phi = \sqrt{1/2}$. But, in this case the maximum value of the geometric mismatch is 0.5. However, even in the region where the concurrence is zero, the GQD and LQU have a certain value and the value of LQU is larger. Thus, even in a separable state, the GQD and LQU have certain values, which means that these quantities are more accurate measures of quantum correlation.

In this paper, we consider the effects of the intensity of the exchange interaction, the intensity of the DM interaction, the intensity of the KSEA interaction and the magnetic field on the evolution of the quantum correlations in the case of $r = 0.9, \sin\phi = 1/2$.

When an external magnetic field is not applied, the density matrix and the time evolution operator of the system at time t can be written as:

$$\rho(t) = U(t)\rho(0)U^{+}(t), U(t) = \exp(-iH_0 t) \tag{15}$$

We first consider the influence of exchange interaction strengths J_x and J_y, and the results are shown in Figure 2. All other parameters are set to zero.

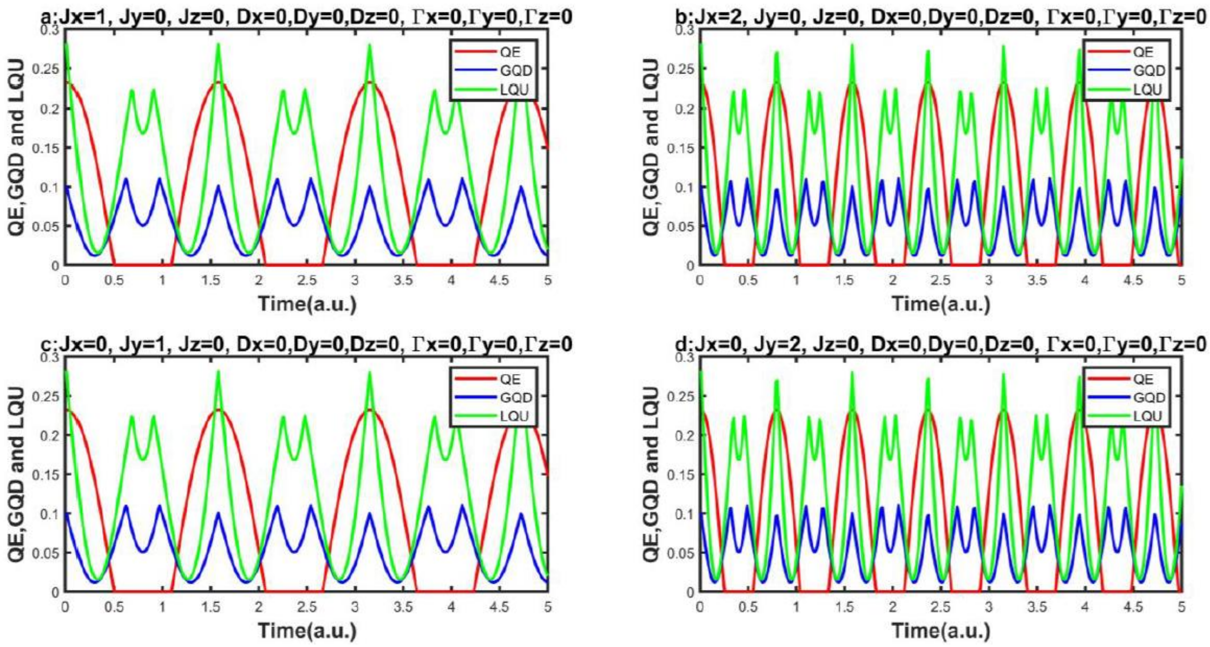

Figure 2. *Quantum correlations vs time for various of exchange interaction.*

As can be seen in Figure 2, the GQD and the LQU have a constant value even for zero concurrence values and exhibit periodic behavior. Also, the increase in the values of J_x and J_y does not affect the maximum of the quantum correlations, but only changes the period. Thus, the greater the strength of the exchange interaction, the smaller the period, which is consistent with the results of previous studies. Also, in the case of only one of the x- or y-direction exchange interactions, the concurrence appears to sudden death and birth during evolution, whereas the GQD and LQU do not appear to sudden death and birth during evolution.

The simulation results for the simultaneous presence of the X-axis and Y-axis exchange interactions are shown in Figure 3. As can be seen in Figure 3, the dynamic evolution of quantum correlations for $J_x = 1, J_y = 2$ and $J_x = 2, J_y = 1$ has the same result. However, for $J_x = 1, J_y = 1$ and $J_x = 2, J_y = 2$, there is no change in the maximum, only a short period, and the concurrence value does not appear to sudden death and birth during evolution, and it has a larger value than GQD. However, in the case of $J_x \neq J_y$, both the quantum correlations, concurrence and GQD and LQU, are sudden death and birth. This is a different result from the XXZ model considering the Heisenberg spin chain.

