

Research Article

Effects of Specific Heat Models on the Performance of the Irreversible Over-Expansion Cycle

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Abstract

Since specific heat variation of working fluid has a great influence on the performance of the engine cycle, it is indispensable to evaluate the effect of different specific heat models on the cycle performance. Different specific heat models including linear, logarithmic, and polynomial ones had already been applied to calculation of the power and efficiency of internal combustion engine cycle, of which polynomial specific heat reflects its detailed variation with temperature. This study aims at deriving the analytic equation with respect to ecological function of the irreversible Dual-Miller cycle (DMC) by using the specific heat model of polynomial, and comparing results of performance calculation based on constant, linear, logarithmic specific heat models with that of polynomial specific heat model. In order to reflect the characteristics of DMC engine in more detail, the cycle involving heat transfer loss, friction loss, and internal irreversible loss of heat expansion as well as variation of specific heat is considered here. In addition, the effects of cycle parameters such as compression ratio and miller cycle ratio, cut-off ratio on the maximum power output and the maximum efficiency conditions are analyzed. The results indicate that logarithmic specific heat model is simpler than polynomial one, but it gives nearly approximate result to polynomial one in calculation of the performance of DMC and AC (Atkinson cycle). The presented models and results are expected to provide guidelines for the design and optimization of DMC engines.

Keywords

Over-Expansion Cycle, Specific Heat, Polynomial, Logarithmic

1. Introduction

In the recent years, numerous research works on the performances of air standard ICE cycles are focused on the reduction of exhaust emission and improvement of the engine performance. Numerous investigations have been conducted to improve engine performance and reduce NO_x emissions using the Miller cycle. [1-5]. The Miller cycle was proposed by R. H. Miller. The Miller cycle is a modern modification of the Atkinson cycle (i.e. a complete expansion cycle).

So far, many investigations have been published to confirm the thermodynamic merits of Miller cycles and study the effects of important engine parameters on the performance of the over-expansion cycle. The performance analysis and optimization of the Air-standard ICE cycles can be divided into three classes according to the specific heat (SH) of WF models, including the constant SH model, variable SH model and variable specific heat ratio (SHR) model [1].

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For the Miller cycle, many scholars studied the power output, thermal efficiency, and ecological function performance characteristics of the Air-standard Miller cycle models with constant SH. Ge [6] optimized power output and efficiency for an irreversible DDC with heat transfer loss, friction loss. Zhao et al. [7] optimized power output and efficiency of the irreversible OMC with heat transfer loss. They optimized power output and efficiency of the cycle with respect to the pressure ratio (maximum/minimum) of the working substance and analyzed optimum criteria of some important parameters such as the power output, efficiency. Mousapour et al. [8] analyzed the power and efficiency of the irreversible OMC and investigated the influences of the Miller cycle ratio and initial temperature, heat transfer on compression ratio. Gonca et al. [9] made a performance analysis based on the power output, thermal efficiency, maximum power output and maximum thermal efficiency criteria for an air standard irreversible DMC with late inlet valve closing (LIVC) version which covers internal irreversibility owing to the irreversible-adiabatic processes. They studied the influences of cycle temperature ratio and cycle pressure ratio of the DMC on the optimum performances as well. Gonca et al. [10] investigated the performance of the irreversible Dual-Miller cycle with LIVC, taking into consideration the influences of heat transfer loss and internal irreversibility. Gonca et al. [11] carried out an ecological performance analyses and optimization of irreversible gas cycle engines such as Atkinson cycle, Otto cycle, Diesel cycle, Miller cycle, DDC, DMC engines based on the ecological coefficient of performance criterion which covers internal irreversibility, heat leak and finite-rate of heat transfer. Gonca et al. [12] carried out the comparative performance analyses of the irreversible OMC engine, Diesel Miller cycle engine and DMC engine based on the maximum dimensionless power output, maximum dimensionless power density. Ebrahimi [13] showed that the work output the DMC has linear relationship with the intake temperature, friction coefficient, working fluid, chemical energy release by combustion and heat-transfer coefficient. Wu et al. [14] investigated power output, thermal efficiency and ecological function characteristics of an endo-reversible DMC with finite speed of the piston. Ust et al. [15] carried out comparative performance analysis and optimization based on the non-dimensional power output and thermal efficiency criteria for an irreversible DMC. The effects of the design parameters such as cycle compression ratio, cut-off ratio and Miller cycle ratio were investigated. You et al. [16] described the model of the DMC with two polytropic processes and heat transfer loss and analyzed influences of compression ratio, cut-off ratio, polytropic exponent on the performance. S. Abedinneshad et al. [17] investigated the performance and the effect of critical parameters on the performance of the DMC with two polytropic processes. Multi-objective optimization was performed to obtain the best point of performance of the DMC. Researches mentioned above investigated an air standard Miller cycle with a constant specific heat.

