American Journal of Mechanics and Applications
2025, Vol. 12, No. 2, pp. 33-39
https://doi.org/10.11648/j.ajma.20251202.12

Research Article

Determination of Forces Acting on a Helical Gear Transmission with an Elastic Element

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Abstract

This paper presents a theoretical analysis of the forces acting on a helical gear transmission with an elastic element. The research investigates the effects of elastic deformation on the load distribution within the gear system, focusing on the influence of the elastic component on axial, radial, and tangential forces. The study employs mathematical models that account for the complex interactions between the helical gears and the elastic element, aiming to optimize the force transmission and enhance the system's efficiency. Key findings reveal that introducing an elastic element in the helical gear transmission can significantly reduce impact loads, leading to improved durability and operational lifespan of the system. Additionally, the research identifies optimal ranges for the stiffness coefficient of the elastic component, ensuring effective load amortization and minimizing the risk of excessive wear or failure. The proposed model provides a comprehensive understanding of how elastic deformation influences the performance of helical gear systems, particularly under varying loads and operational conditions. The results presented offer valuable insights for the design and optimization of gear mechanisms in engineering applications where durability and efficiency are critical. These findings contribute to the broader field of mechanical transmission systems, particularly in applications where gear longevity and reliability are essential.

Keywords

Helical Gear Transmission, Elastic Element, Force Analysis, Torque Distribution, Stiffness Coefficient, Deformation, Gear Meshing, Load Reduction, Mechanical Efficiency

1. Introduction

In cylindrical gear transmissions, when a driving gear is subjected to a driving torque M1, the driven gear experiences a resisting torque. In general, the interaction between the gear teeth generates normal and frictional forces. However, in helical cylindrical gear transmissions, the resultant interaction force is inclined and consists of three components: axial force along the shaft, radial force perpendicular to the shaft, and tangential force responsible for rotation [1].

2. Experimental Procedures

The schematic diagram of the acting forces is shown in Figure 1. In general, when the effect of elastic elements is not considered, the total force Fn acting on the helical tooth is divided into three components. [2, 3]:

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Received: 18 April 2025; **Accepted:** 27 April 2025; **Published:** 29 May 2025

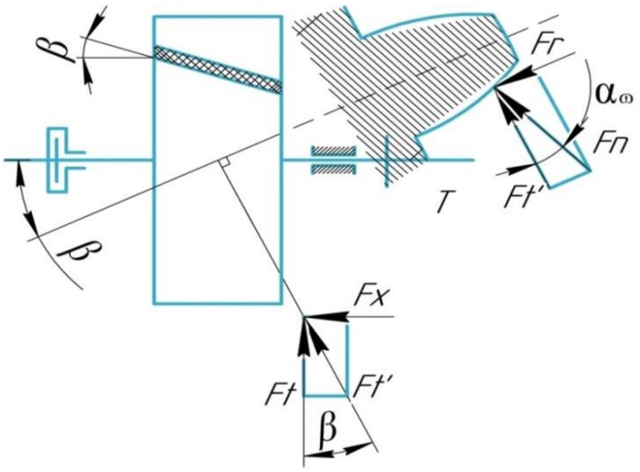


Figure 1. Schematic representation of the forces acting on the helical tooth in the proposed helical cylindrical gear transmission with an elastic element.

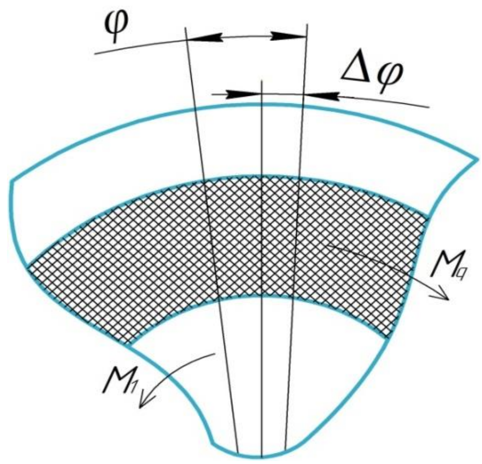


Figure 2. Deformation scheme of the elastic element in the composite helical gear under the influence of the torsional moment M1.

