American Journal of Modern Physics
2025; 14(2): 44-51
http://www.sciencepublishinggroup.com/j/ajmp
doi: 10.11648/j.ajmp.20251402.11
ISSN: 2326-8867 (Print); ISSN: 2326-8891 (Online)

A Representation of an Octonionic Interaction of Color Quarks with the Application of Feynman Diagram

Arun Kumar Rathore*, **Bhupesh Chandra Chanyal**

Department of Physics, G. B. Pant University of Agriculture and Technology, Pantnagar, India

Email address:

arunkumarrathore706@gmail.com (Arun Kumar Rathore), bcchanyal@gmail.com (Bhupesh Chandra Chanyal)
*Corresponding author

To cite this article:

Arun Kumar Rathore, Bhupesh Chandra Chanyal. (2025). A Representation of an Octonionic Interaction of Color Quarks with the Application of Feynman Diagram. *American Journal of Modern Physics, 14*(2), 44-51. https://doi.org/10.11648/j.ajmp.20251402.11

Received: 21 January 2025; **Accepted:** 20 February 2025; **Published:** 3 March 2025

Abstract: The SU(3) flavour symmetry for quarks and antiquarks has been demonstrated via the complexified octonion space, where the six complex octonion operators are essentially identical to the SL(3,C) group generators. It has been developed an extensive analysis of the quark flavour theory in the context of complex-octonion space by analyzing the connection between octonions and the SU(3) group. Therefore, it is argued that the extended theory of quark flavors, which preserves the property of non-commutativity, is the complexified variant of octonions. This theoretical model may be further extended to the SU(3) color symmetry, which is regarded as an exact symmetry. In this work, to gain a complete understanding of quark color theory in the framework of complex octonionic space, we have derived the relationship between octonions and the *SU(3)c* color group. It has been studied that only eight possibilities of paired gluons are available to provide colorless states of hadrons in order to represent theoretically the octonion glueballs. With the help of Feynman diagrams, we examined the octonionic interaction of color quarks (such as quark-quark, quark anti-quark, and anti-quarks anti-quarks interactions). For the interactions, we have obtained the complex octonion algebraic form of the interaction term, propagator, vertex factor, and color factor. Most importantly, we have examined the conditions for valid and invalid interactions for the complex-octonion formalism.

Keywords: Octonion Algebra, Complex Octonion Space, Quark Color, Feynmann Diagram, SU(3) Color Group

1. Introduction

In theoretical physics, division algebras are used to represent physical phenomena via an isomorphism. At present, there are four such algebraic mathematical models [1] namely real number model, the complex number model, the quaternionic model, and the octonion model. In contrast, quaternions made up of complex numbers are not commutative while octonions made up of quaternions are both non-commutative and non-associative. Out of the four normed division algebras, the largest is the octonions [1, 2]. Octonions exhibit interesting algebraic properties beyond their non-commutativity and non-associativity, like not having a multiplicative inverse for each nonzero element. A novel method is applied to study the octonion algebra, also called Cayley algebra [3], on a split basis, which discloses the quark theory [4, 5]. In the same perspective, there has long been conjecture that there is a relationship between octonions and the Standard Model [6], which clarifies a lot of the issues that come with the algebraic model of the Standard Model. In mathematical physics, the SU(3) Lie group can be described by the octonion algebra. Taking into consideration the theory of octonion variables, Pushpa et al. [7, 8] developed the correlations between different isospin components and quark states to the case for the SU(3) group. The octonionic structure of the exceptional groups includes the SU(3) colour group. Quantum chromodynamics (QCD) and quantum electrodynamics (QED) seem to share a lot of similarities. The QED interaction is mediated by a massless photon that corresponds to the single generator of the U(1) local gauge symmetry, while the QCD interaction is mediated by eight massless gluons that correspond to the eight generators of the SU(3) local gauge symmetry. Three conserved colour charges R, B, and G take the place of the single charge found in QED. In the SU(3) colour space, these colour charges serve as a label for the orthogonal states [9]. Particles only couple to gluons when their colour charge is non-zero. Because of this, the colourless leptons are immune to the powerful force. There are three orthogonal colour states for the quarks, which are charged particles. Chanyal et al. [10, 11] proposed split octonions as a basis for the field equations of hot and cold dark matter in their study of the role of octonions in quark chromodynamics and the dark matter field. The hyper-complex division algebras have been used in the Dirac operator [12]. The octonion analysis has also played an important role in the context of various physical problems [13] of higher dimensional supersymmetry, supergravity, superstrings, etc. Likewise, the octonions are extensively studied for the description of color quarks and play an important role in the unification of fundamental interactions in terms of successful gauge theories. It is shown that the three quaternion units explain the structure of Yang-Mill's field while the seven octonion units provide the consistent structure of SU(3) color gauge symmetry of quantum chromodynamics. The SU(3) colour symmetry is exact, in contrast to the approximate SU(3) flavour symmetry, and QCD remains invariant under unitary transformations in colour space. The colour group SU(3) is isomorphic to octonion. We tried to write a connection between octonion algebra and SU(3) colour group generators in light of recent advances [14]. The SU(3) colour symmetry group is based on three colours: R, B, and G. There are eight additional gluons that could be swapped. As a result, 648 different quark colour and gluon combinations are possible to contribute to the process. Thankfully, a single colour component can absorb the effect of adding up all the colour and gluon combinations. The Feynman diagram is used to illustrate how colour quark particles interact. A thorough explanation of the strong interactions among quarks is provided by QCD. The hyper-complex division algebras have been applied by some authors [17-19], and [20, 21] to predict many phenomenological physical theories in high-energy physics. Recently, octonion spaces have been utilized to examine several aspects of physical phenomena like Torques and angular momenta of fluid elements in the octonion spaces, Some spin textures relevant to magnetic moments in the octonion spaces, and Conserved quantities of vectorial magnitudes within the material media [22–24]. Additionally, octonions are very effective in studying unbroken gauge symmetry, supersymmetric quantum field theories, generalized Pauli, and Yang-Mills-like field theory [25-28].