In addition, studies of an air standard Miller cycle have been performed on the performance analysis and optimization of the cycle with the variable specific heat of WF. Al-Sarkhi et al. [18] investigated the influences of the linearly variable specific heat characteristic on the OMC. Al-Sarkhi et al. [19] analyzed the performance of a Miller engine under different specific heat models (i.e., constant, linear, and fourth order polynomial). Their results show that an accurate model such as fourth order polynomial is essential for accurate prediction of cycle performance. Lin and Hou [20] investigated the effects of heat loss characterized by a percentage of fuel's energy, friction and variable specific heats of working fluid on the performance of an air-standard Miller cycle. Ebrahimi et al. [21] investigated the performance of the irreversible Miller cycle with the nonlinear specific heats of a working fluid, the frictional loss related to the mean velocity of the piston, and heat transfer loss through the cylinder wall. Lin et al. [22] investigated the performance of an irreversible air standard Miller cycle in a four-stroke free-piston engine with the variable specific heats, the heat transfer loss as a percentage of fuel's energy and the friction loss. Dobrucali [23] carried out the performance analysis for an irreversible OMC has been presented by taking into consideration heat transfer effects, frictions, variable specific heats. The results demonstrate that the engine design and running parameters have considerable effects on the cycle thermodynamic performance. Gonca [24] carried out the performance investigation of the power, power density, effective efficiency of a DMC engine with nonlinear specific heats. In this study, influences of the engine design and operating parameters on the performance characteristics and energy losses of a diesel engine have been investigated. G. Gonca and B. Sahin [25] numerically and empirically investigated the influences of the combination of the steam injection method, Miller cycle and turbo charging methods on the performance of a direct injection diesel engine. This report also showed a comprehensive comparison of the results of experiments. Ge et al. [26] investigated the ecological function performances of an irreversible Otto cycle considering internal irreversibility loss, friction loss and heat transfer loss, nonlinear specific heats. Ebrahimi [27] analyzed the effect of the variable specific heats and engine speed on the power output and the efficiency of cycle. Wu et al. [28] investigated an air-standard irreversible DMC with linearly variable SHR, and with heat transfer loss, friction loss and other internal irreversible losses.

In most of the previous publications related to the FTT methods, the losses for the heat-transfer and friction were not considered or treated as particularly simple forms. In [29], Zhao et al. demonstrated that the engine design parameters and cycle conditions had significant effects on the heat-transfer coefficient, thus the heat losses through the cylinder wall; and the boundary friction primarily related to the cylinder pressure also had non-negligible influences on the engine cycle performances.

From previous studies, several models of specific heat, such as constant, linear, logarithmic [30, 31], and polynomial, have been used to calculate the performance of the internal combustion engine cycle. It can be seen that the variation of the specific heat of the working fluid has a significant effect on the performance of the cycle, where the polynomial specific heat reflects the variation of the specific heat with temperature in detail. However, a comprehensive comparison of the performance of several specific heat models including logarithmic specific heat with the results of polynomial specific heat model and the cyclic parameter effect analysis to maximize performance of DMC and AC in the logarithmic

specific heat model do not appear to have been published. Thus, in this paper the effect of different specific heat models (i.e., constant, linear, and polynomial, logarithmic) on the power output (P), efficiency (η) of the irreversible DMC and AC have been evaluated.

2. Cycle Model and Performance Analysis

P-v and T-s diagrams of the considered irreversible air-standard DMC (1-2-3-4-5-6-1) are shown in Figure 1.