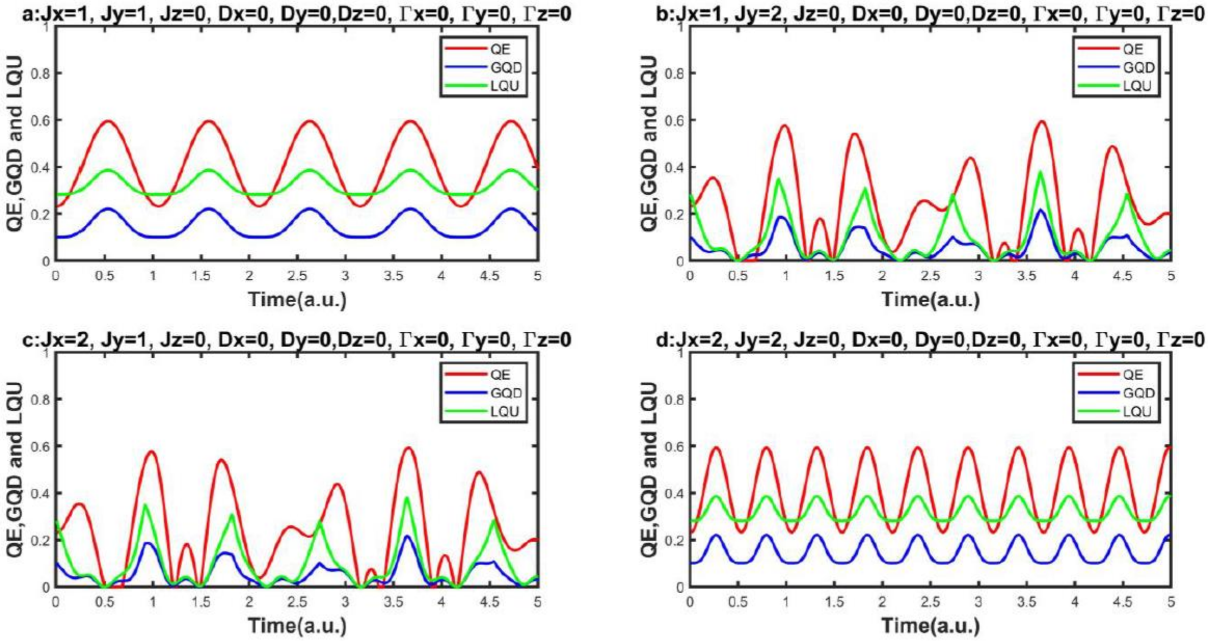

Figure 3. The dynamics evolution of the concurrence, GQD and LQU vs time.