2. Octonion and $SU(3)_c$ Symmetry

The octonions constitute an 8-dimensional unital, distributive algebra over R with basis e_A $(A = 1, 2, ...7)$ and e_0 is the multiplicative unit element. A hypercomplex normed division algebra over real numbers, octonion has twice as many dimensions as quaternions. This algebra is non-associative [3]. An octonion variable may be expressed using eight real integers as [29-31]

$$\begin{aligned}\mathcal{O} =& e_0\mathcal{O}_0 + e_1\mathcal{O}_1 + e_2\mathcal{O}_2 + e_3\mathcal{O}_3 + e_4\mathcal{O}_4 \\ &+ e_5\mathcal{O}_5 + e_6\mathcal{O}_6 + e_7\mathcal{O}_7, \\ &= e_0\mathcal{O}_0 + [A = 1]7\sum e_A\mathcal{O}_A,\end{aligned} \tag{1}$$

These octonion basis elements satisfy some following relations:

$$\begin{aligned} e_0 = 1,\ e_0e_A = e_Ae_0 = e_A, \\ e_Ae_B = -\delta_{AB}e_0 + f^{ABC}e_c,\ (A, B, C = 1, 2, ...7) \\ [e_A, e_B] = 2f^{ABC}e_c, \quad [e_A, e_B] = -2\delta_{AB}e_0, \end{aligned} \tag{2}$$

where $(A, B, C = 1, 2, ..., 7)$, and brackets [ ] indicated the commutation relations, respectively, while δ_{AB} is the standard Kronecker delta-Dirac symbol. For the following permutations, the structure constant f_{ABC} is entirely antisymmetric and take the value 1, i.e.$f_{ABC} = +1$ for $ABC=$ (123), (471), (257), (165), (624), (543), (736). The norm of the octonion algebra is expressed as

$$N(\mathcal{O}) = \overline{\mathcal{O}}\mathcal{O} = \mathcal{O}\overline{\mathcal{O}} = [\beta = 0]7\sum \mathcal{O}_\beta^2 e_0. \tag{3}$$

The fano plane also allows one to display the algebraic structure of octonions and their multiplications. The triangle's three sides, the incircle, and the three altitudes are the seven quaternions subsets that are shown in *Figure 1*.

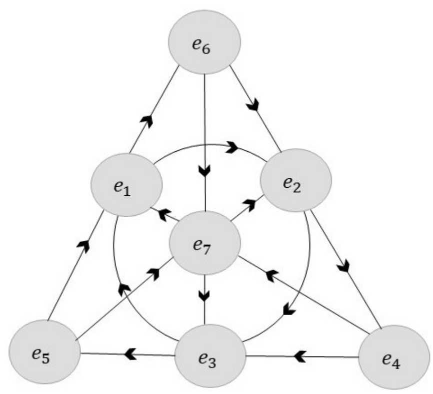


***Figure 1.** Fano plane.*

***Table 1.** Octonion multiplication table.*

	e_1	e_2	e_3	e_4	e_5	e_6	e_7
e_1	-1	e_3	$-e_2$	e_7	$-e_6$	e_5	$-e_4$
e_2	$-e_3$	-1	e_1	e_6	e_7	$-e_4$	$-e_5$
e_3	e_2	$-e_1$	-1	$-e_5$	e_4	e_7	$-e_6$
e_4	$-e_7$	$-e_6$	e_5	-1	$-e_3$	e_2	e_1
e_5	e_6	$-e_7$	$-e_4$	e_3	-1	$-e_1$	e_2
e_6	$-e_5$	e_4	$-e_7$	$-e_2$	e_1	-1	e_3
e_7	e_4	e_5	e_6	$-e_1$	$-e_2$	$-e_3$	-1

The Gell-Mann matrices are as follows:

$$\lambda_1 = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},\ \lambda_2 = \begin{bmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},\ \lambda_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix},\ \lambda_4 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

$$\lambda_5 = \begin{bmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{bmatrix},\ \lambda_6 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix},\ lambda_7 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{bmatrix},\quad \lambda_8 = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{bmatrix}. \tag{4}$$

It satisfy some properties

$$\begin{aligned} &(\lambda_A)^\dagger = \lambda_A, Tr(\lambda_A) = 0,\ Tr(\lambda_A\lambda_B) = 2\delta_{AB}\,[\lambda_A, \lambda_B] = 2iF^{ABC}\lambda_C,\ (\forall A, B, C = 1, 2, ...8) \\ &[\lambda_A, \lambda_B] = \frac{4}{3}\delta_{AB} I + 2\sum_c d^{ABC}\lambda_c, \\ &e_A e_B = -\delta_{AB} e_0 + f^{ABC} e_c,\ (A, B, C = 1, 2, ...7). \end{aligned} \tag{5}$$

The Gell-Mann matrices of SU(3) symmetry in terms of SU(3) Lie algebra and it consists of antihermitian traceless matrices. Hence the consequence, the SU(3) group, $G_2 \sim SU(3)$, is an automorphism of the octonion algebra.