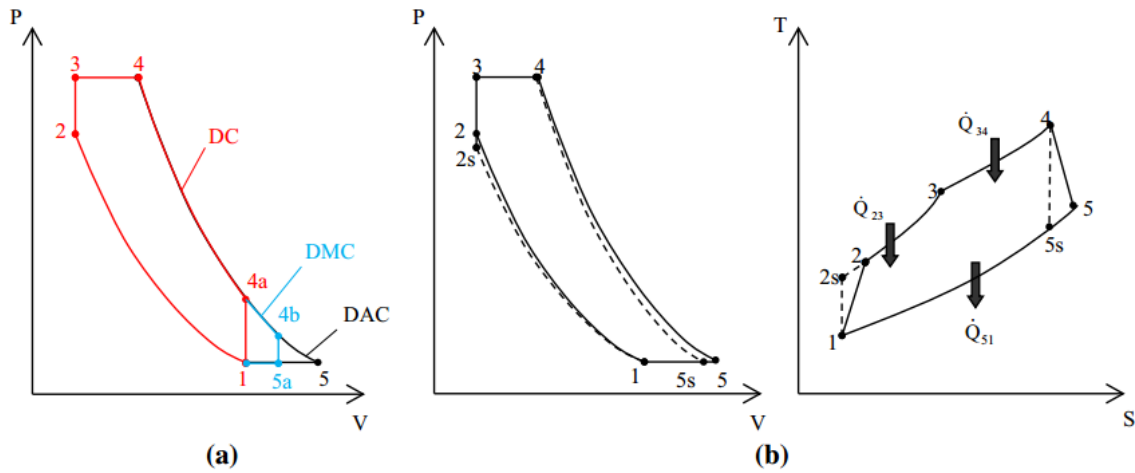


Figure 1. P-v and T-s diagrams for an air-standard irreversible DMC.

In the diagrams, the process 1-2s is an isentropic compression, while the process 1-2 takes internal irreversibilities into account. The heat addition is done in two steps: processes 2-3 and 3-4, they are constant volume and constant pressure heat addition processes respectively. Process 3-4 also makes up the first part of the power stroke. Process 4-5s is an isentropic expansion while the process 4-5 takes into account internal irreversibilities. The heat rejection occurs in two steps: 5-6 and 6-1 are constant volume and constant pressure heat rejection processes respectively.

The design parameters of compression ratio r_c , cut-off ratio ρ and Miller cycle ratio r_M for the DMC and AC are defined as follows:

$$r_c = v_1/v_2 \quad (1)$$

$$\rho = v_4/v_3 = T_4/T_3 \quad (2)$$

$$r_M = v_6/v_1 = T_6/T_1 \quad (3)$$

$$\alpha = T_4/T_1 \quad (4)$$

$$r_A = V_5/V_1 = T_5/T_1 \quad (5)$$

According to Ref. [23], for the temperature range of 300 ~ 3500 K, the specific heats of air can be written as;

$$C_p = a_1T^2 + a_2T^{1.5} + a_3T + a_4T^{0.5} + a_5 + a_6T^{-1.5} + a_7T^{-2} + a_8T^{-3} \quad (6)$$

$$C_v = a_1T^2 + a_2T^{1.5} + a_3T + a_4T^{0.5} + a'_5 + a_6T^{-1.5} + a_7T^{-2} + a_8T^{-3} \quad (7)$$

$$R = C_p - C_v \quad (8)$$

where C_p - specific heat at constant pressure, C_v - specific heat at constant volume, R is gas constant and equals to 0.287 kJ/kgK, and T is the absolute temperature.

$$a_1 = 2.506 \times 10^{-11}, \quad a_2 = 1.454 \times 10^{-7}, \quad a_3 = -4.246 \times 10^{-7}, \quad a_4 = 3.162 \times 10^{-5}, \\ a_5 = 1.3303, \quad a'_5 = 1.0433, \quad a_6 = -1.512 \times 10^4, \quad a_7 = 3.063 \times 10^5, \quad a_8 = -2.212 \times 10^7$$

The heat added to the working fluid, during constant volume process 2→3 and constant pressure process 3-4, is expressed as

$$\dot{Q}_{in} = \dot{m} \left(\int_{T_2}^{T_3} C_v dT + \int_{T_3}^{T_4} C_p dT \right) \quad (9)$$

$$\dot{Q}_{in} = \dot{m} \left[\left(\frac{a_1}{3} T^3 + \frac{a_2}{2.5} T^{2.5} + \frac{a_3}{2} T^2 + \frac{a_4}{1.5} T^{1.5} + a_5 T - \frac{a_6}{0.5} T^{-0.5} - a_7 T^{-1} - \frac{a_8}{2} T^{-2} \right) \right]_{T_2}^{T_4} - (a_5 - a'_5) T \Big|_{T_2}^{T_3} \quad (10)$$

where m is flow rate of working fluid, kg/s or mol/s.