Tangential force,

$$F_t = \frac{2M_1}{d_1}; \quad (1)$$

Axial force

$$F_x = F_t \cdot \tan\beta; \quad (2)$$

Radial force perpendicular to the shaft,

$$F_r = F'_t \cdot \tan\alpha_\omega = F_t \cdot \frac{\tan\alpha_\omega}{\cos\beta}; \quad (3)$$

Total force,

$$F_n = \frac{F'_t}{\cos\alpha_\omega} = \frac{F_t}{\cos\alpha_\omega \cdot \cos\beta} \quad (4)$$

Here, M_1 is the driving torque in the helical gear, d_1 is the pitch circle diameter of the driving helical gear, β is the helix angle of the teeth, and αω is the transmission angle.

The proposed composite elastic element gears, when engaged, partially absorb the force due to the deformation of the elastic element. Although the overall deformation occurs, the deformations in the direction of each force component partially dampen these forces. Therefore, we determine the methods for calculating forces considering the deformation and amortization of the elastic element in each force direction. Figure 2 presents the deformation scheme of the elastic element in the composite helical gear under the influence of the torsional moment M_1 [11, 12].

According to the analysis of this scheme, under the influence of the torsional moment, the elastic element deforms by a value of Δφ in the rotational direction of the gear. That is, part of the torsional moment is absorbed by the deformation. Therefore, the generated rotational force in the engagement process of the helical gear with an elastic element can be determined using the following expression [13]:

$$F_{t1} = \frac{2}{d_1} \cdot (M_1 - C_{a1} \cdot \Delta\varphi_1) \quad (5)$$

here, C_{a1} is the rotational stiffness coefficient of the elastic element, and $\Delta\varphi_1$ is the angular deformation of the elastic element.

Similarly, during the operation of helical gears with elastic elements, the forces acting along the gear shafts cause their elastic elements to deform both axially and radially.

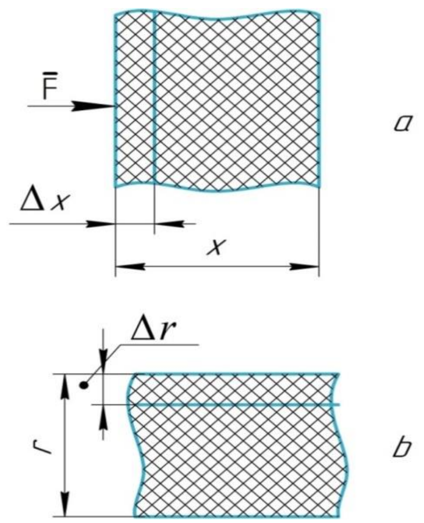


Figure 3. Deformation schemes of the elastic element under the influence of axial (a) and radial (b) forces in helical gear engagement.

During engagement, the flexible elements partially absorb the overall force due to their deformation, distributing it into axial and radial components. That is, the impact of the forces is partially reduced.

Taking into account the given equations (3), (4), and (5), we can write the following:

Axial force component,

$$F_{x1} = (F_{t1} - C_{a1} \cdot \Delta\varphi_1) \cdot \tan\beta - C_{x1} \cdot \Delta X_1 \tag{6}$$

radial force component perpendicular to the axis,

$$F_{r1} = (F_{t1} - C_{a1} \cdot \Delta\varphi_1) \cdot \frac{\tan\alpha_\omega}{\cos\beta} - C_{r1} \cdot \Delta r_1 \tag{7}$$

and the total acting force.

$$F_{n1} = \frac{2 \cdot (M_1 - C_{a1} \cdot \Delta\varphi_1)}{d_1 \cdot \cos\alpha_\omega \cdot \cos\beta} \tag{8}$$

here, C_{x1} and C_{r1} are the stiffness coefficients of the elastic element in the axial and radial directions, respectively, while ΔX_1 and Δr_1 represent the deformation values of the elastic element in the X_1 and r_1 directions [14, 15].

