



Research Article

Magnetic Properties of One-dimensional Helical Spin System with Isotropic Spin Exchanges Under the Effect of DM and KSEA

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Abstract

In the present work, we report on a theoretical investigation of the magnetoelectric parameters of the helical Heisenberg multiferroic spin chain model. The spin-wave approximation in the bosonization process is used to evaluate the energy spectrum of the quadratic form of the model as that of regular quantum gases. The quantized-form of the model is conveniently treated using the canonical ensemble in terms of the free energy developed by the use of Landau theory. Following the Fermi-Dirac statistics of quantum gases, the joint effect of Dzyaloshinskii-Moriya (DM) and Kaplan-Shekhtman-Entin-Wohlman-Aharony (KSEA) interactions on the magnetoelectric properties of the helical multiferroic spin structure controlled by a static electric field in the y-direction and a magnetic field in the z-direction is quantified. The magnetization, susceptibility, and electric polarization are used as measurable parameters. The work convincingly establishes that the juxtaposition of KSEA interactions to DM interactions is worthy since, on the one hand, it amplifies the magnetic property of the system, and on the other hand makes it possible to control the phase transition dynamics induced in such materials by the symmetric inversion due to the DM interactions. At low temperature, the measurable parameters are closely related to ferroelectricity and ferromagnetism in multiferroic materials, where all effects favor the symmetrization mechanism and hence magnetoelectric properties. These properties are critical in spintronics and information storage control.

Keywords

Helical Spin, Isotropic Spin, Magnetoelectric, DM Interaction, KSEA Interaction, Quantum Phase Transition, Spin-orbit Coupling

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1. Introduction

Spin-orbit interactions in spin chains have been a long-standing subject of condensed matter research, particularly in spintronics. These interactions with distinct characteristics are generally structured as confinement potentials, with the antisymmetric interactions of Dzyaloshinskii and Moriya (DM) being the most studied [1, 2]. This antisymmetric exchange interaction can explain, among other properties, the weak ferromagnetism observed in antiferromagnetic crystals with non-symmetric magnetic characteristics and unusual spin alignment [3]. The DM interactions are surface interactions that cause the skyrmion effect (spin swirl), which is typical of spiral and sinusoidal spin topologies [4]. They contribute to better storage and transport quality in spintronics. They are employed as semiconductors to control transition dynamics [5, 6]. The magnetoelectric meta-effect has been demonstrated in multiferroic and antiferromagnetic materials thanks to the antisymmetric characteristic of the DM interaction, which causes symmetric inversion under thermal stress or applied fields [7-9]. The exchange interactions between spins are frequently described using the Heisenberg model. However, the Heisenberg model alone is insufficient for investigating the consequences of spin-orbit interactions, such as the DM interaction caused by spin-orbit coupling.

The DM interaction induces spin canting, which results in the formation of cycloidal spin spirals in antiferromagnetic systems, and it can induce a magnetoelectric effect in multiferroic ferroelectrics. However, research into spin dynamics in one-dimensional multiferroic materials revealed the presence of additional spin-orbit interactions with symmetric features, known as Kaplan-Shekhtman-Entin-Wohlman-Aharony (KSEA) interactions [10-13]. Their impact should not be underestimated, especially when researching nanostructured materials.

The KSEA and DM interactions can be used in information processing and transit to manipulate (control) spin states in spin systems [14]. The symmetry property of the KSEA term opposes the inversion symmetry caused by DM interactions, which are responsible for the phase transition and phase transition dynamics that contribute to system instability (metastability) [9]. Given the increased interest in developing new materials with novel properties and the recognized importance of these interactions in spin chains, research into multiferroic materials with DM and KSEA interactions is required. Multiferroic systems are materials that have at least two distinct ferroic orders [15-17]. Because of the significant interaction between ferroic orders, multiferroics have received a lot of attention. This is because an external electric or magnetic field can be used to manipulate magnetization (or polarization). In addition to strong coupling between distinct ferroic orders, multiferroic materials can also feature isotropic, anisotropic, symmetric, antisymmetric, and superexchange interactions as state coupling [18]. Understanding and controlling these many

forms of interactions is typically utilized to improve the properties (conduction, storage, thermal, etc.) of these materials in preparation for anticipated technological usage. It may also necessitate changing the structure of a compound or complex to preserve the system's features, making it undeniably beneficial for processing and conveying quantum information [19-21].

Significant breakthroughs in multiferroicity have been made recently due to its magnetoelectric (ME) characteristics. These findings clearly demonstrate the importance of magnetoelectricity and multiferroicity in spintronics and condensed matter physics. Multiferroic materials with ME coupling, in particular, are a subset of materials whose reaction to an external electric or magnetic field is responsible for the control of electronic devices [22, 23]. Because of its different domain walls, this class is distinguished by permanent magnetization, which is useful for the storage of quantum information [20]. The quantum information storage phenomenon has also gained momentum with the inclusion of spin-orbit interactions [24]. Spin-orbit coupling causes both symmetric (KSEA interactions) and antisymmetric (DM interactions) anisotropy in the exchange coupling between nearest neighbor spins. A DM interaction vector perpendicular to the layer results in an easy-plane spin anisotropic Hamiltonian for the honeycomb lattice, breaking the model's SU(2) symmetry and reducing it to U(1) symmetry around the axis perpendicular to the honeycomb plane. The DM interaction, which results from spin-orbit coupling, is critical for magnetoelectricity in many multiferroics and heterostructures [25, 26].