The heat rejected of DMC, during constant volume process 5-6 and constant pressure process 6-1, is as follows

$$\dot{Q}_{out,M} = \dot{m} \left(\int_{T_6}^{T_5} C_v dT + \int_{T_1}^{T_6} C_p dT \right) \quad (11)$$

$$\dot{Q}_{out,M} = \dot{m} \left[\left(\frac{a_1}{3} T^3 + \frac{a_2}{2.5} T^{2.5} + \frac{a_3}{2} T^2 + \frac{a_4}{1.5} T^{1.5} + a_5 T - \frac{a_6}{0.5} T^{-0.5} - a_7 T^{-1} - \frac{a_8}{2} T^{-2} \right) \right]_{T_1}^{T_5} - (a_5 - a'_5) T \Big|_{T_6}^{T_5} \quad (12)$$

The heat rejected of AC is as follows

$$\dot{Q}_{out,A} = \dot{m} \int_{T_1}^{T_5} C_p dT \quad (13)$$

$$\dot{Q}_{out,A} = \dot{m} \left(\frac{a_1}{3} T^3 + \frac{a_2}{2.5} T^{2.5} + \frac{a_3}{2} T^2 + \frac{a_4}{1.5} T^{1.5} + a_5 T - \frac{a_6}{0.5} T^{-0.5} - a_7 T^{-1} - \frac{a_8}{2} T^{-2} \right) \Big|_{T_1}^{T_5} \quad (14)$$

Because Cp and Cv, the specific heat ratio k are dependent on temperature, the equation often used for a reversible adiabatic process with constant k cannot be used for a reversible adiabatic process with a variable k.

According to Refs. [28], the equation for the reversible adiabatic process with variable specific heat can be written as

$$\Delta S = S_j - S_i = \int_i^j C_v \frac{dT}{T} + R \ln \frac{v_j}{v_i} = 0 \quad (15)$$

During the isentropic processes 1 to 2s and 4 to 5s Eq. (10) can be used to get a relation between T₁ and T_{2s} and another one between T₄ and T_{5s}.

Therefore, after substituting the value for Cv from Eq. (6) and performing the integration of Eq. (12), equations describing processes 1→2s and 4→5s can be respectively expressed as follows

$$\left(\frac{a_1}{2} T^2 + \frac{a_2}{1.5} T^{1.5} + a_3 T + 2a_4 T^{0.5} + a'_5 \ln T - \frac{a_6}{1.5} T^{-1.5} - \frac{a_7}{2} T^{-2} - \frac{a_8}{3} T^{-3} \right) \Big|_{T_1}^{T_{2s}} = R \ln \frac{V_1}{V_{2s}} = R \ln r_c \quad (16)$$

$$\left(\frac{a_1}{2} T^2 + \frac{a_2}{1.5} T^{1.5} + a_3 T + 2a_4 T^{0.5} + a'_5 \ln T - \frac{a_6}{1.5} T^{-1.5} - \frac{a_7}{2} T^{-2} - \frac{a_8}{3} T^{-3} \right) \Big|_{T_{5s}}^{T_4} = R \ln \frac{V_{5s}}{V_4} = R \ln r_e \quad (17)$$

The internal irreversibilities in irreversible adiabatic processes 1- 2s and 4- 5s can be defined as follows:

$$\eta_c = \frac{(T_{2s} - T_1)}{(T_2 - T_1)} \quad (18)$$

$$\eta_e = \frac{(T_4 - T_5)}{(T_4 - T_{5s})} \quad (19)$$

where η_c and η_e are irreversible compression and expansion efficiencies, respectively, which can reflect all irreversibilities including FL for the processes 1- 2s and 4- 5s.

The heat loss through the cylinder wall to the coolant is very complicated in a real combustion process. It is assumed that the cylinder wall temperature is constant. Thus, the heat loss through the cylinder wall per second can be evaluated as [32]:

$$\dot{Q}_{tr} = h_c A_{cc} (T_m - T_w) \quad (20)$$

where the h_c is the convective heat-transfer coefficient; the A_{cc} is the approximate surface area of combustion chamber; T_m and T_w are mean cylinder gas temperature and cylinder wall temperature, respectively. The heat-transfer coefficient h_c can be expressed as [29]:

$$h_c = CB^{-0.2} P^{0.8} w^{0.8} T_m^{-0.55} \quad (21)$$

In Eq. (12), the T is mean gas temperature, which is the same as the T_m in Eq. (11). The average in-cylinder charge velocity w in Eq. (12) may be expressed as:

$$w = \left[C_1 \bar{S}_p + C_2 \frac{V_d T_r}{P_r V_r} (p - P_m) \right] \quad (22)$$