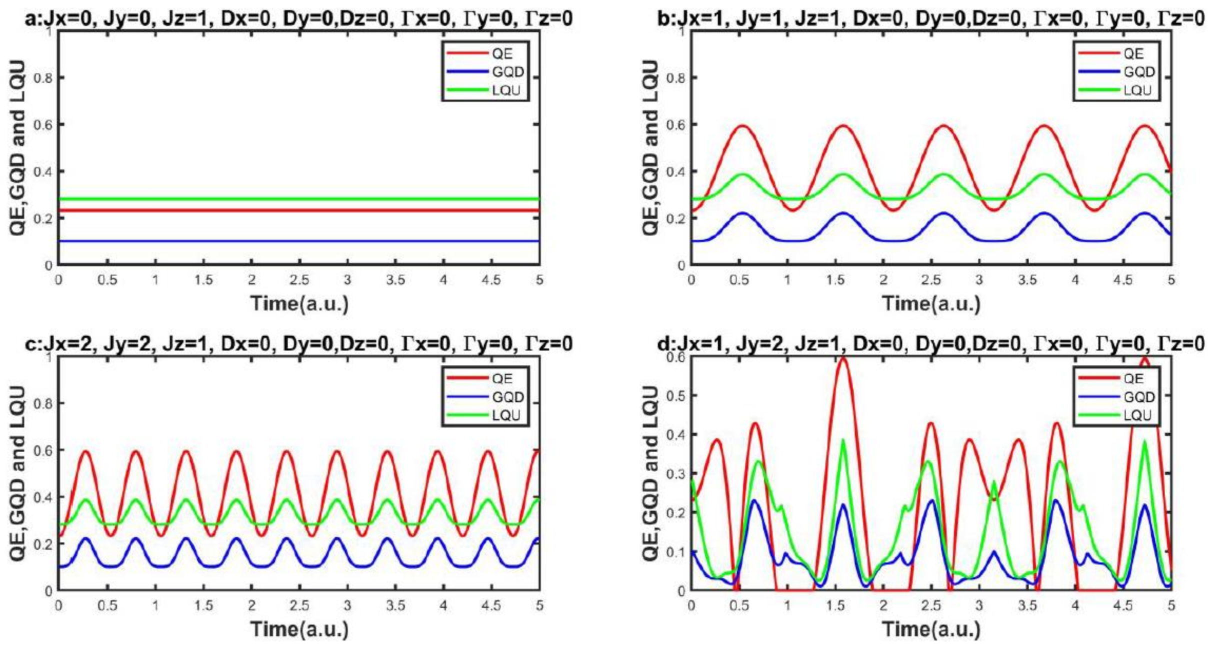

Figure 4. The dynamics evolution of the concurrence, GQD and LQU vs time.

Next, the influence of the exchange interaction in the z-direction is considered and this result is shown in Figure 4. As shown in Figure 4, when $J_x = J_y = 0$ and $J_x = J_y = 1, J_x = J_y = 2$, the exchange interaction along the z-axis has no effect on the evolution of the quantum correlation, given by Eq. (13), for the initial state. Figure 4-b is in perfect agreement with Figure 3-a and Figure 4-c with Figure 3-d. However, as shown in Figure 4-d, when $J_x \neq J_y$, the quantum correlations will behave in a disordered manner and do not affect the maximum value of the quantum correlation. Compared with Figure 3-b, the concurrence shows sudden death and birth, but the GQD and LQU do not exhibit sudden death and birth. Also, the numerical simulations show that, however large the intensity of the exchange interaction in the z-direction, the GQD and LQU do not vanish, although the value is small. It can also be seen that the period of sudden and birth in the concurrence is shorter than that of the case without considering the intensity of exchange interaction in the z-direction (Figure 3b).

This result is only the case when the initial state is given by Eq. (13), and in other cases, although $J_x = J_y$, J_z affects the evolution of quantum correlations, leading to chaotic behavior, shows sudden death and birth. For example, in the case of initial state $|\psi\rangle = \sqrt{1/2}\left(|110\rangle + |000\rangle\right)$, although $J_x = J_y$, both the concurrence, GQD, and LQU shows sudden death and birth.

Next, we consider the effect of DM interaction on quantum correlations.

When the DM interaction in the z-direction is not considered, i.e., $D_z = 0$, the effects of D_x and D_y are the same as the effects of J_x and J_y on the exchange interaction. Therefore, we consider the effect of DM interaction in two ways. Thus, in comparison with the XXZ model and the XYZ model, we first consider the influence of and show this result in Figure 5.