3. Complex Octonion Reformulation of SU(3) Color Symmetry

The new elements will not necessarily be anti-hermitian when characterising the complexified SU(3) group, the complex linear combinations of the SU(3) components, but they will be traceless, which is precisely the property of the Lie algebra of SL(3, C). The basis of SL(3, C)–group i.e., $E_{12}, E_{23}, E_{13}, E_{21}, E_{32}, E_{31}$ will be equivalent of octonion operators in octonion representation as [14, 16],

$$\begin{aligned} u_{12} &= -\frac{i}{2}(e_1 + ie_2), u_{45} = -i(e_4 + ie_5), u_{67} = i(e_6 + ie_7), \\ \bar{u}^*_{12} &= -\frac{i}{2}(e_1 - ie_2), \bar{u}^*_{45} = -i(e_4 - ie_5), \bar{u}^*_{67} = i(e_6 - ie_7). \end{aligned} \tag{6}$$

Where $u_{12}, \bar{u}^*_{12}, u_{45}, \bar{u}^*_{45}, u_{67}, \bar{u}^*_{67}$ are the complex octonion shift operators. These are the complexified form of elements of SU(3) Lie algebra. The two generators may be used as Cartan generators ($u_3 = \frac{-ie_3}{2}$, $u_0 = \frac{2}{3}e_0$). Let us write basic triplet for color quarks [15].

$$|\ \Phi\rangle = \begin{bmatrix} \Phi_R \\ \Phi_B \\ \Phi_G \end{bmatrix},$$

which transform as $\Phi \longrightarrow \Phi' = U\Phi$, where U are unitary matrices of unity determinant and the set of unitary matrices as follows

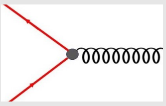
$$U(\vartheta) = e^{(-\frac{i}{2}\sigma_\alpha u_\alpha)}, \tag{7}$$

where u_α are the complex octonion valued matrices and ϑ_α are the eight parameters of SU(3) group form the fundamental representation. Here, $\alpha = 12, 45, 67$ and the conjugates of the octonionic shift operators have values of $\alpha = 12, 45, 67$ while $\alpha = 0, 3$ are non-shift operators. Colour SU(3) refers to the eight conserved charges that make up the QCD symmetry group. As is well known, quantum field theory explains how charged particle interactions are described by quantum electrodynamics (QED); in this theory, coloured particle interactions are described by quantum chromodynamics (QCD). We can write the relation between $\lambda-$matrices with the complex-octonion operators as

$$\begin{aligned} &\lambda_1 = [u_{12} + \bar{u}^*_{12}], \quad \lambda_4 = [u_{45} + \bar{u}^*_{45}], \quad \lambda_6 = [u_{67} + \bar{u}^*_{67}] \\ &\lambda_2 = -i[u_{12} - \bar{u}^*_{12}], \quad \lambda_5 = -i[u_{45} - \bar{u}^*_{45}], \quad \lambda_7 = -i[u_{67} - \bar{u}^*_{67}] \\ &\lambda_3 = [u_3], \lambda_8 = [u_0]. \end{aligned} \tag{8}$$

The evaluation of a Feynman diagram, which shows a particular decay or scattering process is done by using the Feynman rules which help to calculate the matrix element. The matrix element contains the dynamical information of particles. Feynman rule for QCD are shown in *Table 2*.

Table 2. *Table for QCD Feynman rules.*

Description	Graphical representation	Factor
incoming quark		$C_i u_f^{(s)}(E,p)$
outgoing quark		$C_i \bar{u}_f^{(s)}(E,p)$
incoming anti-quark		$C_i \bar{v}_f^{(s)}(E,p)$
outgoing anti-quark		$C_i v_f^{(s)}(E,p)$
gluon propogator		$-i\frac{g_{\mu\nu}\delta^{ab}}{(p_i-p_f)^2}$
quark-gluon vertex		$-ig_s\frac{u_\beta}{2}\gamma^\mu$

where $u_f^{(s)}(E,p)$ and $v_f^{(s)}(E,p)$ are Dirac spinor in terms of energy and momentum, s index $=(1,2)=$ (up, down).

4. Complex Octonionic Color Quark Interactions

As we already know there is three distinct form (i.e., states) of single quark (*R,G,* and *B*). The QCD color quark states are represented in terms of color spinors as

$$|R\rangle=\begin{pmatrix}1\\0\\0\end{pmatrix},\quad |G\rangle=\begin{pmatrix}0\\1\\0\end{pmatrix},\quad |B\rangle=\begin{pmatrix}0\\0\\1\end{pmatrix},\tag{9}$$

and the anti-color quark states are given by

$$\begin{aligned}\langle R| &= \begin{pmatrix}1 & 0 & 0\end{pmatrix},\\ \langle G| &= \begin{pmatrix}0 & 1 & 0\end{pmatrix},\\ \langle B| &= \begin{pmatrix}0 & 0 & 1\end{pmatrix}.\end{aligned}\tag{10}$$

these are also known as colored and anti-colored charges for complex octonion based SU(3)$_c$ -symmetry, now, let us define the eight octonionic gluons as

