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Research Article

Dust Ion Acoustic Solitary Waves in a Magnetized Plasma with Super-thermal Electrons

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Abstract

By considering a magnetized dusty plasma system which is composed of inertial negatively charged dust particles, positively charged warm ions, and inertia less κ-distributed electrons, the obliquely propagating dust ion acoustic solitary waves (DIASWs) are thoroughly examined. The shape of nonlinear electrostatic excitations is significantly altered by the external magnetic field. A Zakharov–Kuznetsov equation is derived by utilizing well known reductive perturbation method. The basic characteristics (amplitude, width, phase speed, etc.) that related to the DIASWs are examined. It is found that for the considered plasma system the fundamental features of DIASWs changes significantly. It is correspondingly analyzed that the amplitude of positive solitary waves changes significantly for different plasma parameters. The results of this work can be used to comprehend the properties of DIASWs and localized electrostatic structures in different astrophysical plasmas. Numerous physical parameters, including the temperature ratio, electron superthermality, and dust to ion mass ratio, have a substantial impact on the propagation characteristics of DIASWs. An increase in dust content enhances the overall mass loading, which tends to reduce phase speed and broaden the solitary structures, while also modifying the balance between dispersion and nonlinearity. A brief discussion is given of the implications of this work for laboratory plasmas and space.

Keywords

Dust Ion Acoustic Solitary Waves, Zakharov Kuznetsov Equation, Magnetized Dusty Plasma, Super Thermal Distribution

1. Introduction

A considerable amount of research work has been done by many researchers in the field of dusty plasmas from the last few decades. [1, 2]. Basically, the existence of dust-acoustic waves (DAWs) in an unmagnetized dusty plasma as well as the dispersion relationship of DAWs predicted in 1990 by Rao et al. theoretically [3]. Shukla et al. further proposed that there is another type of waves in dusty plasma, namely dust ion acoustic waves (DIAWs) [4]. In 1995, Barkan et al. observed the presence of DAWs and DIAWs in experiments and confirmed the theoretical predictions [5].

It was observed that dust grains play an important role for the variation of the dynamics of the dusty plasma medium (DPM) [6, 7]. The majority of astrophysical surroundings contain such dust grains and also in laboratory plasmas. A great deal of observations has shown that in space plasmas, such as the Saturn's magnetosphere plasma, the solar wind, Jupiter's magnetosphere, and Earth's magnetosphere plasma sheet contain highly energetic particles which can exhibit high energy tails [8-11].

A significant number of authors considered an external

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magnetic field to investigate theoretically nonlinear electrostatic solitary and shock waves in variant plasma system [12-16]. This external magnetic field has a crucial impact for changing the shape of nonlinear electrostatic excitations [17]. Unlike previous studies, which focused on Maxwellian or weakly nonthermal electron populations, our study investigates a strongly superthermal κ-electron component for a broad range of κ values extending well below those considered previously. This allows us to examine how extreme superthermality modifies dispersion and nonlinearity in magnetized dusty plasmas, a regime that remains insufficiently explored. The concept of super thermal kappa (κ) distribution was first inaugurated by Vasyliunas [18]. This distribution is applicable for plasma containing highly energetic particles which can exhibit high energy tails [8, 9, 19-22]. On the other hand, for highly energetic particles Maxwellian distribution fails to describe of their dynamics [23]. So, in this particular case κ-distribution play an effective role to understand and analyze the high energetic particles in specific areas.

It should be mentioned that the reductive perturbation method is applied to small-amplitude solitary waves. On the other hand, the Sagdeev pseudopotential approach works well for examining SWs with large amplitude [23]. The objective of our research, however, is to use the reductive perturbation method to investigate small amplitude SWs. Prior works generally treated the effects of electron superthermality and magnetic field obliqueness separately or only in limited parameter domains. In contrast, our analysis systematically evaluates their combined influence on the amplitude, polarity, and width of DIASWs. This leads to several new results, including conditions under which compressive and rarefactive structures can switch, which were not reported in earlier analyses.

Therefore, we have both analytically and numerically studied the DIASWs and used the reductive perturbation method to obtain the ZK equation in order to achieve a more generalized study on magnetized dusty plasma (which is composed of inertial negatively charged dust particles, positively charged warm ions, and inertia-less κ-distributed electrons).