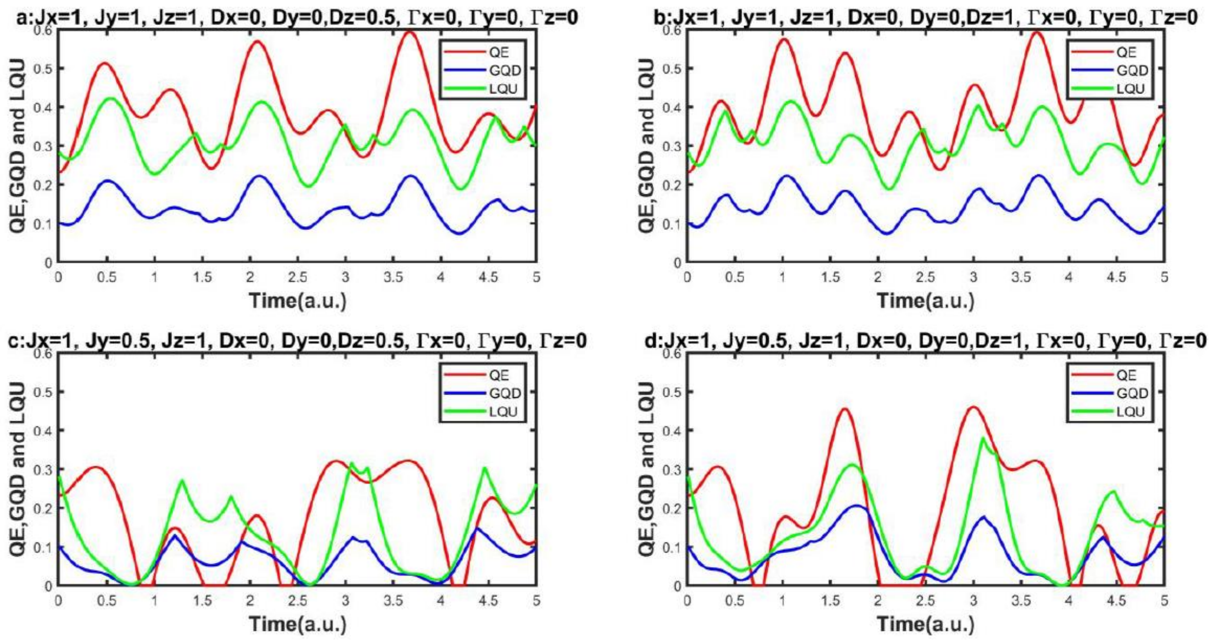

Figure 5. *The dynamics evolution of the concurrence, GQD and LQU vs time.*

Comparing Figure 5-a with Figure 5-b and Figure 4-b, it can be seen that D_z acts to reduce the minimum of the quantum correlation. Thus, in Figure 4-b, without considering DM interactions, we see that quantum correlations such as concurrence GQD, and LQU oscillate and evolve without falling below the initial value. However, in the case of DM interaction, it can be seen that the minimum drops below the initial quantum correlation. However, in the XXZ model with $J_x = J_y$, no sudden death or birth occurs, however large D_z is. However, as can be seen in Figures 5-c and 5-d, the quantum correlations in the XYZ model show sudden death and birth in evolution.

In Figure 6, we consider the effect of D_x and D_y on the dynamics of quantum correlation, with $D_z = 0$. When $D_z = 0$, the individual effects of D_x and D_y are the same. Thus, the evolution of quantum correlation in the case of $J_x = 1, J_y = 1, J_z = 1, D_x = a, D_y = 0, D_z = 0$ and $J_x = 1, J_y = 1, J_z = 1, D_x = 0, D_y = a, D_z = 0$ $(a = const)$ has the same character. Therefore, the effects of D_x and D_y on the XXZ and XYZ models are considered. As can be seen in Figure 6, the Heisenberg XXZ model does not generate sudden death and birth even considering the DM interaction of the x- and y-axes. However, under the influence of D_x and D_y, quantum correlation in the XYZ model is expected to occur rapidly during evolution. Also, during evolution, the quantum correlation becomes smaller than the maximum value in the XXZ model.

Next, the evolution of quantum correlation in the case of considering KSEA interaction is shown in Figure 7.