Many authors reported on spin-orbit interaction in spin systems, showing that a symmetric anisotropy term of the form $\vec{S}\vec{A}\vec{S}$ can also be obtained. However, Moriya [2] believed that this factor would be minor when compared to the antisymmetric anisotropy term of DM. More recently, it was reported that if the microscopic model that underpins spin-spin interactions has an SU(2) spin symmetry, this assumption is incorrect [12, 13, 27]. The effective spin-spin interactions exhibit the same SU(2) invariance as the microscopic model. The symmetric term ensures that the spin anisotropy generated by the DM component is correctly compensated. This class of symmetric anisotropy terms must be combined with the DM, as indicated by KSEA [12, 13, 27]. However, it has been demonstrated that this non-negligible symmetric helical interaction can explain the weak ferromagnetism in La_2CuO_4 . As a result, the symmetric helical interaction may be easily identified as KSEA type [28]. The effect of KSEA interaction has also been demonstrated experimentally using neutron diffraction and inelastic neutron scattering studies. The indirect KSEA interaction between two local spins can be manufactured and regulated by manipulating the topological insulator and the system when Rashba spin-orbit coupling is not in equilibrium [29, 30]. Thus, studying the effects of KSEA interaction in a spin system is intriguing. The magnetization results show that Yb_4As_3 [30],

$\text{K}_2\text{V}_3\text{O}_8$ [31], La_2CuO_4 [32], and $\text{Ba}_2\text{CuGe}_2\text{O}_7$ [28] have KSEA interaction.

A multiferroic quantum spin system with DM interactions has previously been studied in terms of thermodynamics and ME [31-33]. The Lanczos numerical technique was used to investigate the impact of DM and KSEA interactions on the dimerization of a spin-Peierls system [34]. The purpose of this study is to look at the combined impact of DM and KSEA interactions on ME multiferroics with anisotropic crystals.

2. Materials and Methods

To investigate the joint effect of the DM and KSEA interaction on the magnetic properties of a helical spin structure of a multiferroic system, we consider a vibration state of a Heisenberg spin-1/2 chain with nearest neighbor (NN) and next nearest neighbor (NNN) at low temperature, driven by the electric field in the y-direction and the magnetic field in the z-direction. Given the above discussion, we suggest the typical model Hamiltonian, which is built on the anisotropic 1D dimerized Heisenberg spin chain, as follows [35].

$$H = H_{ex} + H_z + H_{SIA} + H_{Pol} + H_{DM} + H_{KSEA} \quad (1)$$

with

$$H_{ex} = \sum_i (J_x S_i^x S_{i+1}^x + J_y S_i^y S_{i+1}^y) + \sum_i (J'_x S_i^x S_{i+2}^x + J'_y S_i^y S_{i+2}^y) \quad (2)$$

the spin-exchange model Hamiltonian part, where J_x , J'_x , and, J_y , J'_y are the interaction parameters in the x- and y-directions, respectively, i.e., the intrachain Heisenberg interaction parameter with nearest and NNN responsible for the helical structure in the spin system,

$$H_z = -\Omega \sum_i S_i^z \quad (3)$$

the Zeeman energy, with Ω the transverse magnetic field,

$$H_{SIA} = -\Delta \sum_i (S_i^z)^2 + K \sum_i \left((S_i^x)^2 - (S_i^y)^2 \right) \quad (4)$$

the contribution of the single ion-anisotropy (SIA) in the three directions of space, Δ , and, K being the magnetocrystal-line anisotropy constants [7],

$$H_{POL} = -\mathbf{E} \cdot \mathbf{P} \quad (5)$$

the electric polarization part, with $\mathbf{P}_i = \frac{1}{N} \sum_j [\mathbf{e}_{ij} \times (\mathbf{S}_i \times \mathbf{S}_{i+1})]$ a net electric polarization well described by the spin-current

model and found under spiral and conical spin structures [24, 36], where $\mathbf{e}_{ij}$ is the propagation direction of the spin spiral and $\mathbf{S}_{ij}$ the moments on neighboring spins. $\mathbf{E}(0, \zeta, 0)$ is the external electric field. The sum is over the sites $i \in [0, N[$, with N the number of spins per unit volume,

$$H_{DM} = D_z \sum_i (S_i^x S_{i+1}^y - S_i^y S_{i+1}^x) \quad (6)$$

denotes the DM interaction due to the antisymmetric exchange interactions of spins in the plane (with D_z , the DM factor taken along the z-direction),

$$H_{KSEA} = \Gamma_z \sum_i (S_i^x S_{i+1}^y + S_i^y S_{i+1}^x) \quad (7)$$

The KSEA interaction term is associated with amplitude Γ_z .