$$g_1 :\longrightarrow u_{12}\,,\, g_2 :\longrightarrow \bar{u}^*_{12}\,,\, g_3 :\longrightarrow u_{45}\,,\, g_4 :\longrightarrow \bar{u}^*_{45}\,,\, g_5 :\longrightarrow u_{67}\,,\, g_6 :\longrightarrow \bar{u}^*_{67}\,,\, g_7 :\longrightarrow u_0\,,\, g_8 :\longrightarrow u_3\,.$$

where $\{u_{12}\,,\bar{u}^*_{12}\,,u_{45}\,,\bar{u}^*_{45}\,,u_{67}\,,\bar{u}^*_{67}\,,u_0\,,u_3\}$ are the color octet operators resembles with gluons

$$\left\{R\bar{G}, G\bar{R}, R\bar{B}, B\bar{R}, G\bar{B}, B\bar{G}, \frac{1}{\sqrt{6}}\left(R\bar{R}+G\bar{G}-2B\bar{B}\right), \frac{1}{\sqrt{2}}\left(R\bar{R}-G\bar{G}\right)\right\}.$$

These octonionic gluons are corresponding to field associated with generator of SU(3) color symmetry. It is noted that the complex octonion states u_3 and u_0 are colorless. Since only eight gluons are coloured and the others are not, hence, there are eight octonionic physical gluons that can be exchanged. The matrix form of the eight octonionic gluons with states is shown below.

$$u_{12}=|R\rangle\langle G|=\begin{pmatrix}0&1&0\\0&0&0\\0&0&0\end{pmatrix},\bar{u}^*_{12}=|G\rangle\langle R|=\begin{pmatrix}0&0&0\\1&0&0\\0&0&0\end{pmatrix} u_{67}=|G\rangle\langle B|=\begin{pmatrix}0&0&1\\0&0&0\\0&0&0\end{pmatrix},$$

$$\bar{u}^*_{67}=|B\rangle\langle G|=\begin{pmatrix}0&0&0\\0&0&0\\1&0&0\end{pmatrix},\,,u_{45}=|R\rangle\langle B|=\begin{pmatrix}0&0&0\\0&0&1\\0&0&0\end{pmatrix},\bar{u}^*_{45}=|B\rangle\langle R|=\begin{pmatrix}0&0&0\\0&0&0\\0&1&0\end{pmatrix},$$

$$u_3=[|R\rangle\langle R|-|G\rangle\langle G|]=\frac{1}{2}\begin{pmatrix}1&0&0\\0&-1&0\\0&0&0\end{pmatrix},$$

$$u_0=\frac{1}{\sqrt{3}}[|R\rangle\langle R|+|G\rangle\langle G|-2|B\rangle\langle B|]=\frac{1}{3}\begin{pmatrix}1&0&0\\0&1&0\\0&0&-2\end{pmatrix}.\tag{11}$$

In the complexified octonion field, these are alternatively referred to as octonionic shift operators, which are used to convert the color and anti-color charges. The local gauge theory of color SU(3) group gives the theory of quantum chromodynamics (QCD), which is close to Yang-Mills (non-Abelian) gauge theory. We already discussed the fundamental color triplet, whereas and the unitary transformation matrix for color transformation. Now, we will see how complex octonion QCD and color-quarks interactions in terms of complex octonion color field. The locally gauge invariant Lagrangian density for COQCD is written as [14].

$$\mathcal{L}_{QCD}^{\mathcal{O}} = -\frac{1}{4}G_a^{\mu\nu}G_{\mu\nu}^a + \overline{\Phi}_R(i\gamma^\mu D_\mu - m)\Phi_R - g_s\overline{\Phi}_R u_\alpha\gamma^\mu\Phi_R G_\mu^\alpha + \overline{\Phi}_B(i\gamma^\mu D_\mu - m)\Phi_B$$
$$- g_s\overline{\Phi}_B u_\alpha\gamma^\mu\Phi_B G_\mu^\alpha + \overline{\Phi}_G(i\gamma^\mu D_\mu - m)\Phi_G - g_s\overline{\Phi}_G u_\alpha\gamma^\mu\Phi_G G_\mu^\alpha \,. \tag{12}$$

G_μ^α represents the complex octonion gluon, which is indicated by the adjoint representation of gauge group of SU(3) with α; γ^μ denotes the Dirac matrices or Gamma matrices for $\mu = 1, 2, 3, 4$. Φ_R, Φ_G, Φ_B and $\overline{\Phi}_R, \overline{\Phi}_G, \overline{\Phi}_B$ represents the complex octonion color and anticolor states. It is noted that the complex octonion shift operators can be used by replacing the Gell-Mann matrices when the QCD Lagrangian density is written in complex octonion formalism. In COQCD, the shift operators can be treated as gluons. The term $[-g_s\overline{\Phi}_j u_\alpha\gamma^\mu\Phi_j G_\mu^\alpha]$ represents the interaction of quark state with the gluons. While, $[-\frac{1}{4}G_\alpha^{\mu\nu}G_{\mu\nu}^\alpha]$ represents the kinetic term for the eight massless gauge fields G_μ^α. The Dirac fields constitute eight complex-octonionic currents of gluons, which can be written as

$$j_\alpha^\mu = -\frac{1}{2}\left(g_s\overline{\Phi}_j\gamma^\mu u_\alpha\Phi_j\right) \quad , \tag{13}$$

Where j_α^μ acts as the source of gluonic fields G_μ^α, which is analogous to the current in electromagnetic interaction acting as the source of electric current. The complex octonion covariant derivative can be written as [14]

$$(D_\mu^{\mathcal{O}})_\alpha \simeq \delta_\alpha\partial_\mu - i\sum_\alpha u_\alpha A_\mu^\alpha \,, \tag{14}$$

The above complex octonion quantum chromodynamics Lagrangian density is invariant under the gauge transformations of SU(3) group.