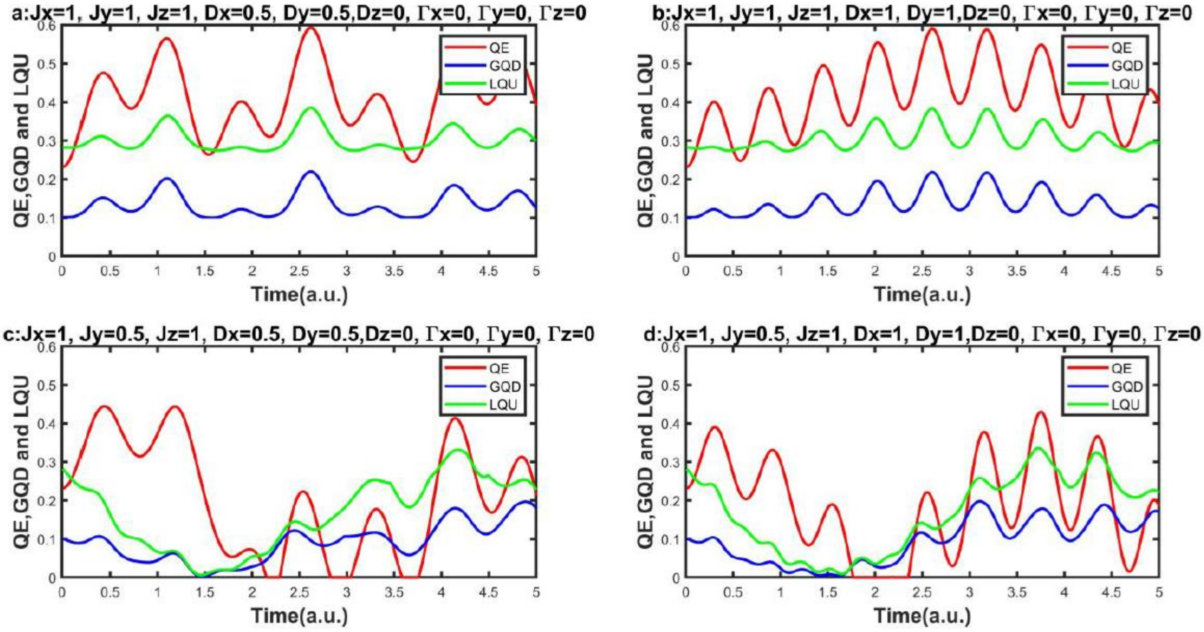

Figure 6. *The dynamics evolution of the concurrence, GQD and LQU vs time.*

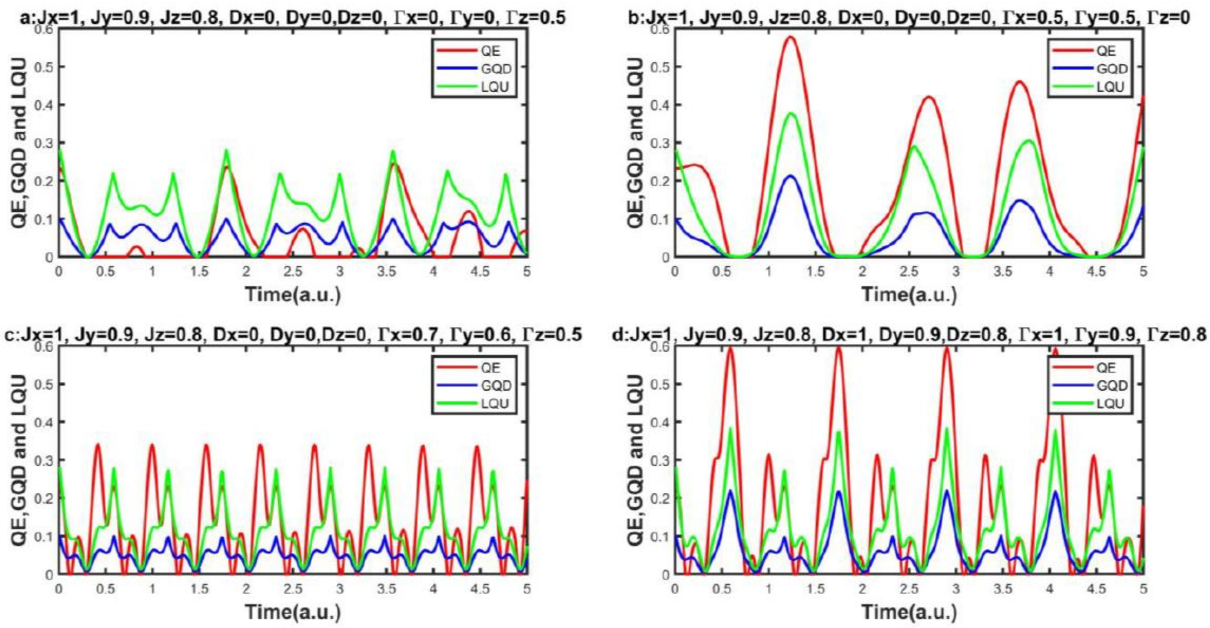

Figure 7. *The dynamics evolution of the concurrence, GQD and LQU vs time.*

Figure 7-a shows the dynamic evolution of the quantum correlation with only the z-direction KSEA interaction, and Figure 7-b considers the dynamic evolution with only the x- and y-direction KSEA interaction. Also, Figure 7-c shows the dynamic evolution of the quantum correlation when considering all the KSEA interactions in the x, y, and z directions, and Figure 7-d, when considering both the DM and KSEA interactions in the x, y, and z directions.

As can be seen from Figures 7-a, 7-b, and 7-c, the quantum correlation, whether considering the KSEA interaction separately or together, is expected to occur rapidly during evolution. Also, in the case of only considering the KSEA interaction in the z-direction, the maximum value of the quantum correlations is smaller than that of the case considering the KSEA interaction in the x- and y-direction.

Overall, in the case of considering both the x-, y-, and z-direction DM and the KSEA interactions in the Heisenberg XYZ spin chain, the quantum correlations appear sudden death and birth and evolve chaotically, as shown in Figure 7-d.