The given expressions (5), (6), (7), and (8) are derived for the helical gear with a flexible element in the driving transmission. Correspondingly, for the driven gear, the following expressions are applicable:

$$F_{t2} = \frac{2}{d_2} \cdot (M_2 - C_{a2} \cdot \Delta\varphi_2) \ ;$$

$$F_{x2} = (F_{t2} - C_{a2} \cdot \Delta\varphi_2) \cdot \tan\beta - C_{x2} \cdot \Delta X_2 \ ;$$

$$F_{r2} = (F_{t2} - C_{a2} \cdot \Delta\varphi_2) \cdot \frac{\tan\alpha_\omega}{\cos\beta} - C_{r2} \cdot \Delta r_2 \ ;$$

$$F_{n2} = \frac{2 \cdot (M_2 - C_{a2} \cdot \Delta\varphi_2)}{d_2 \cdot \cos\alpha_\omega \cdot \cos\beta} \tag{9}$$

here, C_{a2}, C_{x2}, and C_{r2} are the stiffness coefficients of the elastic element in the driven helical gear for different force directions; M_2 and d_2 represent the torsional moment and pitch circle diameter of the driven helical gear, respectively; and $\Delta\varphi_2$, ΔX_2, and Δr_2 correspond to the deformation values of the elastic element in the helical gear for torsional, axial, and radial directions [9, 10].

3. Materials and Methods

It is well known that gear transmissions are widely used in technological machines. One of their main drawbacks is the wear and breakage of the teeth. The primary reason for this is the low number of paired teeth engaged at the same time. This phenomenon is expressed through the contact ratio coefficient. [7, 8];

$$\varepsilon_\alpha = \frac{L_{AB}}{t} \tag{10}$$

here, L_{AB} - line of action, t - pitch.

Typically, the contact ratio coefficient should have a value of $\mathcal{E}\alpha \geq 1.0$. In spur gear transmissions, $\mathcal{E}\alpha$ can range between (1.0÷1.25). Due to this, the force acting on each tooth during meshing is relatively high. To increase $\mathcal{E}\alpha$ and reduce the acting force, helical or herringbone gear transmissions are used. According to [9], in a helical gear transmission, the contact ratio coefficient is:

$$\varepsilon_\beta = \frac{b_\omega \cdot \tan\beta}{t_\omega} \tag{11}$$

here, bω- represents the gear width, and

t_ω - denotes the line of action or axial pitch.

For the recommended helical gear transmission with an elastic element, a formula has been derived to determine the transmission contact ratio coefficient, considering the deformation of the elastic element:

$$\varepsilon'_\alpha = \left[1.88 - 3.2 \cdot \left(\frac{1}{z_1} + \frac{1}{z_2}\right)\right] \cdot cos\,\beta + \Delta\varepsilon_\alpha; \tag{12}$$

here, Z_1 and Z_2 represent the number of teeth on the gears, while Δεα is the coefficient accounting for the deformation of the elastic elements in the gears. The values 1.88 and 3.2 are derived from the geometric dimension limits of the gears, as stated in [4].

Considering the length of the contact line of the meshing teeth (11):

$$l_k = \frac{b_\alpha}{\cos\beta} \cdot \varepsilon_\alpha \tag{13}$$

Therefore, the bending stress calculation for helical gears with elastic elements is determined using the following recommended formula:

$$6_e = (F_{t1} \cdot K_F \cdot \cos\beta) \cdot \frac{K_{F\alpha} \cdot K_{F\beta} \cdot K_{FV}}{K_C \cdot \varepsilon'_\alpha \cdot m \cdot b_\alpha} \leq [6_e]; \tag{14}$$

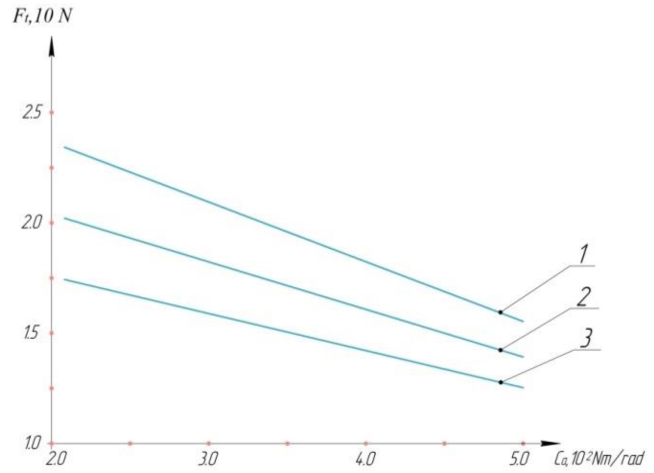
here: $K_{F\alpha}$ - axial contact ratio coefficient; K_F - force variation coefficient; $K_{F\beta}$ - inclination angle variation coefficient; K_{FV} - velocity variation coefficient; $K_{F\alpha}$ - coefficient representing the accuracy of the transmission; K_c - coefficient considering the elastic element; [Ϭe] - allowable bending stress.