5. Complex Octonion Based Quark-gluon Interactions

In order to formulate the mathematical approaches of quark-gluon interactions in terms of complex octonionic algebra, we can write the interactions of color quarks with complex octonion gluon as

$$\begin{aligned}
&u_{12}|G\rangle = |R\rangle \,, \quad && \bar{u}_{12}^*\,|R\rangle = |G\rangle \,, \\
&u_{45}|B\rangle = |G\rangle \,, && \bar{u}_{45}^*\,|G\rangle = |B\rangle \,, \\
&u_{67}\,|B\rangle = |R\rangle \,, && \bar{u}_{67}^*\,|R\rangle = |B\rangle \,, \\
&\langle R|u_{12} = \langle G| \,, && \langle G|\,\bar{u}_{12}^* = \langle R| \,, \\
&\langle G|\,u_{45} = \langle B| \,, && \langle B|\,\bar{u}_{45}^* = \langle G| \,, \\
&\langle R|\,u_{67} = \langle B| \,, && \langle B|\,\bar{u}_{67}^* = \langle R| \,.
\end{aligned} \tag{15}$$

For example, we can see that the interaction of green color quark (G) and red color quark (R) can be interacted with complex octonion gluon (u_{12}). The complex octonion based interaction operators are used to convert the color quark states into other color quark states by interacting with gluons. On the other hand, the octonion Cartan generator (u_3^c, u_0^c) do not change the state of color quark, which is given by

$$\begin{aligned}
&u_3^c\,|R\rangle = \frac{1}{2}\,|R\rangle \,, \; u_0^c\,|R\rangle = \frac{1}{3}\,|R\rangle \,, \\
&u_3^c\,|G\rangle = -\frac{1}{2}\,|G\rangle \,, \; u_0^c\,|G\rangle = \frac{1}{3}\,|G\rangle \,, \\
&u_3^c\,|B\rangle = 0, \; u_0^c\,|B\rangle = -\frac{2}{3}\,|B\rangle \,, \\
&\langle R|\,u_3^c = -\frac{1}{2}\,\langle R| \,, \; \langle R|\,u_0^c = -\frac{1}{3}\,\langle R| \,, \\
&\langle G|\,u_3^c = \frac{1}{2}\,\langle G| \,, \; \langle G|\,u_0^c = -\frac{1}{3}\,\langle G| \,, \\
&\langle B|\,u_3^c = 0, \; \langle B|\,u_0^c = \frac{2}{3}\,\langle B| \,.
\end{aligned} \tag{16}$$

Here, we analyse that in the interaction between two red color quarks, the mediated complex octonion gluon is (u_3^c). Similarly, we can also analyse for the other interactions of color quarks in (16). Now, we can tabulated the interacting octonionic gluons facilitating all possible interactions between the octonionic color and anti-color quarks states as given in *Table 3.* We note that for some of these octonionic interactions, there can be mediated more than one gluons that can participate in the strong interaction, though only one of them will participate at a time.

Table 3. *Interactions of color and anti-color quarks in terms of octonionic form.*

	$\boldsymbol{R}$	$\boldsymbol{G}$	$\boldsymbol{B}$	$\bar{\boldsymbol{R}}$	$\bar{\boldsymbol{G}}$	$\bar{\boldsymbol{B}}$
R	u_3, u_0	u_{12}	u_{67}	u_3, u_0	u_{12}	u_{67}
G	$\bar{u}_{12}^*$	u_3, u_0	u_{45}	$\bar{u}_{12}^*$	u_3, u_0	u_{45}
B	$\bar{u}_{67}^*$	$\bar{u}_{45}^*$	u_3, u_0	$\bar{u}_{67}^*$	$\bar{u}_{45}^*$	u_3, u_0
$\bar{R}$	u_3, u_0	u_{12}	u_{67}	u_3, u_0	$\bar{u}_{12}^*$	u_3, u_0
$\bar{G}$	$\bar{u}_{12}^*$	u_3, u_0	u_{45}	u_{12}	u_3, u_0	$\bar{u}_{67}^*$
$\bar{B}$	$\bar{u}_{67}^*$	$\bar{u}_{45}^*$	u_3, u_0	u_{67}	u_{45}	u_3, u_0

In the *Table 3*, we can see u_{12} is the mediator between the interaction of red and green color quarks.