The model Hamiltonian described in Eq. (1), with the term DM (D_z) in charge of SU (2) symmetry breaking on its own, which includes NN spins and NNN, has been shown to be suitable to explain meta-magnetoelectric excitations due to its significant polarization in the ferroelectric phase and strong ME coupling [35]. In the absence of KSEA interaction, this model has been utilized to accurately represent the physical properties of CuGeO_3 [37, 38]. However, in systems with full SU (2) symmetry, the DM and KSEA interactions are inextricably coupled [8, 13, 28]. The symmetric nature of the KSEA interaction, in turn, maintains the antisymmetric nature of the DM interaction and leads to magnetic domains and ME and ferroelectric phase transitions. To avoid any ambiguity, the KSEA term is also taken in the same z-direction as DM.

In order to reduce the model characterized by the SU (2) algebra, where the spin vectors are linked to the Pauli matrices by $\mathbf{S}_i = \frac{\hbar}{2} \boldsymbol{\sigma}_i$ ($\boldsymbol{\sigma}_i^\alpha$ the Pauli matrix defined at site i in the three space directions $\alpha = x, y, z$) into the solvable model, the spin wave theory, based successively on the Jordan-Wigner (JW) transformations, Fourier transformations and thus Bogoliubov transformations, in the adiabatic limit, is used in this work as a diagonalization procedure. The NNN interactions, represented by a set of four fermions in JW representations, can result in 2l-fermions vertices for spin exchanges between NN of rank 1. The system deals with non-vanishing four-fermion order terms implying that the NNN term turns the model into deep paramagnetic phase for all positive value of Ω where all spins are oriented in the direction of the transverse field such that $\langle S^z \rangle = 1$ (i.e. $1 - 2C_{i+1}^\dagger C_{i+1} = -1$).

Following the methods used in [39-42], the JW transformation provides an exact mapping between one-dimensional fermions and hardcore bosons, spin-1/2 magnetic moments.

Given that, Eq. (1) can be reduced using the Bogoliubov transformations in the following from [7]:

$$H = \sum_q \omega_q \left(\eta_q^+ \eta_q - \frac{1}{2} \right) - \frac{N}{2} \left(\frac{\Delta}{2} + \Omega \right) \quad (8)$$

with

$$\omega_q = \left(\left(J \cos q - J' \cos 2q + \left(\frac{ND_z - \zeta}{N} \right) \sin q - \Omega \right)^2 + \Gamma_z^2 \sin^2 q \right)^{1/2} \quad (9)$$

the energy spectrum. η is the Bogoliubov quasiparticle expressed in terms of the Fourier transform operator $c_q = \sigma_i^- \left[\prod_{j \leq i-1} (-\sigma_j^z) \right]$ and q be the wave numbers in the range $-\pi \leq q \leq \pi$.

3. Results and Discussions

In this section, we present the cumulative influence of DM and KSEA interactions on the nature of the ME parameters of the multiferroic XXZ spin chain model, as determined from the XYZ spin chain model in Eq. (1). Single-phase magnetoelectric multiferroics, as studied in this paper, are a subset of multiferroics that maintain direct coupling between magnetic and electrical order when compared to composite multiferroics such as piezoelectric and magnetostrictive layers, which have multiple phases [7]. This type of material preserves the material's magnetic characteristics while sacrificing electrical

conductivity, guaranteeing advancing spintronics and significant information storage.

Historically, in multiferroic crystals, the polarization P and magnetization M are related to the electric ζ and magnetic fields Ω by the following relations [15]:

$$P_i(\zeta, \Omega) = -\frac{\partial F}{\partial \zeta_i} = P_i^s + \epsilon_0 \chi_{ij}^e \zeta_j + \alpha_{ij} \Omega_j + \dots \quad (10)$$

and

$$M_i(\zeta, \Omega) = -\frac{\partial F}{\partial \Omega_i} = M_i^s + \mu_0 \chi_{ij}^m \Omega_j + \alpha_{ij} \zeta_j + \dots \quad (11)$$

where P_i^s and M_i^s are the spontaneous polarization and magnetization components, respectively, ϵ_0 and μ_0 are the electrical permittivity and magnetic permeability constants in the vacuum, χ_{ij}^e and χ_{ij}^m are the second-order tensors of electric and magnetic susceptibilities of the substance. ζ_i and Ω_i are the electric field and magnetic field components in the medium. α_{ij} are the tensors used to determine the magnitude of the linear ME effect in the material.