Even when a homogeneous and inhomogeneous magnetic field is applied to spin chain, quantum correlations evolve into chaotic behavior, and sudden death and birth occur during evolution (Figure 8). Figure 8-a shows the dynamic behavior of quantum correlations when all three spins are subjected to a homogeneous constant magnetic field B. Figure 8-b shows the dynamic evolution of quantum correlations when the first spin is subjected to a constant magnetic field B+b and the third spin is subjected to a constant magnetic field B-b.

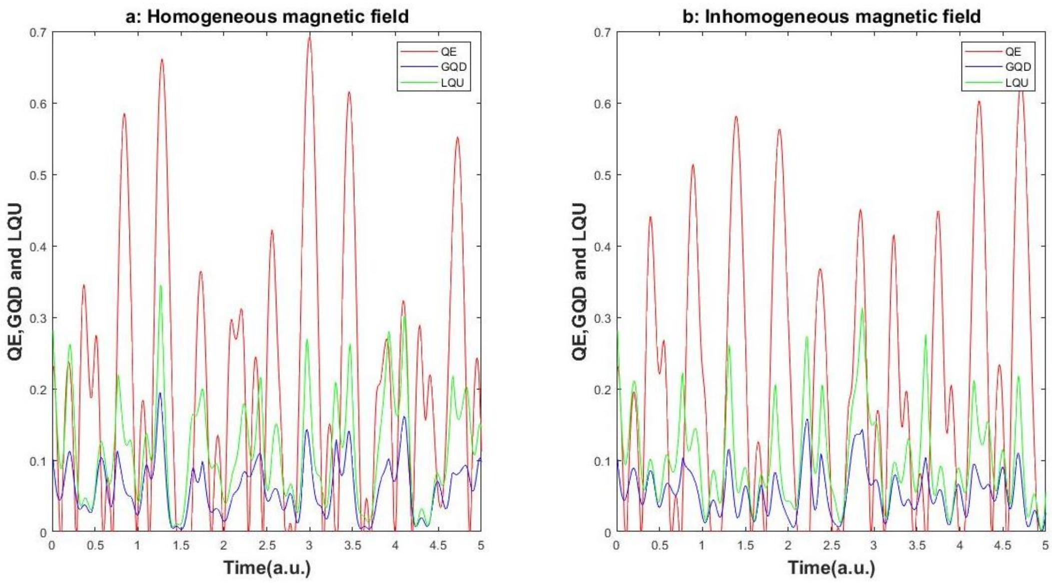

Figure 8. The dynamics evolution of the concurrence, GQD and LQU vs time.

(a: $J_x = 1, J_y = 0.9, J_z = 0.8, D_x = 1, D_y = 0.9, D_z = 0.8, \Gamma_x = 1, \Gamma_y = 0.9, \Gamma_z = 0.8, B = 1$)

(b: $J_x = 1, J_y = 0.9, J_z = 0.8, D_x = 1, D_y = 0.9, D_z = 0.8, \Gamma_x = 1, \Gamma_y = 0.9, \Gamma_z = 0.8, B = 1, b = 0.2$)

As can be seen from Figure 8, the Heisenberg XYZ model exhibits sudden death and birth, and random behavior under the action of homogeneous and inhomogeneous magnetic fields.

3.2. The Dynamic Evolution of Quantum Correlations Under Time-dependent Magnetic Fields

We apply a time-dependent magnetic field to keep the quantum correlations constant. In other words, at each time of evolution, we design a magnetic field by considering the intensity of the magnetic field as a virtual particle and evolving these particles using a particle swarm optimization algorithm so that quantum correlations can always increase without disturbing behavior.

The method of determining the magnetic field strength at each time point based on the particle swarm optimization algorithm is as follows:

Step 1: At the initial time, we randomly select n virtual particle swarm. Thus, the initial values $B_x(i), i = 1, \ldots, n$ and $v(i), i = 1, \ldots, n$ of the magnetic field strength of each particle are randomly chosen within the maximum allowable strength.

Step 2: A suitable estimation function is chosen to measure the quantum correlation for the state evolved for each particle. We choose the local quantum uncertainty (LQU) as an estimation function.

Step 3: For each particle $B_x(i), i = 1, \ldots, n$, a comparison with the self-optimum (pbest) is made. If $LQU(B(i)) > LQU(p(i))$, replace the current $LQU(B(i))$ by $LQU(p(i))$ and replace $B_x(i)$ by $p(i)$.

Step 4: Similarly, we perform a comparison with the swarm optimum (*gbest*) for each particle. If $LQU(B(i)) > LQU(p(g))$, then $LQU(B(i))$ is replaced by $LQU(p(g))$.