In Eq. (13), V_d is the displaced volume; p is the instantaneous cylinder pressure; and P_r , V_r , T_r are the working-fluid pressure, volume and temperature at a reference state. P_m is the motored cylinder pressure at the same crank angle as p . As shown in Figure 1, under ideal conditions P_m is equal to P_2 . Select the point 2 as a reference state; and use mean incylinder pressure to substitute for the instantaneous cylinder gas pressure. Thus, these values are given as:

$$P_m = P_2 \quad (23)$$

$$p = \frac{(P_3 + P_2)}{2} \quad (24)$$

$$P_r = P_2 \quad (25)$$

$$V_r = V_2 = \frac{1}{4} \pi B^2 \frac{L}{r_c - 1} \quad (26)$$

$$T_r = T_2 \quad (27)$$

applying the ideal gas equation:

$$PV = mRT \quad (28)$$

The combustion chamber surface area is approximately estimated as:

$$A_{cc} = \frac{1}{2} \pi B^2 + \pi BL \frac{1}{r_c - 1} \quad (29)$$

Piston assemblies are the dominant source of engine friction losses, which is primarily dominated by the piston ring. The piston ring tension and the gas pressure p_c behind the compression ring cause radial force and lead to the boundary friction. The piston ring tension is almost constant while the gas pressure p_c is related with the engine design and operation parameters. The boundary friction may be expressed as:

$$(FMEP)_{boundary} = a \times \frac{L}{B^2} \times P_{max} \quad (30)$$

where the coefficient a is multiplied to consider the influence of peak cycle pressure P_{max} ; and can be calibrated to include the pressure-affected friction in camshaft and crankshaft system in a real engine.

The viscous piston friction under hydrodynamic lubrication conditions is primarily related with mean piston speed. The viscous piston friction is correlated by:

$$(FMEP)_{hydrodyn} = b \times \frac{A_{p,eff}}{LB^2} \times \bar{S}_p \quad (31)$$

where $A_{p,eff}$ is the effective area of piston skirt in contact with the cylinder liner. The coefficient b is multiplied to include speed-dependent friction in the camshaft and crankshaft system in a real engine.

Therefore, the friction-lost power for the Atkinson cycle can be expressed as [29]:

$$P_f (kW) = \frac{FMEP (kpa) \times V_d (dm^3) \times N (r/s)}{2 \times 10^3} \quad (32)$$

where the N is engine rotating speed and can be expressed as:

$$N = \bar{S}_p / 2L \quad (33)$$

Therefore, the power output and efficiency of the DMC are

$$P = \dot{Q}_{in} - \dot{Q}_{out} - P_{\mu} \quad (34)$$

$$\eta = \frac{\dot{Q}_{in} - \dot{Q}_{out} - P_{\mu}}{\dot{Q}_{in} + \dot{Q}_{leak}} \quad (35)$$

In order to evaluate the effects of specific heat variation on the performance of DMC, different specific heat models are used. The heat addition and rejection of the working fluid with a constant specific heat (C_p , C_v , $k=\text{const}$) can be written as [16, 17].

$$\dot{Q}_{in} = \dot{m}C_v[(T_3 - T_2) + k(T_4 - T_3)] \quad (36)$$

$$\dot{Q}_{out,M} = \dot{m}C_v[(T_5 - T_6) + k(T_6 - T_1)] \quad (37)$$

$$\dot{Q}_{out,A} = \dot{m}kC_v(T_5 - T_1) \quad (38)$$

$$\frac{T_2}{T_1} = \frac{\eta_c + r_c^{k-1} - 1}{\eta_c} \quad (39)$$

$$\frac{T_5}{T_4} = 1 - \eta_e \left(1 - \frac{1}{r_e^{k-1}} \right) \quad (40)$$

$C_p = 1.005 \text{ kJ/kg K}$, $C_v = 0.718 \text{ kJ/kg K}$, $k = 1.4$, $\alpha = 7$, $\eta_c = 0.9$, $\eta_e = 0.9$

The linear specific heat model of Al-Sarkhi et al. is shown [18, 19]. The heat addition and rejection of the working fluid can be written as;

$$C_p = a + k_1 T, \quad C_v = b + k_1 T \quad (41)$$

$$R = C_p - C_v = a - b \quad (42)$$

$b = 0.6888 \sim 0.8036 \text{ kJ/kg K}$, $k_1 = 0.000133 \sim 0.00034 \text{ kJ/kg K}^2$.