2. Governing Equations

A simple model for DIASWs in a magnetized, three-component dusty plasma medium (DPM), which consists of positively charged warm ions, inertia-less κ-distributed electrons, and negatively charged, inertial dust particles, is considered. In the system, an external magnetic field B_0 oriented along the z-axis has been taken into consideration. For simplicity, we have considered the quasineutrality condition at equilibrium for our plasma model. Here we have considered normalized value for plasma parameter. The following normalized form governs the dynamics of the magnetized DPM:

$$\frac{\partial n_d}{\partial t} + \nabla(n_d.u_d) = 0 \tag{1}$$

$$\frac{\partial u_d}{\partial t} + (u_d \cdot \nabla)u_d = \alpha_1 \nabla\psi - \alpha_1 \Omega_c (u_d \times \hat{z}) \tag{2}$$

$$\frac{\partial n_i}{\partial t} + \nabla(n_i.u_i) = 0 \tag{3}$$

$$\frac{\partial u_i}{\partial t} + (u_i \cdot \nabla)u_i = -\nabla\psi - \Omega_c (u_i \times \hat{z}) - \alpha_2 \nabla n_i^{(\gamma-1)} \tag{4}$$

Here,

$$\alpha_1 = Z_d m_i / Z_i m_d,\ \alpha_2 = \gamma T_i/(\gamma-1) Z_i T_e, \mu_e = n_{e0}/Z_i n_{i0}, and\ \Omega_c = \omega_{ci}/\omega_p$$

Where, $n_d(n_i)$ is the dust (ion) number density, $m_d(m_i)$ is dust (ion) mass, $Z_d(Z_i)$ is the charge state of the dust (ion), e is the magnitude of electron charge.

Here, dust-to-ion mass (or density) ratio alters both the effective inertia and charge balance of the plasma. Cyclotron frequency Ω_c reflects the magnetic-field strength. Stronger superthermality increases electron pressure responsiveness, which enhances nonlinearity.

Now, the number density of electrons that follows the κ-distribution can now be expressed as-

$$n_e = \left[1 - \frac{\psi}{\kappa - 3/2}\right]^{-\frac{\kappa+1}{2}} \tag{5}$$

where the non-thermal characteristics of the electrons are represented by the parameter κ. Poisson's equation can be expressed as follows by extending Eq. (5) to the third order in ψ as,

$$\nabla^2\psi = \mu_e + n_d(1-\mu_e) - n_i + \sigma_1\psi + \sigma_2\psi^2 + \sigma_3\psi^3 \tag{6}$$

Where,

$$\sigma_1 = \mu_e[(\kappa+1)/2(\kappa-3/2)]$$

$$\sigma_2 = \mu_e[(\kappa+1)(\kappa+3)/8(\kappa-3/2)^2]$$

$$\sigma_3 = \mu_e[(\kappa+1)(\kappa+3)(\kappa+5)/48(\kappa-3/2)^3$$

Derivation of Zakharov-Kuznetsov (ZK) Equation

We employed a scaling of the independent variables through the stretched coordinates to obtain the ZK equation [25, 26]:

$$X = \epsilon^{1/2} x \tag{7}$$

$$Y = \epsilon^{1/2} y \tag{8}$$

$$Z = \epsilon^{1/2}(z - V_p t) \tag{9}$$

$$\tau = \epsilon^{3/2} t \tag{10}$$

where ϵ is a smallness parameter that measures the dispersion's weakness ($0 < \epsilon < 1$) and V_p represents the phase speed.

The expanded perturbed quantities and their equilibrium values are as follows

$$n_d = 1 + \epsilon n_d^{(1)} + \epsilon^2 n_d^{(2)} + \ldots \tag{11}$$

$$u_{dx} = \epsilon^{3/2} u_{dx}^{(1)} + \epsilon^2 u_{dx}^{(2)} + \cdots \tag{12}$$

$$u_{dy} = \epsilon^{3/2} u_{dy}^{(1)} + \epsilon^2 u_{dy}^{(2)} + \cdots \tag{13}$$

$$u_{dz} = \epsilon u_{dz}^{(1)} + \epsilon^2 u_{dz}^{(2)} + \cdots \tag{14}$$

$$n_i = 1 + \epsilon n_i^{(1)} + \epsilon^2 n_i^{(2)} + \ldots \tag{15}$$

$$u_{ix} = \epsilon^{3/2} u_{ix}^{(1)} + \epsilon^2 u_{ix}^{(2)} + \cdots \tag{16}$$

$$u_{iy} = \epsilon^{3/2} u_{iy}^{(1)} + \epsilon^2 u_{iy}^{(2)} + \cdots \tag{17}$$

$$u_{iz} = \epsilon u_{iz}^{(1)} + \epsilon^2 u_{iz}^{(2)} + \cdots \tag{18}$$

$$\psi = \epsilon \psi^{(1)} + \epsilon^2 \psi^{(2)} + \cdot\cdot \tag{19}$$

It is now possible to get Poisson's equation, the z constituent of the momentum equation, and the initial order continuity equation using Eqs. (7) to (10) with expended perturbed terms. These equations, when simplified, yield the phase speed,