Step 5: According to Eqs. (11) and (12), the velocity and position of each particle are updated.

Step 6: Finally, we judge whether the estimation value is in accordance with the criterion. If satisfied, stop the iteration and output the optimal solution. If not satisfied, go to step 2 and repeat. The criterion here is generally the maximum number of iterations, or the requirement for the designed estimation value. The gbest value at each time instant is the control field we want to find, i.e., the magnetic field at each time instant.

In this paper, we consider the dynamic evolution of quantum correlations, including local quantum uncertainty under different exchange interaction strengths, DM interaction strengths, and KSEA interaction strengths, to verify the effectiveness of the proposed method. In other words, we choose the local quantum uncertainty as an estimation function to design an x-axis magnetic field so that this value increases without chaotic behavior, and we consider together the dynamic evolution of other quantum correlations such as the concurrence and the geometric quantum discord during such state evolution.

Figure 9 show the dynamic evolution of quantum correlations (QE, GQD, LQU) under time-dependent magnetic fields (Figure 10) under different exchange interaction strengths, DM interaction strengths, and KSEA interaction strengths, and the designed magnetic field. In the numerical simulations, to investigate the effect of the exchange interaction strength on the dynamic evolution of quantum correlation, numerical simulations were carried out in order of fixing the DM and KSEA interactions strength, increasing and decreasing the x, y, and z direction exchange interaction strength.

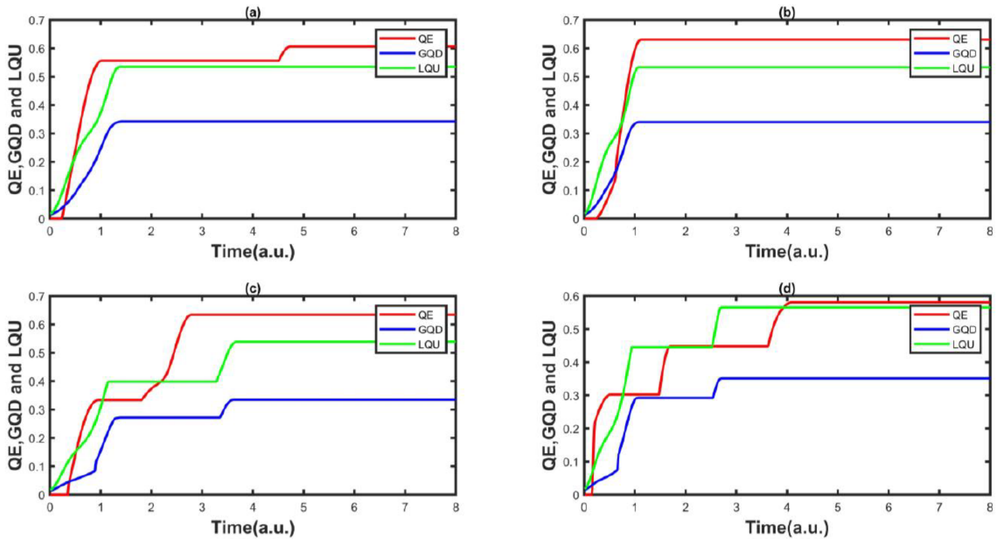

Figure 9. *Dynamic evolution of quantum correlation vs time.*

(a: $J_x = 0.5, J_y = 0.6, J_z = 0.7, D_x = 0.5, D_y = 0.6, D_z = 0.7, \Gamma_x = 0.5, \Gamma_y = 0.6, \Gamma_z = 0.7$)

(b: $J_x = 0.7, J_y = 0.6, J_z = 0.5, D_x = 0.7, D_y = 0.6, D_z = 0.5, \Gamma_x = 0.7, \Gamma_y = 0.6, \Gamma_z = 0.5$)

(c: $J_x = 1, J_y = 0.9, J_z = 0.8, D_x = 1, D_y = 0.9, D_z = 0.8, \Gamma_x = 1, \Gamma_y = 0.9, \Gamma_z = 0.8$)

(d: $J_x = 0.8, J_y = 0.9, J_z = 1, D_x = 0.8, D_y = 0.9, D_z = 1, \Gamma_x = 0.8, \Gamma_y = 0.9, \Gamma_z = 1$)