4. Results

The obtained expressions (5), (6), and (7) were used to construct graphs showing the relationship between the variation laws of the forces generated during meshing and the changes in the stiffness coefficients of the elastic element in the corresponding direction, taking into account the numerical values of the parameters. [5].

Figure 5 presents graphs illustrating the relationship between the torque generated during meshing in a helical gear transmission with an elastic element and the variation of the rotational stiffness of the elastic element.

Based on the analysis of the constructed graphs, it is observed that as the rotational stiffness of the elastic element increases from 2.45 × 10² Nm/rad to 5.0 × 10² Nm/rad, the meshing torque Ft decreases linearly. Specifically, when the gear diameter is d = 125 × 10⁻³ m, Ft decreases from 2.32 × 10 N to 1.53 × 10 N. [6].



1- d=100 mm; 2- d=125 mm; 3- d=150 mm

Figure 4. *Graphs of the relationship between the torque generated during meshing in a helical gear transmission with an elastic element and the variation of the rotational stiffness of the elastic element.*

If the pitch circle diameters are $d_1 = 150 \times 10^{-3}$ m, the meshing torque values decrease linearly from 1.74 × 10 N to 1.25 × 10 N (Figure 5).

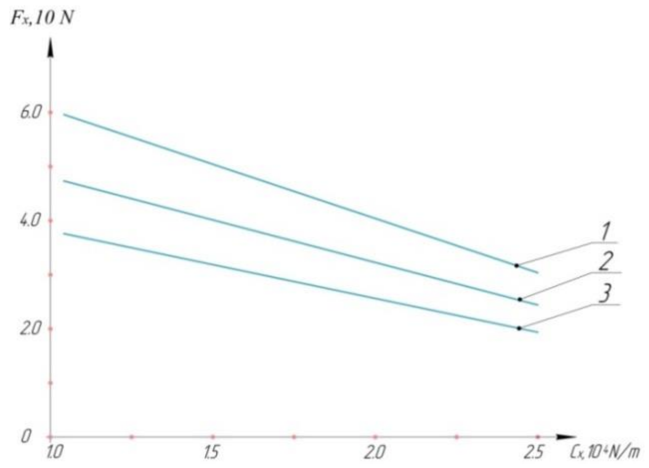
Thus, to maintain a higher torque, it is advisable to select the rotational stiffness coefficient of the composite gear in the range of (300–350) Nm/rad. In this case, the resistance moments on the gear shafts and their rotational frequencies should be matched with appropriate Ca values. Specifically, for Mq = 100 Nm and na = 1500 rpm, it is recommended that

Ca = (600–700) Nm/rad [7].

Figure 6 presents graphs illustrating the relationship between the variation of the axial force in the meshing of a helical gear with an elastic element and the linear stiffness coefficient of the elastic element in the radial direction.

According to the analysis, as the axial stiffness coefficient of the elastic element increases from 1.25×10^4 N/m to 2.5×10^4 N/m, the axial force Fx decreases linearly from 5.91 × 10 N to 3.05 × 10 N when β = 10°. This occurs because an increase in stiffness reduces deformation.

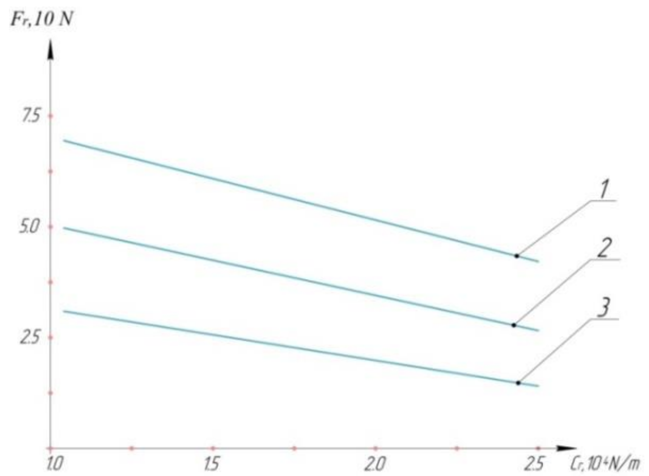
Similarly, when the helix angle decreases to 9.0°, the axial force further decreases to 3.82 × 10 N.