6. Interaction of Octonionic Color Quarks in Generalised Form

The interaction of octonionic color quark can be represented with the help of space-time diagram (Feynman diagram). We can generalize them with the help of general Feynman diagrams shown in *Figures 2-3.*

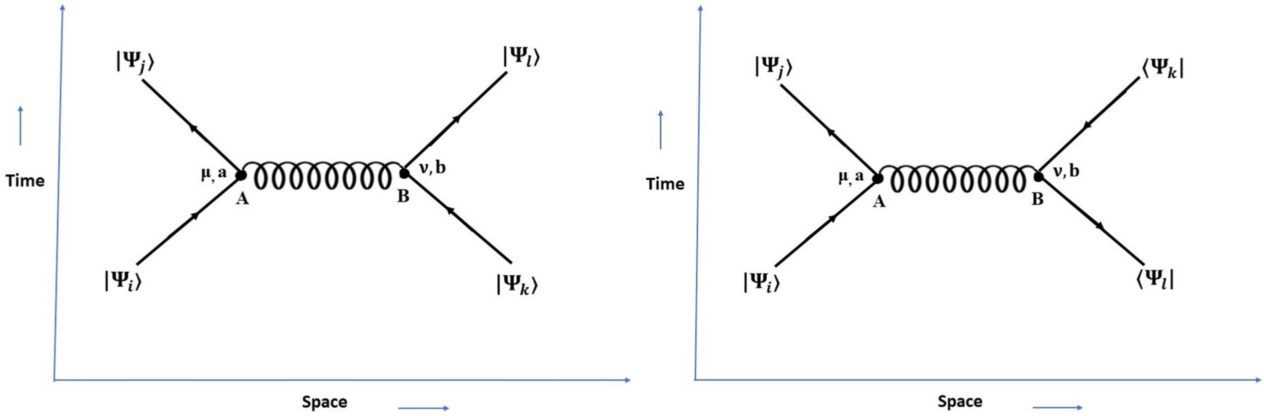


Figure 2. Illustration of general interaction between two color quarks, and color quark and anti-color quark.

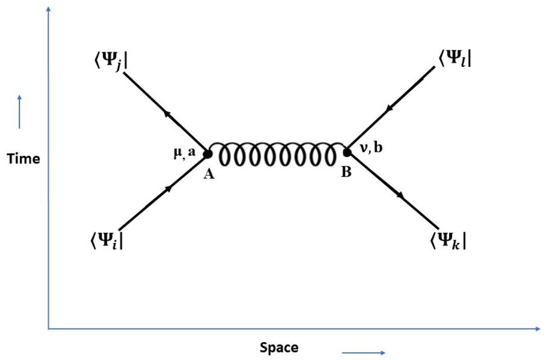


Figure 3. Illustration of general interaction between two anti-color quarks.

These Feynman diagrams are to be interpreted as follows: Two color quark wavefunctions, which represents energy and momentum, are propagating in space-time and interacting by exchange a virtual octonionic gluon. In momentum space, these Feynman diagrams have following amplitudes (i) $c_\beta \Psi_\beta$, $-\frac{ig_{\mu\nu}}{(p_i-p_f)^2}$, (ii) and (iii) $-\frac{1}{2}ig_s u_\alpha \gamma^\mu$, namely, the Dirac spinor, the propagator term and the vertex. Here Ψ_β is the Dirac spinor (where $\beta = i,j,k,l$) that represents the particle interacting with the gluon emitted or absorbed at each vertex. Now we write the generalized matrix element for the different type of interaction of color quarks as

$$-iM = \langle c_j \Psi_j | \left\{ -\frac{1}{2} ig_s u_\alpha \gamma^\mu \right\} | c_i \Psi_i \rangle - \frac{ig_{\mu\nu}}{(p_i - p_f)^2} \langle c_l \Psi_l | \left\{ -\frac{1}{2} ig_s u_\alpha \gamma^\nu \right\} | c_k \Psi_k \rangle \,, \tag{17}$$

The different color indices in equation (17) *i.e.* $(i,j,k,l) = R,G,B$ can determine for the complex octonion gluons are relevant to the interaction. Hence, the generalized octonion color factor is given by $\frac{1}{4}\left[\langle c_j \,|\, u_\alpha \,|\, c_i \rangle \; \langle c_l \,|\, u_\alpha \,|\, c_k \rangle\right]$, where, $\langle c_j \,|\, u_\alpha \,|\, c_i \rangle$ indicates the generalized color part for $(i,j,k,l = 1,2,3)$.

For $i = j = k = l = R = G = B$, $i = j = \bar{k} = \bar{l}$ and $\bar{i} = \bar{j} = \bar{k} = \bar{l}$ We can write the complex octonionic mediator for the interaction of same color quarks or antiquarks *i.e.* u_3 and u_0, while $\frac{-ig_{\mu\nu}}{(p_i-p_f)^2}$ is the propagator term, which is shown in Figure 2. Both the mediator contributes same value color factor in these type of interactions.

1. For $i = l = 1 = R$ and $k = j = 2 = G$, $i = l = 2 = G$ and $k = j = 1 = R$. The complex octonion mediator is given by u_{12} and $\bar{u}^*_{12}$. The value of color factor for this case is zero.
2. For $i = l = 1 = R$ and $k = j = 3 = B$, $i = l = 3 = B$ and $k = j = 1 = R$. Here, the complex octonion mediator is u_{67} and $\bar{u}^*_{67}$.
3. For $i = l = 2 = G$ and $k = j = 3 = B$, $i = l = 3 = B$ and $k = j = 2 = G, i = \bar{k} = 3 = B$ and $\bar{l} = j = 2 = G$.

Hence, the complex octonion mediator for these interactions are same *i.e.* u_{45} and $\bar{u}^*_{45}$. The complex octonion color factor for different color quark interactions are summarized in the *Table 4*.

Table 4. Table for color factor for different octonionic quarks (or anti-quarks) interactions.