Following the traditional description of the single-phase crystal ME effect [15], the free energy F of the system with applied fields (magnetic and electric), can be expressed in Landau theory as

$$F(E, \Omega) = F_0 - P_i^s \zeta_i - M_i^s \Omega_i - \frac{1}{2} \epsilon_0 \chi_{ij}^e \zeta_i \zeta_j - \frac{1}{2} \mu_0 \chi_{ij}^m \Omega_i \Omega_j - \alpha_{ij} \zeta_i \Omega_j - \dots \quad (12)$$

where F_0 is the ground state free energy.

Following the Landau theory in the Fermi-Dirac statistics, the thermal averages are determined in the spin-wave approximation as for ordinary quantum gases, and the model is treated in the canonical ensemble for the partition function associated with Hamiltonian Eq. (8) given by:

$$Z = \text{Tr} \exp \{-H\} = \exp \left\{ \frac{N\mathcal{G}}{2} \left(\frac{\Delta}{2} + \Omega \right) \right\} \prod_q \exp \left(\frac{\mathcal{G}\omega_q}{2} \right) \prod_q (1 + \exp(-\mathcal{G}\omega_q)) \quad (13)$$

from which the free energy is obtained:

$$F = -T \ln Z = -T \left(\frac{N\mathcal{G}}{2} \left(\frac{\Delta}{2} + \Omega \right) + \frac{\mathcal{G}}{2} \sum_q \omega_q \right) - \frac{1}{\beta} \sum_q \ln(1 + \exp(-\mathcal{G}\omega_q)) \quad (14)$$

where $\mathcal{G} = 1/K_B T$ (T is the absolute temperature and $K_B \approx 1$ is the Boltzmann constant).

In the consideration above, Eq.(14) is used as the measurement of uncertainty in the phase of Gibbs, from which the

magnitude of the thermodynamic effects is characterized using ME coefficients:

$$M = \frac{1}{2} \left\{ N - \sum_q A_q \cdot (1 - 2B_q) \right\} \quad (15)$$

$$\chi^m = \frac{1}{T} \sum_q \frac{T}{2\omega_q} (1 - A_q^2) (1 - 2B_q^2) + A_q^2 B_q^2 e^{\frac{\omega_q}{T}} \quad (16)$$

$$P = -\frac{1}{2N} \sum_q \left\{ A_q \cdot (1 - 2B_q) \sin q \right\} \quad (17)$$

$$\chi^e = \Phi \chi^m = \frac{\sin^2 q}{N^2} \chi^m \quad (18)$$

where $A_q = \frac{J \cos q - J' \cos 2q + \left(\frac{ND - \zeta}{N}\right) \sin q - \Omega}{\omega_q}$ and $B_q = \frac{1}{1 + e^{\frac{\omega_q}{T}}}$ are respectively a dimensionless energy and the number of particles with energy ω_q defined according to Fermi-Dirac statistics at thermal equilibrium. Φ the phase factor linking χ^e to χ^m . The coefficients M , χ^m , P ,

and χ^e are respectively the magnetization, magnetic susceptibility, electric polarization and the electric susceptibility of the system.

The behavior of the magnetoelectric parameters with respect to applied external field constraints are depicted in Figures 1 to 8 by varying the values of spin-orbit interaction factors D_z and Γ_z .

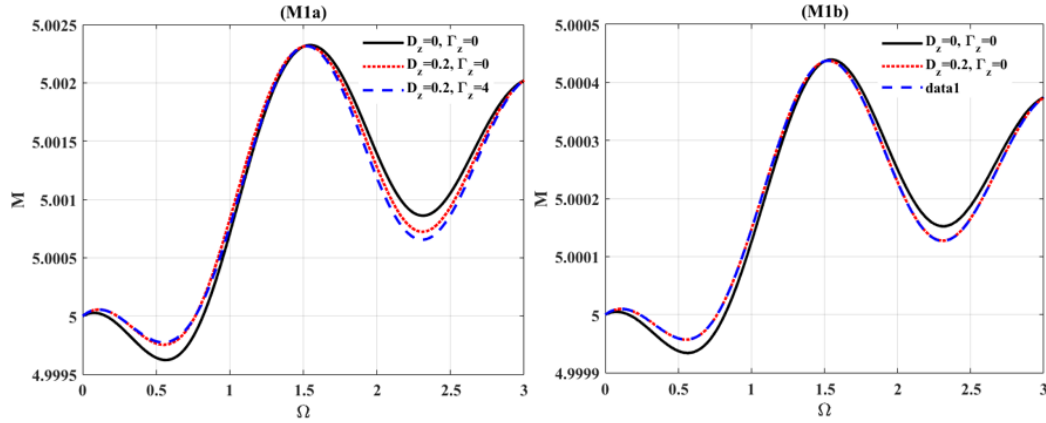


Figure 1. Magnetic field-dependent magnetization $M(\Omega)$ measured under the joint effect of DM and KSEA interaction, with $T = 3.5$ in (M1a) and $T = 20$ in (M1b). The other parameters are $\zeta = 1.5$ and $N = 1000$.