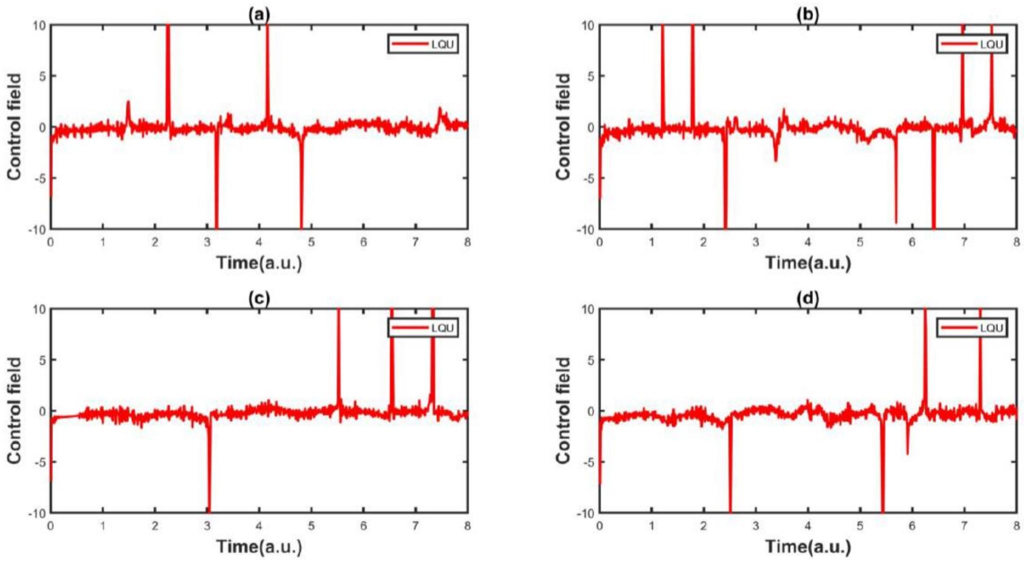

Figure 10. *Control fields.*

As can be seen in Figure 9, quantum correlations are kept constant after a certain time. It can also be seen that the case of $J_x < J_y < J_z$ reaches a maximum faster than the case of $J_x > J_y > J_z$, and this result is similar to the result of Ref. [39]. Only in Ref. 39, they considered the dynamic evolution of entanglement when applying an inhomogeneous magnetic field to space, and in this paper, we applied a homogeneous to space and inhomogeneous magnetic field to time. It can also be seen that the larger the exchange interaction strength, the larger the maximum value of quantum correlations.

4. Conclusion

In this paper, we consider the dynamic evolution of quantum correlation in the three-qubit Heisenberg XYZ model including DM and KSEA interaction. The results show that the intensity of the exchange interaction in the x- and y-directions (J_x, J_y) does not affect the maximum of the quantum correlations, but only the period (Figure 2). However, when considering J_x and J_y together, the quantum correlations in the XXZ model do not appear sudden death and birth during evolution, but sudden death and birth in the XYZ model (Figure 3).

When J_z is considered, in the XXZ model, J_z has no effect on the evolution of quantum correlations (Figure 4-b=Figure 3-a and Figure 4-c=Figure 3-d), but in the XYZ model, it has an effect and thus leads to a disordered behavior (Figure 4-d). Also, in the concurrence, sudden death and births are generated, but in GQD and LQU, although the value is very small, they are not zero (Figure 4-d).

In addition, considering the effect of DM interaction, the XXZ model ($J_x = J_y$) does not generate sudden death and birth in quantum correlations LQU and GQD (Figure 5-a and Figure 5-b), but in the XYZ model, sudden death and birth occur (Figure 5-c and Figure 5-d).

This result is also same for the KSEA interaction (Figure 7).

Overall, the Heisenberg XYZ model, no adjustment of the exchange interaction strength, (J_x, J_y, J_z) DM interaction strength (D_x, D_y, D_z), KSEA interaction strength ($\Gamma_x, \Gamma_y, \Gamma_z$), and external magnetic field (B) can eliminate the sudden death and births in the evolution of quantum correlations.

Thus, we apply a time-varying magnetic field designed using a particle swarm optimization algorithm to increase the quantum correlations at each time of evolution. The result shows that the quantum correlations reach a maximum faster for $J_x < J_y < J_z$ than the case of $J_x > J_y > J_z$ and the larger the intensity of the exchange interaction, the larger the maximum of the quantum correlations.

Abbreviations

DM Interaction	Dzyaloshinskii-Moriya Interaction
KSEA Interaction	Kaplan-Shekhtman-Entin-Wohlman-Aharony Interaction
GQD	Geometric Quantum Discord
LQU	Local Quantum Uncertainty
PSO	Particle Swarm Optimization

Conflicts of Interest

The authors declare no conflicts of interest.

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