1-β=10˚; 2-β=7,0˚; 3-β=4,0˚

Figure 5. *Graphs showing the dependence of the axial force variation in the meshing of a helical gear with an elastic element on the linear stiffness coefficient of the elastic element in the radial direction.*

To minimize the values of the axial forces between the gears as much as possible, it is recommended to select the axial stiffness coefficient of the rubber bushing in the range of $(2.2–2.8) \times 10^4$ N/m.



1-β=10˚; 2-β=7,0˚; 3-β=4,0˚

Figure 6. *Graphs of the variation of the radial force generated during the meshing of teeth in a helical gear transmission depending on the stiffness coefficient of the compliant element in the same direction.*

Figure 6 Graphs showing the dependence of the variation of the radial force generated in the meshing of a helical gear with an elastic element on the stiffness coefficient of the elastic element in the same direction.

Analysis of the constructed graphs shows that when the stiffness coefficient of the elastic element in the direction perpendicular to the gear axis increases from 1.24×10^4 N/m to 2.5×10^4 N/m, and the helix angle is $\beta = 10°$, the radial force decreases linearly from 6.48×10 N to 4.18×10 N. However, when $\beta = 4.0°$, the Fa values decrease from 3.34×10 N to 1.32×10 N.

Thus, to reduce the reaction forces in the bearings of the gear shafts, it is advisable to select the stiffness coefficient of the damping rubber bushing in this direction within the range of $(2.5–3.0) \times 10^4$ N/m.

5. Conclusions

It should be noted that in helical gears, when truncated conical rubber bushings are used as compliant elements, the forces are sufficiently damped due to their deformation in the corresponding directions. This increases the contact ratio. Additionally, considering the damping of forces, it becomes possible to reduce the permissible stress in bending and compression calculations during meshing by (7.0–9.0)%. As a result, the service life of the gear mechanism is extended.

Abbreviations

F_t	Tangential Force
F_x	Axial Force
F_r	Radial Force
F_n	Total Force
d_1	Pitch Circle Diameter of the Driving Helical Gear
β	Helix Angle of the Teeth
α_ω	Transmission Angle
C_{a1}	Rotational Stiffness Coefficient of the Elastic Element (for the Driving Gear)
$\Delta\varphi_1$	Angular Deformation of the Elastic Element (for the Driving Gear)
C_{x1}	Stiffness Coefficient of the Elastic Element in the Axial Direction (for the Driving Gear)
ΔX_1	Deformation Value of the Elastic Element in the X_1 Direction (for the Driving Gear)
C_{r1}	Stiffness Coefficient of the Elastic Element in the Radial Direction (for the Driving Gear)
Δr_1	Deformation Value of the Elastic Element in the r_1 Direction (for the Driving Gear)
F_{t1}	Tangential Force with Elastic Element (for the Driving Gear)
F_{x1}	Axial Force with Elastic Element (for the Driving Gear)
F_{r1}	Radial Force with Elastic Element (for the Driving Gear)
F_{n1}	Total Force with Elastic Element (for the Driving Gear)
M_2	Torsional Moment for the Driven Gear
d_2	Pitch Circle Diameter of the Driven Helical Gear
C_{a2}	Rotational Stiffness Coefficient of the Elastic Element (for the Driven Gear)
$\Delta\varphi_2$	Angular Deformation of the Elastic Element (for the Driven Gear)
C_{x2}	Stiffness Coefficient of the Elastic Element in the Axial Direction (for the Driven Gear)
ΔX_2	Deformation Value of the Elastic Element in the X_2 Direction (for the Driven Gear)
Cr_2	Stiffness Coefficient of the Elastic Element in the Radial Direction (for the Driven Gear)
Δr_2	Deformation Value of the Elastic Element in the r_2 Direction (for the Driven Gear)
F_{t2}	Tangential Force with Elastic Element (for the Driven Gear)
F_{x2}	Axial Force with Elastic Element (for the Driven Gear)
F_{r2}	Radial Force with Elastic Element (for the Driven Gear)
F_{n2}	Total Force With Elastic Element (for the Driven Gear)
$\mathscr{E}_\alpha$	Contact Ratio Coefficient (for Spur Gears)
$\mathscr{E}_\beta$	Contact Ratio Coefficient (for Helical Gears)
b_ω	Gear Width
t_ω	Line of Action or Axial Pitch
$\mathscr{E}\alpha'$	Transmission Contact Ratio Coefficient with Elastic Element
Z_1	Number of Teeth on the Driving Gear
Z_2	Number of Teeth on the Driven Gear
$\Delta\varepsilon_\alpha$	Coefficient Accounting for the Deformation of the Elastic Elements
l_k	Length of the Contact Line of Meshing Teeth
Ϭe	Bending Stress
K_F	Force Variation Coefficient
$K_{F\alpha}$	Axial Contact Ratio Coefficient
$K_{F\beta}$	Inclination Angle Variation Coefficient
K_{FV}	Velocity Variation Coefficient
K_C	Coefficient Considering the Elastic Element