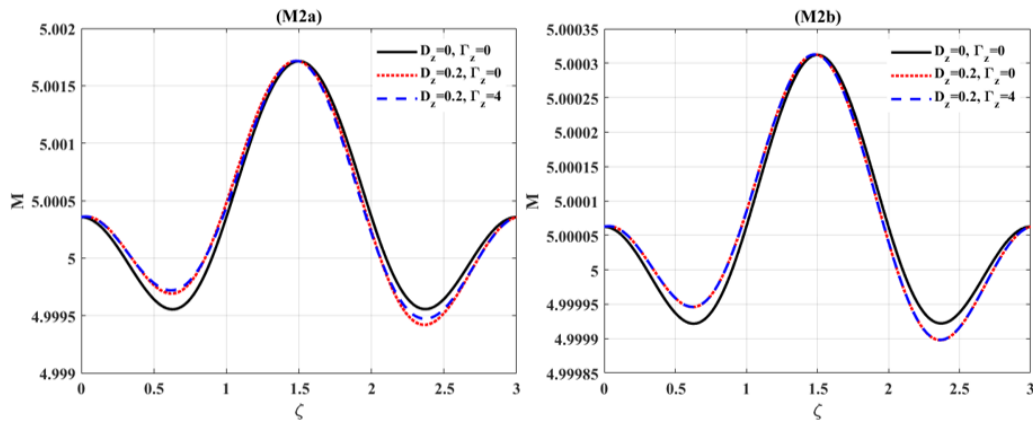


Figure 2. Electric field-dependent magnetization $M(\zeta)$ measured under the joint effect of DM and KSEA interaction, with $T = 3.5$ in (M2a) and $T = 20$ in (M2b). The other parameters are $\Omega = 1.5$ and $N = 1000$.

Magnetization $M(\Omega, \zeta)$ and electric polarization $P(\Omega, \zeta)$ are comparable concepts used to quantify magnetic $\chi^m(\Omega, \zeta)$ and electric field $\chi^e(\Omega, \zeta)$ responses, respectively, indicating magnetic dipole moment density and electric dipole moment density, both of which align spin moment. Magnetic/electric susceptibility are characteristic parameters that indicate how a quantum system reacts to external fields,

resulting in quantum/dynamical quantum phase transitions. In ME multiferroics, ME coupling is a direct result of applied magnetic and electric fields. It is the signature that controls the ME parameters by the action of applied external fields. All influences on the ME coupling, such as spin-orbit interactions investigated in this study, are critical for creating and understanding novel material states devices. Modifying the ME

coupling factor affects the electric structure and thus the energy levels, spin dynamics, and even the material's topology [43]. Figures 1 and 2 illustrate the magnetization behavior

$M(\Omega)$, $M(\zeta)$ and in Figures 5 and 6 the polarization behavior at $T = 3.5$ (for M1a, M2a, P1a and P2.a) and $T = 20$ (for M1b, M2b, P1.b and P2.b) alongside fits obtained under different values of D_z and Γ_z .

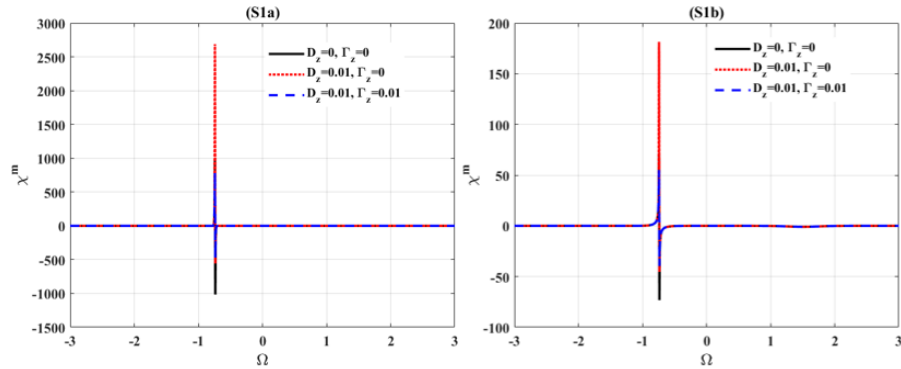


Figure 3. Magnetic field-dependent magnetic susceptibility $\chi^m(\Omega)$ measured under the joint effect of DM and KSEA interaction, with $T = 0.02$ in (S1a) and $T = 0.3$ in (S1b). The other parameters are $\zeta = 0.5$ and $N = 1000$.

