

Research Article

Compensation Method of Characteristic Nonlinearity on Control System

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Abstract

Typical characteristic nonlinearities include insensitivity, saturation, clearance and hysteresis, and various combinations of them, which have a great influence on the system in control systems. For example, Backlash nonlinearity of the reducers and gears has strong effect on the system in some electromechanical tracking systems. Due to the effect of backlash nonlinearity on quality and reliability of the system, many researchers have tried to find out its solutions. In this study, we performed theoretical considerations and simulations to compensate the effects of nonlinearities such as backlash, saturation, hysteresis, etc. present in the control system. This paper shows that, unlike the past linear approximation of the inherent nonlinearities present in control systems, it directly eliminates the inherent nonlinearities and is convenient for physical implementation. First, this paper shows the simple method of compensating nonlinearities by using direct coupling principle. Next, Feedback compensation method is introduced in the systems which involve the saturation nonlinearity element. Finally, through the experiment, the advantages of these methods are confirmed. This is a new approach for nonlinear compensation and more convenient and simpler to implement compared with the previous different control methods. It can also be used to overcome the effect of several nonlinearities in different control systems as well as in the primary static satellite system.

Keywords

Insensitivity, Saturation, Hysteresis, Nonlinearity Compensation

1. Introduction

We can see many electromechanical tracking systems in practice, which generally affect the quality of control systems due to the presence of inherent nonlinearities, which are insensitive, saturated, hysteresis and their arbitrary combination.

In nonlinear control systems where these inherent nonlinearities exist, unlike linear systems, a vibration process with constant amplitude and frequency, i.e., self-oscillation, can occur regardless of the type of external disturbance, and can lead to unstable states.

Hence, the literature considers the method of compensat-

ing the inherent nonlinearity and the analysis and design of the system.

In-depth studies and analyses to eliminate the influence of insensitive nonlinearity have been carried out. [1-5]

In this paper, we propose a solution to overcome this by using various adaptive and fuzzy control method, etc. to cope with insensitivity.

A scheme is proposed to compensate the effect of input insensitivity and saturation. [6, 7]

In this work, an adaptive fuzzy controller is used to over-

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come the effects of the nonlinearity of the system on which the input dead zone and saturation are simultaneously acting.

Also, we have proposed methods of overcoming the nonlinear problem using Multi-Dimensional Taylor Networks (MDTN) based on the combination of backstepping approach and dynamic surface control in nonlinear systems with input constraints.

The effect of input-limited nonlinear is solved by using adaptive saturation compensation control method using neural network state observer. [8-10]

In this work, the state estimation problem for uncertain systems with time-varying state delays and input saturation nonlinearities are solved using neural networks and a new type of Lyapunov-Krasovskii function.

A path integral stochastic optimal control method based on the Hamilton-Jacobi-Bellman Equation was used to overcome the nonlinear effects for nonlinear systems with control input constraints and saturation. [11]

PD controller was used to improve the speed and steady-state error in servomotor position control system with input saturation. [12], adaptive fuzzy dynamic surface feedback was used to improve tracking performance in uncertain systems with input saturation and unobservable states. [13]

The method of active disturbance rejection control (ADRC) using nonlinear extended state observer is described in pneumatic actuator with actuator saturation. [14].

The stability analysis and solution of uncertain systems with actuator saturation using STC is presented. [15, 16]

They showed an example of improving the effect of nonlinearities of insensitivity and saturation by using model predictive control and improving the system quality. [17, 18].

Also described a low-order observer (LOO) design method to compensate the nonlinear effect by observing the disturbance of a nonlinear feedback system acting on the input. [19, 20].

As discussed, different approaches have been proposed to compensate the inherent nonlinearities such as insensitivity, saturation, hysteresis, etc., but they also include some problems in terms of complexity and feasibility.

In this paper, we apply the direct coupling and feedback compensation method to compensate the inherent nonlinearities in control system design, and verify its effectiveness through theoretical considerations and simulation analysis to compensate the inherent nonlinearities composed of insensitivity, saturation, hysteresis and their series of combinations.

2. Compensation by Direct Coupling

This method is a direct coupling of the compensating element with the nonlinearity one for nonlinearity compensation.