$Q_c^{\mathcal{O}} \leftrightarrow Q_c^{\mathcal{O}} \longmapsto Q_c^{\mathcal{O}} \leftrightarrow Q_c^{\mathcal{O}}$	$Q_c^{\mathcal{O}} \bar{Q}_c^{\mathcal{O}} \longmapsto Q_c^{\mathcal{O}} \bar{Q}_c^{\mathcal{O}}$	$\bar{Q}_c^{\mathcal{O}} \bar{Q}_c^{\mathcal{O}} \longmapsto \bar{Q}_c^{\mathcal{O}} \bar{Q}_c^{\mathcal{O}}$	Color factor
$A \leftrightarrow A \Leftrightarrow A \leftrightarrow A$	$A \leftrightarrow A \Leftrightarrow A \leftrightarrow A$	$A \leftrightarrow A \Leftrightarrow A \leftrightarrow A$	$\frac{1}{4}$
$A \leftrightarrow B \Leftrightarrow B \leftrightarrow A$	$A \leftrightarrow A \Leftrightarrow B \leftrightarrow B$	$A \leftrightarrow B \Leftrightarrow B \leftrightarrow A$	0
$A \leftrightarrow B \Leftrightarrow A \leftrightarrow B$	$A \leftrightarrow B \Leftrightarrow A \leftrightarrow B$	$A \leftrightarrow B \Leftrightarrow A \leftrightarrow B$	0

Here, $A \leftrightarrow A$ or $B \leftrightarrow B$ show the same color quark interaction while $A \leftrightarrow B$ or $B \leftrightarrow A$ show the two different color quark interaction. The color charge is conserved at each vertex, the red quark changes into green quark emmiting complex octonion gluons and this gluons absorbed by green quark changes into red quark in $RG-GR$ interaction. Because of SU(3) color symmetry, the value of complex octonion color factor is same for $RG-RG, GR-GR, GB-GB, BR-BR$, and $BG-BG$ as $RB-RB$ interaction as well as for $RG-GR, GR-RG, GB-BG, BR-RB$, and $BG-GB$ is same as $RB-BR$ interaction. Similarly, we can study the octonion color quark-anti-quark and anti-quark-anti-quark interaction. No color factor can involved for unexpected color-interactions.

7. Conclusion

Complex octonion algebra is used to study a variety of fields. We have shown the connection of Gell-Mann matrices with complex octonion gluons. We have shown the possible interactions of color-color, color-anticolor and anticolor-anticolor quarks interactions with help of Feynman diagram in complex octonionic formalism. All the interactions are constructed in complex octonion space. We have concluded that in complex octonion formalism, there may be more than one possible mediator for interactions of color quarks, which implies that they have more than one vertex factor. Consequently, we have calculated the generalized matrix elements for these interactions by multiplying the Dirac spinor, vertex factor and the propagator term, also, calculated the color factor. We have extended the reformulation of QCD in complex octonion formalism. The Dirac color spinor represented the state of color quarks, which interact with the complex octonion gluon emitted or absorbed at each vertex. Hence, we made use of the complex octonion operator to convert the color quark states into another color quark states by mediating with complex octonionic gluon. Most importantly, we have discussed the conditions for valid and invalid interactions for complex octonion formalism. Complex octonion formalism of QCD or SU(3) colour group directly established the one to one mapping between the non-associativity and the theory of the strong interaction. These interactions are crucial in the formation of hadrons and hence we provided a firm reformulated theoretical framework in complex octonion formalism. The SU(3) color group is a part of the exceptional groups' octonionic structure. The SU(3) color symmetry for quarks and antiquarks has been demonstrated via the complexified octonion space, where the six complex-octonion operators are essentially identical to the SL(3, C) group generators. In SU(3) group theory, octonion plays a important role, because antisymmetric structure constant and octonionic structure constant are isomorphic. Quark theory's octonion analysis will provide us with a novel method in particle and theoretical physics.

Abbreviations

QCD	Quantum Chromodynamics
QED	Quantum Electrodynamics
COQCD	Complex Octonion Quantum Chromodynamics

ORCID

0009-0007-3621-5088 (Arun Kumar Rathore)
0000-0003-3652-1601 (Bhupesh Chandra Chanyal)

Author Contributions

Arun Kumar Rathore: Conceptualization, Formal Analysis, Methodology, Writing - original draft, Writing - review & editing

Bhupesh Chandra Chanyal: Conceptualization, Formal Analysis, Methodology, Writing - original draft, Writing - review & editing

Acknowledgments

I would like to express my sincere gratitude to Dr. B.C. Chanyal for their invaluable guidance and support throughout this research. I also extend my thanks to a conducive environment for my study.

Conflicts of Interest

Authors declare no conflicts of interest.

References

[1] Dickson, L. E. On quaternions and their generalization and the history of the eight square theorem. Annals of Mathematics. 1919, 20(3), 155-171. https://doi.org/10.2307/1967865

[2] P. G. Tait, "An elementary Treatise on Quaternions", Ed. 3, Cambridge Univ. Press (1875).

[3] Cayley, A. On Jacobi's Elliptic functions, in reply to the Rev. Brice Bronwin; and on Quaternions: To the editors of the Philosophical Magazine and Journal. The London, Edinburgh, and Dublin Philosophical Magazine and Journal of Science. 1845, 26(172), 208-211.