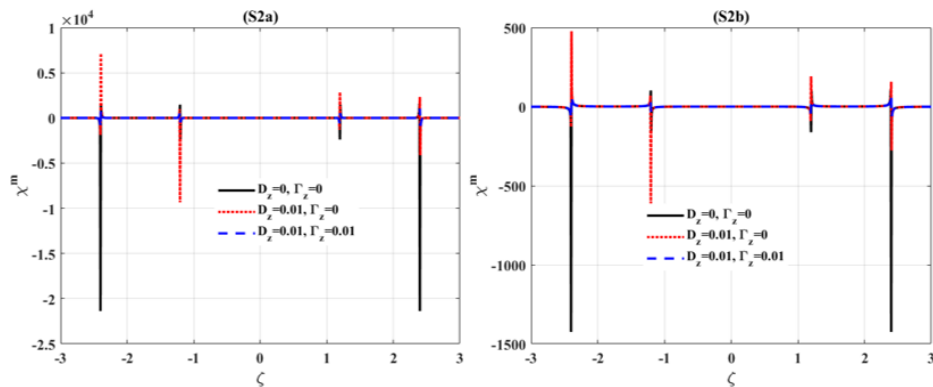


Figure 4. Electric field-dependent magnetic susceptibility $\chi^m(\zeta)$ measured under the joint effect of DM and KSEA interaction, with $T = 0.02$ in (S2a) and $T = 0.3$ in (S2b). The other parameters are: $\Omega = 0.5$ and $N = 1000$.

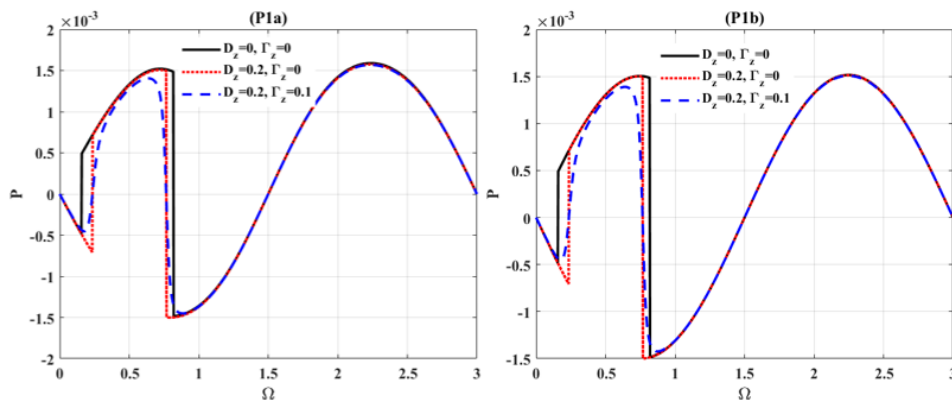


Figure 5. Magnetic field-dependent electric polarization $P(\Omega)$ measured under the joint effect of DM and KSEA interaction, with $T = 3.5$ in (P1a) and $T = 20$ in (P1b). The other parameters are $\zeta = 1.5$ and $N = 1000$.

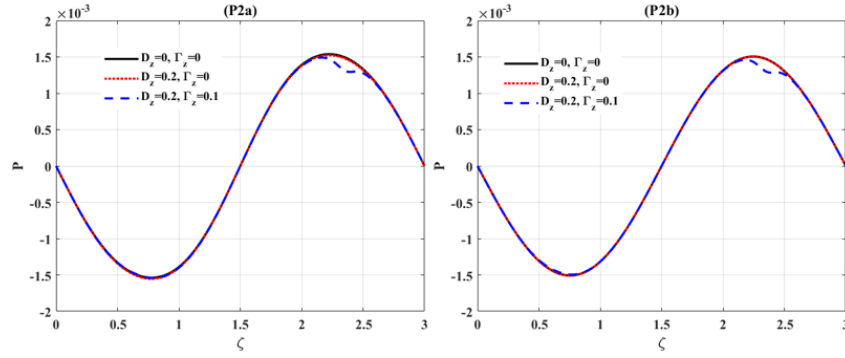


Figure 6. Electric field-dependent electric polarization $P(\zeta)$ measured under the joint effect of DM and KSEA interaction, with $T = 3.5$ in (P2a) and $T = 20$ in (P2b). The other parameters are $\Omega = 1.5$ and $N = 1000$.

In the absence of spin-orbit interactions, magnetization and electric polarization evolve antisymmetrically and symmetrically with respect to the magnetic field (Figures 1 and 5) and electric field respectively (Figures 2 and 6). The interaction parameter D_z , which is responsible for symmetry breaking, preserves the observed behavior in the evolution of magnetization and polarization with respect to the magnetic field (Figures 1 and 5), while

breaking the symmetric behavior observed in their evolution when they depend on the electric field (Figures 2 and 6). However, the factor Γ_z tends to resist the action needed by the interaction parameters D_z , diminishing its influence and providing greater control over the magnetic and electric domains.

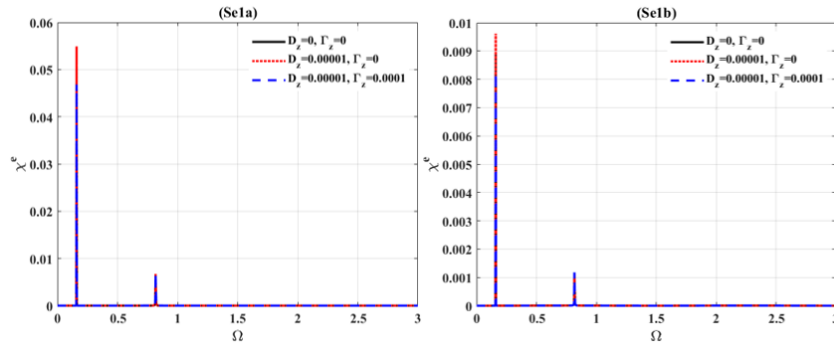


Figure 7. Magnetic field-dependent electric susceptibility $\chi^e(\Omega)$ measured under joint effect of DM and KSEA interaction, with $T = 3.5$ in (Se1a) and $T = 20$ in (Se1b). The other parameters are: $\zeta = 1.5$ and $N = 1000$.