Author Contributions

Anvar Djuraev: Conceptualization, Data curation, Formal Analysis, Funding acquisition, Methodology, Resources, Software, Validation, Writing – original draft, Writing – review & editing

Dilshod Abduvakhobov: Conceptualization, Supervision, Visualization, Writing – review & editing

Davron Khabibullaev: Conceptualization, Data curation, Formal Analysis, Funding acquisition, Investigation, Methodology, Resources, Software, Validation, Writing – original draft, Writing – review & editing

Conflicts of Interest

The authors declare no conflicts of interest.

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Biography

Anvar Djuraev is a highly respected Doctor of Technical Sciences and a professor, renowned for his extensive contributions to the field of machine details. He has authored numerous articles and books, which are considered fundamental references in the industry. His expertise and profound knowledge have earned him recognition both locally and internationally. Currently, he serves as a department professor at the prestigious Tashkent Institute of Textile and Light Industry, where he imparts his valuable knowledge to the next generation of engineers. Throughout his distinguished career, Professor Djuraev has mentored numerous students, many of whom have gone on to achieve great success in their professional fields. His dedication to research, education, and the advancement of machine design has made him a key figure in his field, with a lasting impact on both academia and industry.

Dilshod Abduvakhidovich is a Doctor of Philosophy (PhD) in Technical Sciences and an Associate Professor at the Department of Mechanics of Namangan Engineering-Construction Institute. His scientific activity is focused on the fields of agricultural machinery and mechanics, and he is the author of numerous research papers and publications in these areas. Abduvakhobov's research primarily covers the vibration analysis of agricultural machinery and the enhancement of their operational efficiency. His work has gained international recognition and has been published in reputable databases such as Scopus and Google Scholar. He is the author of 3 Scopus-indexed articles and has received 175 citations on Google Scholar. His h-index is 7, and his i10-index is 4. His significant contributions to science and his dedication to educating and mentoring the younger generation have established him as a leading expert in his field.

Davron Khabibullaev is a PhD candidate and a junior lecturer at the Department of Mechanics. He is currently working on his doctoral research and contributing to the field through teaching and academic involvement. He graduated Namangan Engineering-Construction Institute in 2020, and his Master of Ground vehicles and systems (by mode of transport) from the same institution in 2022. He has participated in multiple international research collaboration projects in recent years.

Research Fields

Anvar Djuraev: Theory of Machines and Mechanisms, Mechanical Engineering, Elastic Elements in Gear Transmissions, Dynamics and Kinematics of Technological Machines, Vibration Analysis in Gear Systems, Reliability and Efficiency of Machine

Components.

Dilshod Abduvakhobov: Agricultural Machinery, Mechanical Vibrations and Oscillations, Optimization of Soil Tillage Equipment, Vibro-dynamics of Agricultural Machines, Analysis and Improvement of Operational Stability, Application of Elastic Elements in Mechanical Systems.

Davron Khabibullaev: Helical Gear Transmission with Elastic Elements, Applied Mechanics, Machine Design and Analysis, Dynamic Behavior of Mechanical Systems, Torsional and Axial Vibration Analysis, Structural Optimization in Mechanical Drives.