$$\dot{Q}_{in} = \dot{m} [a(T_4 - T_3) + b(T_3 - T_2) + 0.5k_1(T_4^2 - T_2^2)] \quad (43)$$

$$\dot{Q}_{out,M} = \dot{m} [a(T_6 - T_1) + b(T_5 - T_6) + 0.5k_1(T_5^2 - T_1^2)] \quad (44)$$

$$\dot{Q}_{out,A} = \dot{m} [a(T_5 - T_1) + 0.5k_1(T_5^2 - T_1^2)] \quad (45)$$

Equations describing processes 1→2s and 4→5s can be respectively expressed as follows;

$$k_1(T_{2s} - T_1) + b \ln \frac{T_{2s}}{T_1} = R \ln r_c \quad (46)$$

$$k_1(T_4 - T_{5s}) + b \ln \frac{T_4}{T_{5s}} = R \ln r_e \quad (47)$$

According to NISTJANAF thermochemical tables [30, 31], temperature-dependent changes of specific heats for air were obtained in form of logarithmic function.

$$C_v = a_0 + k_0 \ln T, \quad C_p = b_0 + k_0 \ln T \quad (48)$$

$$R = C_p - C_v = b_0 - a_0 \quad (49)$$

$a_0 = -0.118$, $k_0 = 0.142$

The heat addition and rejection of the working fluid can be written as;

$$\dot{Q}_{in} = \dot{m} \left[a_0(T_3 - T_2) + b_0(T_4 - T_3) + k_0(T \ln T - T) \right]_{T_2}^{T_4} \quad (50)$$

$$\dot{Q}_{out,M} = \dot{m} \left[a_0(T_5 - T_6) + b_0(T_6 - T_1) + k_0(T \ln T - T) \right]_{T_1}^{T_5} \quad (51)$$

$$\dot{Q}_{out,A} = \dot{m} \left[b_0(T_5 - T_1) + k_0(T \ln T - T) \right]_{T_1}^{T_5} \quad (52)$$

Equations describing processes 1→2s and 4→5s can be respectively expressed as follows

$$a_0 \ln T \Big|_{T_1}^{T_{2s}} + \frac{1}{2} k_0 (\ln T)^2 \Big|_{T_1}^{T_{2s}} = R \ln r_c \quad (53)$$

$$a_0 \ln T \Big|_{T_{5s}}^{T_4} + \frac{1}{2} k_0 (\ln T)^2 \Big|_{T_{5s}}^{T_4} = R \ln r_e \quad (54)$$

3. Numerical Example

An investigation has been performed on the effects of specific heat variation and heat transfer on the performance of the irreversible DMC based on the power output, thermal efficiency, and ecological function. Performance of the DMC has been compared with different specific heat models (i.e., constant, linear, and nonlinear polynomial). The effect of the design parameters such as compression ratio, pressure ratio, cut-off ratio and Miller cycle ratio on the performance of the irreversible DMC have also been investigated.

When the values of T_1 , r_c , r_m , α , ρ , η_c and η_e are given, temperatures T_{2s} , T_2 , T_3 , T_{5s} , T_5 and T_6 can be calculated by using Eqs. (7)-(8) and (15)-(18). Then, substituting those temperatures into Eqs. (24), (25) and (32) yields P , η and E of the DMC. Assuming the ambient temperature T_0 is 300 K, the intake temperature T_1 should be higher than T_0 . so it can be assumed to be 340~400 K [28], $\alpha=7$. According to Ref. [28, 29], the optimal r_m for MP is between 2 and 3, thus, ρ and r_m can be both taken as 1~4. To compare the present study with

previous studies, in the calculations, the parameters are taken as; $\mu = 1.1 \text{ kg/s}$, $L = 0.06 \text{ m}$, $n = 30 \text{ s}^{-1}$, $\dot{m} = 1 \text{ mol/s}$, $\eta_c = 0.9$, $\eta_e = 0.9$

The variance of power and efficiency according to variance of compression ratio in constant, linear, polynomial and log-

arithmic specific heat models with fixed cut-off ratio and Miller cycle ratio is shown in Figure 2. The results show that the power output for linear specific heat model is always greater than a constant and polynomial and logarithmic model.