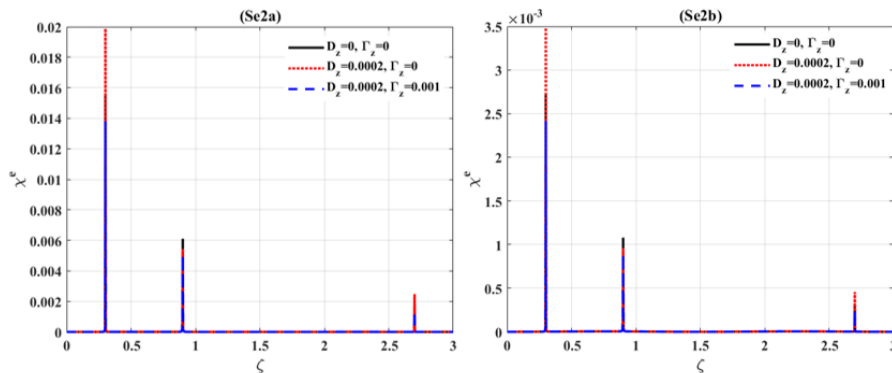


Figure 8. Electric field-dependent electric susceptibility $\chi^e(\zeta)$ measured under joint effect of DM and KSEA interaction, with $T = 3.5$ in (Se2a) and $T = 20$ in (Se2b). The other parameters are: $\Omega = 1.5$ and $N = 1000$.

The magnetic susceptibility $\chi^m(\Omega, \zeta)$ shows a sudden change in concavity at the critical values of the applied field (Figures 3 and 4). These points correspond to the phase transition points. There are several such points depending on the electric field (Figure 4). In contrast, the electric susceptibility $\chi^e(\Omega, \zeta)$ exhibits progression with a number of positive transitions at the critical points of the applied field (Figures 7 and 8). The multiplicity of critical points as a function of the electric field refers to the variation of the action of the transverse electric field with respect to that of the anisotropy D_z and that of the magnetic field applied along the z-axis. The variation of the DM interaction generates quantum transition dynamics [44]. The magnetic field and DM interaction increase the magnitude of magnetic parameters (Figures 1-4).

The KSEA interaction counteracts the symmetrical inversion caused by DM; it reduces the impact of DM interactions (Figures 1-8) and delays electrical (Figures 7, 8) and magnetic (Figures 3, 4) phase transitions. By delaying the quantum phase transition, the KSEA interaction enhances the broadening of magnetic and electric domains, as seen by a rise in magnetic and electric fields. The rise in magnetic domains benefits information storage, but the growth in electrical domains benefits electrical transmission. The electrical distribution as a function of magnetic weight, described by Eq. (18), has a non-linear evolution (see Figure 9), reducing with increasing particle number and increasing with increasing magnetic weight. This trend demonstrates a direct relationship between electrical and magnetic properties in type II multiferroics.