[4] Günaydin, M., & Gürsey, F. Quark structure and octonions. Journal of Mathematical Physics. 1973, 14(11), 1651-1667. https://doi.org/10.1063/1.16662 40

[5] Günaydin, M., & Gürsey, F. Quark statistics and octonions. Physical Review D. 1974, 9(12), 3387. https://doi.org/10.1103/PhysRevD.9.3387

[6] O'Neill, C. Octonions, the three flavours of matter and new kind of super-symmetry. Canadian Journal of Pure & Applied Sciences. 2021, 15(2), 5261-5268.

[7] Pushpa, Bisht, P. S., Li, T., & Negi, O. P. S. Quaternion octonion reformulation of quantum chromodynamics. International Journal of Theoretical Physics. 2011, 50, 594-606.

[8] Pushpa, Bisht, P. S., Li, T., & Negi, O. P. S. Quaternion-Octonion SU (3) Flavor Symmetry. International Journal of Theoretical Physics. 2012, 51, 1866-1875. https://doi.org/10.1007/s10773-011-1062-x

[9] Weng, Z. H. Color Confinement and Spatial Dimensions in the Complex-Sedenion Space. Advances in Mathematical Physics. 2017, 2017(1), 9876464-9876490.

[10] Chanyal, B. C., Bisht, P. S., Li, T., & Negi, O. P. S. Octonion quantum chromodynamics. International Journal of Theoretical Physics. 2012, 51, 3410-3422.

[11] Chanyal, B. C., Sharma, V. K., & Negi, O. P. S. Octonionic gravi-electromagnetism and dark matter. International Journal of Theoretical Physics. 2015, 54, 3516-3532.

[12] Wang, H., & Bian, X. The right inverse of Dirac operator in octonionic space. Journal of Geometry and Physics. 2017, 119, 139-145.

[13] S. Catto, Y. Gürcan, A. Khalfan, and Kurt, L. Algebraic formulation of hadronic supersymmetry based on octonions: new mass formulas and further applications. J. Phys. Conf. Ser. 2014, 563, 012006.

[14] Chanyal, B. C. On octonion quark confinement condition. Modern Physics Letters A. 2021, 36(37), 2150264. https://doi.org/10.1142/S0217732321502643

[15] Joshi, D. C. Introduction to Quantum Electrodynamics and Particle Physics. I. K. International Publishing House Pvt. Ltd. New Delhi, India. 2006, pp 126-129.

[16] Rathore, A. K., Sharma, V. K., & Chanyal, B. C. Complex octonion-based SU (3) flavor symmetry with the concept of octet representation for quark composite particles. The European Physical Journal Plus. 2024, 139(3), 210. https://doi.org/10.1140/epjp/s13360-024-05021-5

[17] Chanyal, B. C., & Pathak, M. Quaternionic approach to dual magnetohydrodynamics of dyonic cold plasma. Advances in High Energy Physics. 2018, 2018(1), 7843730-7843743. https://doi.org/10.1155/2018/7843730

[18] Chanyal, B. C. A relativistic quantum theory of dyons wave propagation. Canadian Journal of Physics. 2017, 95(12), 1200-1207. https://doi.org/10.1139/cjp-2017-0080

[19] Chanyal, B. C., Bisht, P. S., & Negi, O. P. S. Generalized octonion electrodynamics. International Journal of Theoretical Physics. 2010, 49, 1333-1343.

[20] Weng, Z. H. Color Confinement and Spatial Dimensions in the Complex-Sedenion Space. Advances in Mathematical Physics. 2017, 2017(1), 9876464-9876491.

[21] Wang, H., & Bian, X. The right inverse of Dirac operator in octonionic space. Journal of Geometry and Physics. 2017, 119, 139-145.

[22] Weng, Z. H. Torques and angular momenta of fluid elements in the octonion spaces. Mathematical Methods in the Applied Sciences. 2023, 46(5), 5810-5820.

[23] Weng, Z. H. Some spin textures relevant to magnetic moments in the octonion spaces. AIP Advances. 2023, 13(8), 085016.

[24] Weng, Z. H. Conserved quantities of vectorial magnitudes within the material media. International Journal of Modern Physics A. 2023, 38(01), 2350001.

[25] Furey, C. Three generations, two unbroken gauge symmetries, and one eight-dimensional algebra. Physics Letters B. 2018, 785, 84-89.

[26] Kugo, T., & Townsend, P. Supersymmetry and the division algebras. Nuclear Physics B. 1983, 221(2), 357-380.

[27] Chanyal, B. C. Octonion generalization of Pauli and Dirac matrices. International Journal of Geometric Methods in Modern Physics. 2015, 12(01), 1550007-1550031.

[28] Marques-Bonham, S., Chanyal, B. C., & Matzner, R. Yang–Mills-like field theories built on division quaternion and octonion algebras. The European Physical Journal Plus. 2020, 135(608), 1-34. https://doi.org/10.1140/epjp/s13360-020-00626-y

[29] Chanyal, B. C. Octonion symmetric Dirac-Maxwell equations. Turkish Journal of Physics. 2014, 38(2), 174-186. https://doi.org/10.3906/fiz-1401-14

[30] Chanyal, B. C. Octonionic matrix representation and electromagnetism. Journal of the Korean Physical Society. 2014, 65, 1715-1728.

[31] Chanyal, B. C., Bisht, P. S., & Negi, O. P. S. Octonion and conservation laws for dyons. International Journal of Modern Physics A. 2013, 28(26), 1350125.