$$V_p = \sqrt{\frac{m_2 \pm \sqrt{m_2^2 - 4m_1 m_3}}{2m_1}} \tag{20}$$

Here,

$$m_1 = \sigma_1$$

$$m_2 = 1 + \alpha_1 - \alpha_1 \mu_e + \alpha_2 \sigma_1 (\gamma - 1)$$

$$m_3 = \alpha_1 \alpha_2 (1 - \mu_e)(\gamma - 1)$$

Now employing higher order equations and after some simplification, we can finally obtain ZK equation as:

$$\frac{\partial \psi}{\partial \tau} + AB\psi \frac{\partial \psi}{\partial Z} + \frac{1}{2} A \frac{\partial}{\partial Z} \left[\frac{\partial^2}{\partial Z^2} + D\left(\frac{\partial^2}{\partial X^2} + \frac{\partial^2}{\partial Y^2}\right)\right] \psi = 0 \tag{21}$$

Here,

$$A = \frac{V_p^3 \{V_p^2 - \alpha_2(\gamma - 1)\}^2}{V_p^4 + \alpha_1(1 - \mu_e) V_p^2 - \alpha_2(\gamma - 1)^2}$$

$$B = \frac{3V_p^2}{2V_p^2 - \alpha_2(\gamma - 1)^2} - \frac{3{\alpha_1}^2(1 - \mu_e)}{2V_p^4} - \sigma_2$$

$$D = 1 + \frac{(1 - \mu_e)}{\alpha_1 \Omega_c^2} - \frac{V_p^4}{\Omega_c^2 V_p^2 - \alpha_2(\gamma - 1)^2}$$

Here A is the nonlinearity coefficient, B is the dispersion coefficient, and D is the obliqueness coefficient.

The ZK equation, shown by equation (1), describes the nonlinear DIA wave propagation in a magnetized plasma.

In order to study the unique features of SWs propagating in a direction that forms an angle δ with the Z-axis, i.e. while lying in the (Z-X) plane and subjected to an external magnetic field, the Y-axis remains fixed while the coordinate axes (X, Z) rotate at an angle δ.

Consequently, we convert our independent variables into

$$\rho = X\cos\delta - Z\sin\delta, \ \ \eta = Y$$

$$\xi = X\sin\delta + Z\cos\delta, \quad \tau = t.$$

The transformation of these independent variables (38; 39) helps us to write the ZK equation in the form

$$\frac{\partial \psi}{\partial t} + \delta_1 \psi \frac{\partial \psi}{\partial \xi} + \delta_2 \frac{\partial^3 \psi}{\partial \xi^3} + \delta_3 \psi \frac{\partial \psi}{\partial \rho} + \delta_4 \frac{\partial^3 \psi}{\partial \rho^3} + \delta_5 \frac{\partial^3 \psi}{\partial \xi^2 \partial \rho} + \delta_6 \frac{\partial^3 \psi}{\partial \xi \partial \rho^2} + \delta_7 \frac{\partial^3 \psi}{\partial \xi \partial \eta^2} + \delta_8 \frac{\partial^3 \psi}{\partial \rho \partial \eta^2} = 0.$$

Here,

$$\delta_1 = AB\cos\delta,$$

$$\delta_2 = \frac{1}{2} A(\cos^3\delta + D\sin^2\delta\cos\delta),$$

$$\delta_3 = -AB\sin\delta,$$

$$\delta_4 = -\frac{1}{2} A(\sin^3\delta + D\sin\delta\cos^2\delta),$$

$$\delta_5 = A\left[D\left(\sin\delta\cos^2\delta - \frac{1}{2}\sin^3\delta\right) - \frac{3}{2}\sin\delta\cos^2\delta\right],$$

$$\delta_6 = -A\left[D\left(\sin^2\delta\cos\delta - \frac{1}{2}\cos^3\delta\right) - \frac{3}{2}\sin^2\delta\cos\delta\right],$$

$$\delta_7 = \frac{1}{2} AD\cos\delta,$$

$$\delta_8 = -\frac{1}{2} AD\sin\delta.$$

The following represents the ZK equation's steady state solution [27, 28]:

$$\psi = \psi_0(Z),$$

Where $Z = \xi - U_0 t$.

In this case U_0 represents a constant speed. The ZK equation can now be expressed in steady state form using this transformation as follows:

$$-U_0 \frac{d\psi_0}{dZ} + \delta_1 \psi_0 \frac{d\psi_0}{dZ} + \delta_2 \frac{d^3\psi_0}{dZ^3} = 0.$$

Presently with the proper boundary conditions, viz.