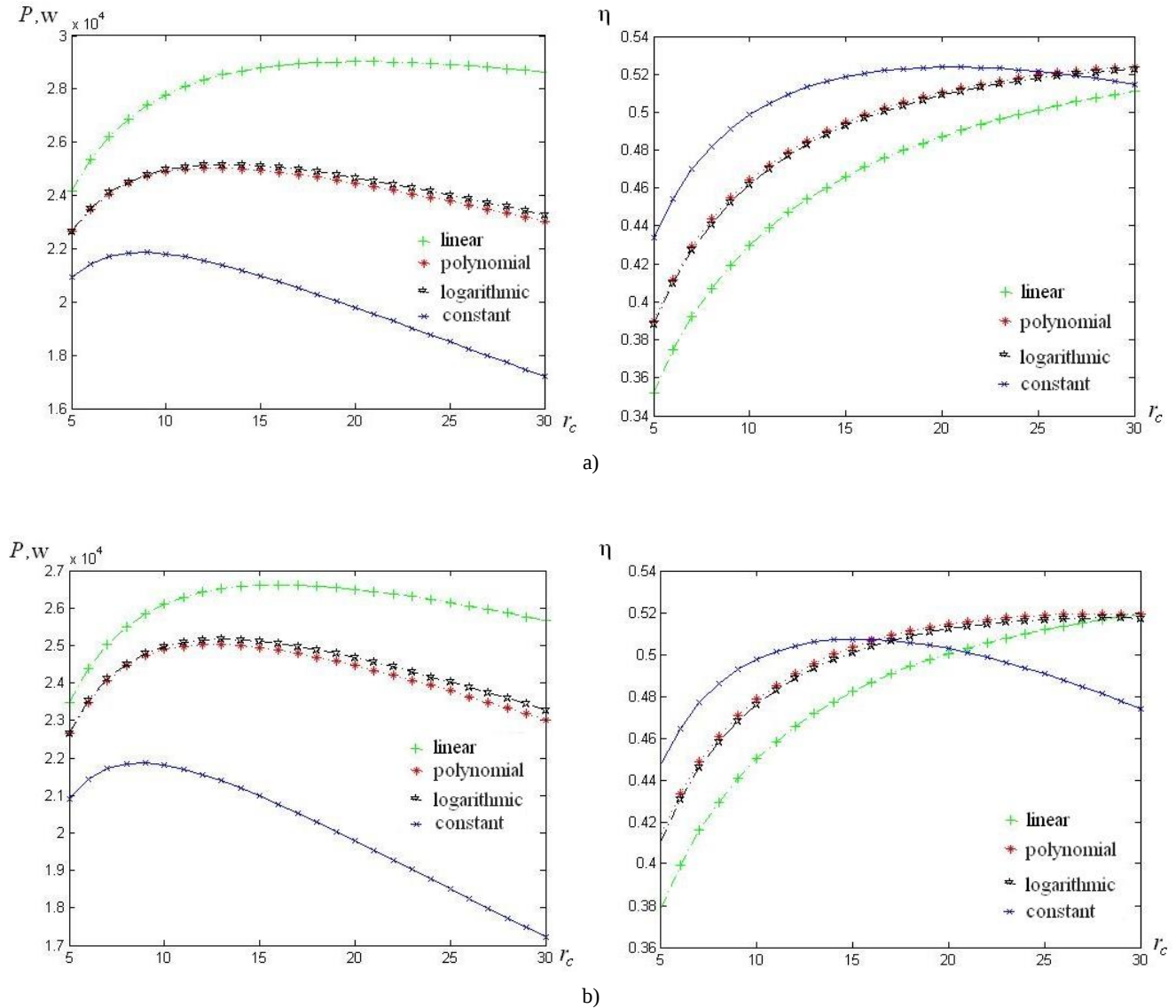


Figure 2. Effect of SH models on the power output and efficiency with compression ratio; (a) $r_M = 2$, $\rho = 2$), (b) $r_M = 2$, $\rho = 1.2$.

The compression ratio in state of maximum power in case of linear is also greater than in case it is logarithmic, polynomial or constant.

$$\text{i.e. } r_c(c = \text{const}) < r_c(c = \text{polynomial}) < r_c(c = \log) < r_c(c = \text{linear})$$

Also, the results show that the constant value and linear model of the specific heat might over or underpredict the power output and efficiency of the Miller cycle compared to polynomial specific heat. This was already mentioned in [18, 19]. Therefore, the constant value and linear model of the

specific heat should not be compromised. The amount of heat added to and rejected from the cycle involves an integration of the specific heat equation of working fluid. Even if the difference between values of the specific heats in the different models is small but after the integration operation this leads to larger differences. Consequently, there exists a considerable difference in power and efficiency in linear and constant SH models compared to polynomial and logarithmic models because of the increase of specific heat caused by increase of temperature due to increase of compression ratio.