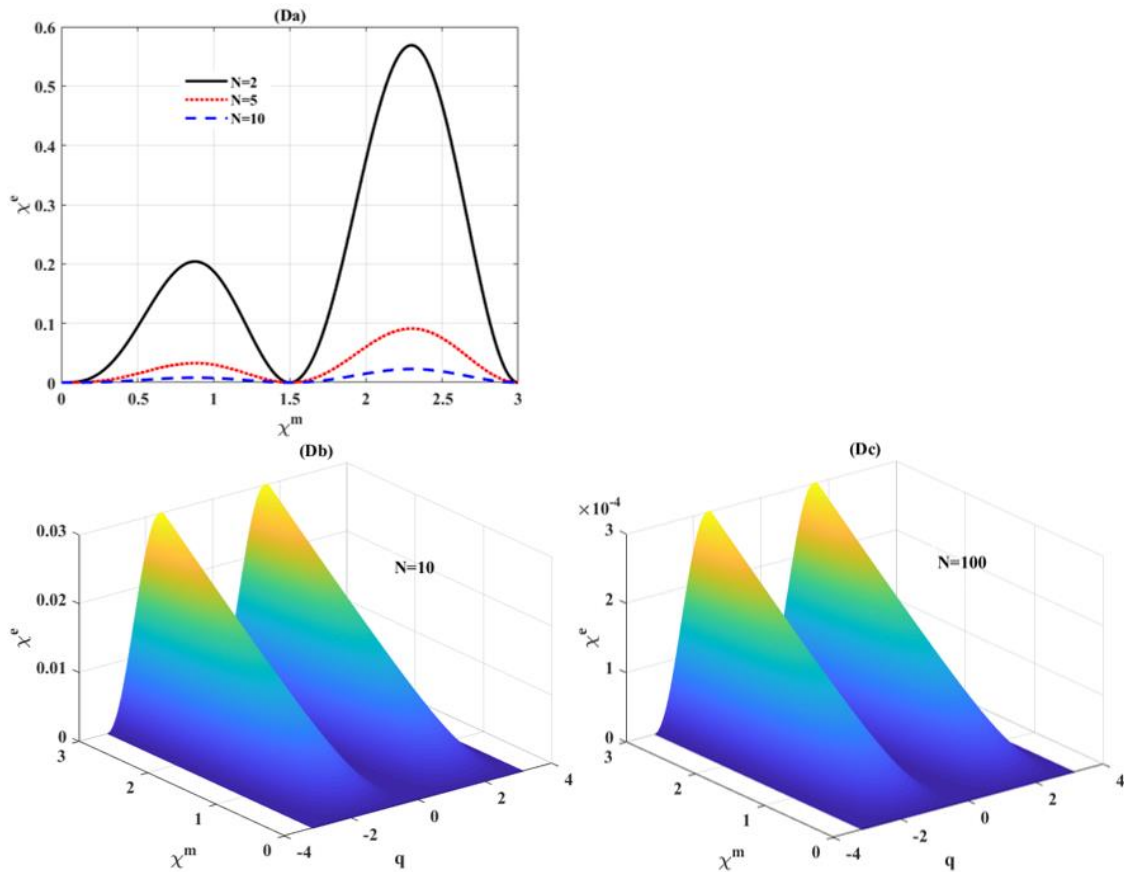


Figure 9. Magnetic susceptibility-dependent electric susceptibility $\chi^e(\zeta)$ phase diagram.

The linear ME effect is linked to electrical polarization and magnetization by ME coupling $\alpha = \frac{\partial P}{\partial \Omega} = \frac{\partial M}{\partial \zeta}$ [8]. The shape of this factor preserves the antiferromagnetic symmetry of the material. Similarly, electrical and magnetic susceptibility are related to polarization and magnetization, respectively, by the

following relationships $\chi^e = \frac{\partial P}{\partial \zeta}$ et $\chi^m = \frac{\partial M}{\partial \Omega}$. With reference to the result obtained in Eq. (18), we can read the possibility of coupling the two susceptibilities by a non-linear factor whose profile is represented in Figure 9.

4. Concluding Remarks

In this work, we investigated the combined effect of DM and KSEA interactions on the magnetoelectric characteristics of multiferroic materials with helical configurations. We developed the partition function from Fermi-Dirac statistical theory, which allowed us to establish analytical formulations for magnetization, magnetic susceptibility, polarization, and electric susceptibility, respectively. The sign, amplitude, and behavior of each of these quantifiable parameters, as shown in the illustrative curves, appropriately reflect the intrinsic features of each DM and KSEA anisotropy factor on the system's topology. Because of its antisymmetric nature, the DM term generates symmetric inversion, promoting a dynamic transition around the critical points. This dynamic transition also maintains the original nonlinearity produced by the model's antiferromagnetic characteristics. The positive or negative sign of each numerical curve indicates the system's symmetry inversion, or helicity. In contrast, the KSEA term encourages delay in a specific system phase while opposing symmetry inversion. Similarly, the effect of KSEA interaction reduces the DM factor. Our findings sustain a remarkable conclusion, as already reported by other authors, on the possibility to influence the phase transition dynamics generated by the continuous symmetry inversion in the helix-shaped spin chain model; which we achieved here through a joint effect of DM and KSEA interactions. This paves the way to many potential applications in spintronics.

Abbreviations

DM	Dzyaloshinskii
KSEA	Kaplan-Shekhtman-Entin-Wohlman-Aharony
NN	Nearest Neighbor
NNN	Next Nearest Neighbor
ME	Magnetoelectric
JW	Jordan-Wigner

Author Contributions

Blaise Ndakom: Data curation, Formal Analysis, Resources, Funding acquisition, Investigation, Methodology, Resources, Software

Erna Laeticia Tchinda Ngounou: Formal Analysis, Funding acquisition, Investigation, Methodology, Resources, Software

Bertrand Sitamtze Youmbi: Conceptualization, Visualization, Validation, Writing - original draft, Writing - review & editing

Ngarmaim Nadjitonon: Supervision, Validation, Writing - review & editing

Georges Collince Fouokeng: Conceptualization, Resources, Software, Supervision, Validation, Writing - original draft, Writing - review & editing

Aurelien Kenfack Jitsa: Supervision, Visualization, Validation

Nirina Randrianantoandro: Conceptualization, Supervision, Validation

Conflicts of Interest

The authors declare no conflicts of interest.

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