$\psi \rightarrow 0$, $(d\psi/\mathrm{dZ}) \rightarrow 0$, $(d^2\psi/\mathrm{d}Z^2) \rightarrow 0$ as $Z \rightarrow \pm\infty$, the solution to this equation in solitary waves is provided by

$$\psi_0(Z) = \psi_m \mathrm{sech}^2(kZ),$$

Where $\psi_m = 3U_0/\delta_1$ is the amplitude and $k = \sqrt{\frac{U_o}{4\delta_2}}$ is the inverse of the width of the SWs. Here, the SWs will exist with only positive potential $(\psi_m > 0)$ since $U_0 > 0$.

With the exception of δ_1 and δ_2, all δ have vanished in the steady state solution of ZK Eq. (21) in one dimension. This indicates that the solution solely contains the functions δ_1 and δ_2, which are functions of δ. As a result, we have illustrated how the SWs widths change with δ (shown in Figure 3).

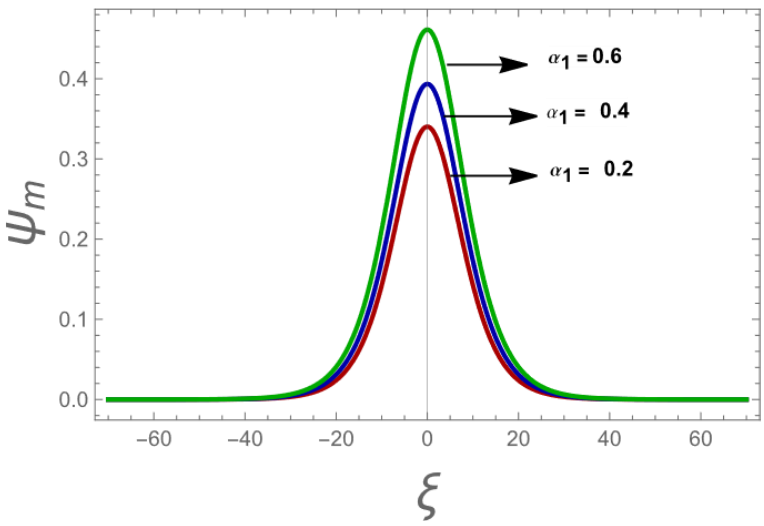

Figure 1*. Variation of the amplitude of the SWs with ξ for different values of α_1. Other plasma parameters are σ_1 =0.3, $\sigma_2 = 0.1$, $\mu_e = 0.2$, $\gamma = 0.3$, and $\alpha_2 = 0.03$.*

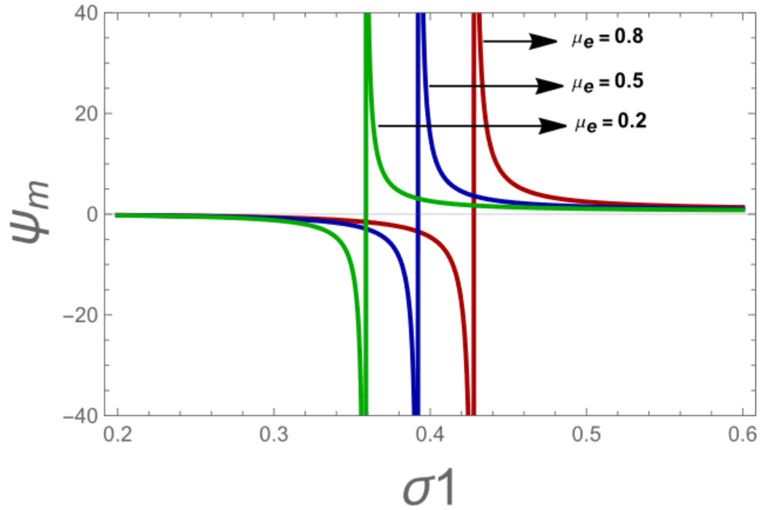

Figure 2*. Variation of the amplitude of the SWs with $\sigma_1$for different values of μ_e. Other plasma parameters are α_1 =0.3, $\alpha_2 = 0.03$, $\gamma = 0.3$, and $\sigma_2 = 0.5$.*