Especially, the results show that logarithmic specific heat model is simpler than polynomial one, but it gives nearly approximate result in calculation of output power and efficiency to polynomial one. As in Table 1, compression ratio which provides maximum output power (or maximum efficiency) has the differences up to 10% in polynomial and logarithmic model. Thus, the polynomial and logarithmic model should be used.

Table 1. Compression ratios at maximum power output or maximum efficiency r_c ($r_M = 2$, $p = 2$).

	$r_c (P_{\max})$	$r_c (\eta_{\max})$
polynomial	12.65	10.95
logarithmic	13.05	12.05
polynomial	12.53	11.02
logarithmic	13.15	12.15

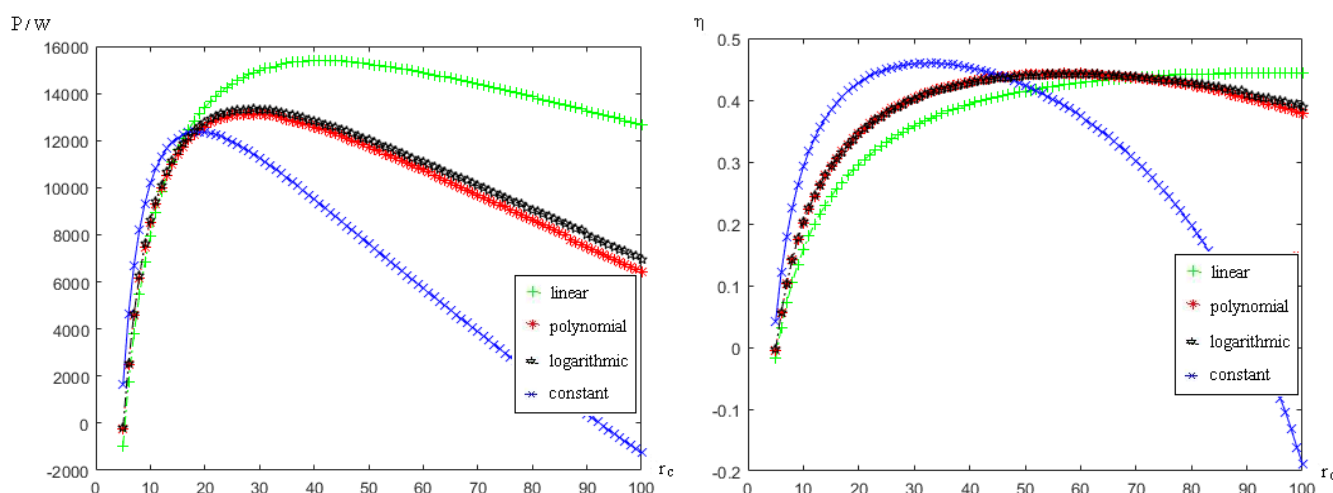


Figure 3. Effects of specific heat models on the performance of the irreversible AC.

4. Conclusions

We compared the effects of specific heat models on the performance of the overexpansion cycle (irreversible dual Miller cycle, irreversible dual Atkinson cycle). The relations between the power output and the compression ratio and between the thermal efficiency and the compression ratio of the cycle are derived.

The results of the study can be concluded as follows.

- 1) It can be seen that the polynomial specific heat model reflects the performance of the cycle in detail, especially the logarithmic specific heat model is simpler than the polynomial specific heat, but it reflects a close approximation in the performance prediction.
- 2) The calculated results for the power are larger than those for the linear specific heat model, the logarithmic specific heat, the polynomial specific heat and the constant specific heat model.

Also, the compression ratio that maximizes the power is larger than that of the logarithmic, polynomial and constant specific heat models.

The results show that logarithmic specific heat model is simpler than polynomial one, but it gives nearly approximate

result in calculation of output power and efficiency to polynomial one. The models and results are expected to be very important in the study of optimization design and choosing the cycle parameters of an irreversible over-expansion cycle.

Abbreviations

AC	Atkinson Cycle
DDC	Dual-Diesel Cycle
DMC	Dual-Miller Cycle
FTT	Finite Time Thermodynamics
HTL	Heat Transfer Loss
ICE	Internal Combustion Engine
LIVC	Late Inlet Valve Closing
MEF	Maximum Thermal Efficiency
MP	Maximum Power Output
MPD	Maximum Dimensionless Power Density
OC	Otto Cycle
OMC	Otto Miller Cycle
SH	Specific Heat
SHR	Specific Heat Ratio

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Author Contributions

Rim Guk Jang: Conceptualization, Methodology, Writing - original draft

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Conflicts of Interest

The authors declare no conflicts of interest.

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