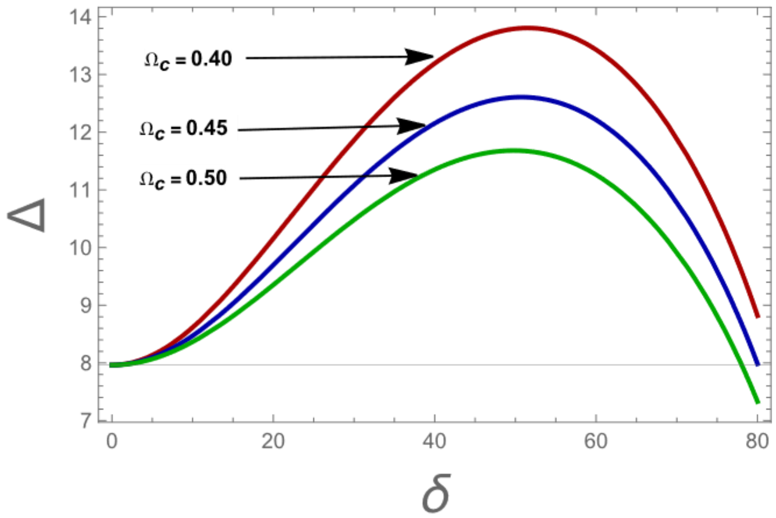

Figure 3. *Variation of the width Δ of the SWs with δ for different values of Ω_c. Other plasma parameters are α_1 =0.1, $\alpha_2 = 0.03$, σ_1 =0.3, $\sigma_2 = 0.1$, $\mu_e = 0.2$, and $\gamma = 0.3$.*

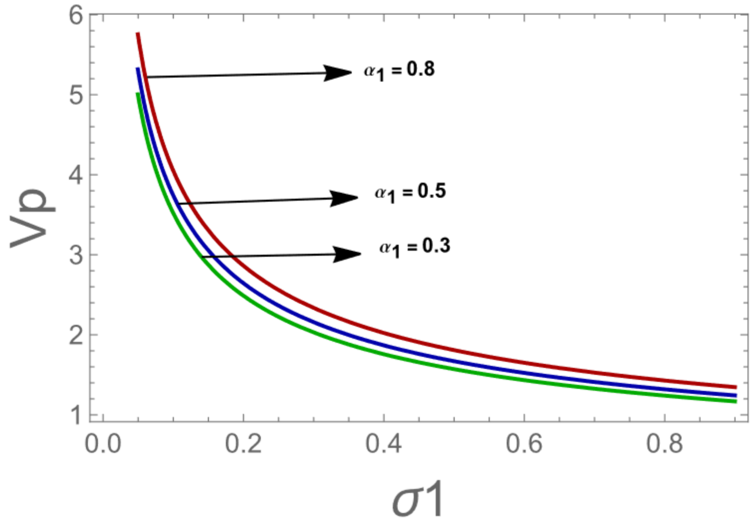

Figure 4. *Variation of the phase speed $V_p$of the SWs with σ_1 for different values of α_1. Other plasma parameters are α_1 =0.3, $\alpha_2 = 0.02$, $\sigma_2 = 0.1$, $\mu_e = 0.2$, and $\gamma = 0.3$.*

3. Discussion

In this work a magnetized plasma system is considered accompained with three-component dusty plasma medium (DPM), which is a combination of dust particles that are negatively charged and inertial, positively charged warm ions, and inertia less κ-distributed electrons. The outcomes of our analysis can be summed up as follows.

1) The amplitude of the positive potential SWs increases with the increase of plasma parameter α_1 as shown in Figure 1. That means there is an impact of $\alpha_1$on the amplitude of SWs.
2) The amplitude of the positive potential SWs varies for different values of plasma parameter μ_e as shown in Figure 2. Which indicates that with the increase of μ_e the amplitude also varies.
3) Figure 3 indicates the variation of width Δ for different values of Ω_c. Here with the increase of Ω_c the width Δ decreases. Figure 3 also indicate that for a certain value of δ the amplitude of the positive potential SWs increases after that value of δ, it started to decrease linearly.
4) The variation of phase speed $V_p$of the SWs with σ_1 for different values of α_1 is shown in Figure 4. This figure indicates the role of plasma parameter α_1 on the phase speed.

Finally we can conclude that how decreasing κ enhances both the nonlinear steepening term and the dispersion term in distinct ways not captured by Maxwellian-based models. This provides fresh physical insight into the mechanism by which superthermal electrons accelerate or suppress the formation of DIASWs in magnetized dusty plasmas.

Abbreviations

DAWs	Dust-acoustic Waves
DIAWs	Dust Ion Acoustic Waves
DPM	Dusty Plasma Medium
ZK	Zakharov Kuznetsov
DIASWs	Dust Ion Acoustic Solitary Waves

Author Contributions

Al Rafat is the sole author. The author read and approved the final manuscript.

Conflicts of Interest

The author declares no conflicts of interest.

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