A schematic diagram for compensation is shown in Figure 1.

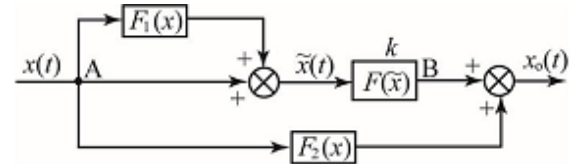


Figure 1. Compensation Schematic diagram by direct coupling.

The elementary equation of the block diagram is given as

$$\begin{cases} \tilde{x}(t) = x(t) + F_1(x) \\ x(t) = F(\tilde{x}) + F_2(x) \end{cases} \quad (1)$$

Now consider the nonlinear compensation problem of a dead-zone nonlinear element.

The static characteristics of the insensitive nonlinear element is defined as follows

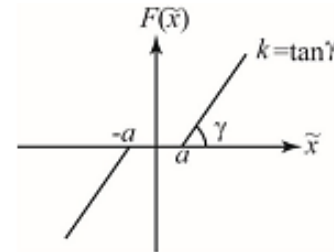


Figure 2. Static characteristic of nonlinear element.

The analytical expression of this characteristic is defined as follows

$$F(\tilde{x}(t)) = \begin{cases} 0; |\tilde{x}(t)| \leq a \\ -k \operatorname{sign} \tilde{x}(t) + k \tilde{x}(t); |\tilde{x}(t)| > a \end{cases} \quad (2)$$

Now, let us find $F_1(x)$ and $F_2(x)$ to provide the following linear relation

$$x_o(t) = kx(t) \quad (3)$$

Substituting (1) into (2), we have

$$F(\tilde{x}) = F_{eq}(x) = \begin{cases} 0; |x(t) + F_1(x)| \leq a \\ -ka + kF_1(x) + kx(t); x(t) + F_1(x) > a \\ ka + kF_1(x) + kx(t); x(t) + F_1(x) < -a \end{cases} \quad (4)$$

Here, $F_{eq}(x)$ is the equivalent property between points A and B.

From (4), with following condition

$$\begin{cases} ka + kF_1(x) = 0; x(t) + F_1(x) < -a \\ -ka + kF_1(x) = 0; x(t) + F_1(x) > a \end{cases} \quad (5)$$

It follows that the nonlinear function is given as

$$F(\tilde{x}) = kx(t); |x(t) + F_1(x)| > a \quad (6)$$

Condition (5) is satisfied when $F_1(x)$ is the sign function of $x(t)$.

$$F_1(x) = \text{sign}(x(t)) \quad (7)$$

If $F_1(x)$ is determined according to (7), with all values of $x(t)$, we have

$$|x(t) + F_1(x)| = |x(t) + \text{sign}x(t)| > a$$

Thus, arbitrary values of input signal- $x(t)$, where $|x(t)| \leq a$ too. We get the following linear relation $F_{eq}(x) = kx(t)$.

Then the output signal is $x_o(t) = F_{eq}(x) + F_2(x) = kx(t) + F_2(x)$, which is linear when $F_2(x) = 0$.

Thus, to compensate the insensitive nonlinearity, it is sufficient to have $F_1(x)$ component in Figure 1.

Consequently, the compensation condition of the insensitive nonlinearity can be written in the following form.

$$\tilde{x}(t) = x(t) + F_1(x) = x(t) + \text{sign}x(t) \quad (8)$$

Finally, we must add a sign function with signal $x(t)$ to the input of the nonlinear element.

Based on this analysis, the simulation circuit and simulation results for insensitive nonlinearity compensation are as follows.

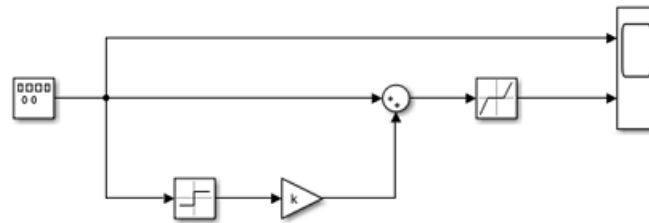


Figure 3. Simulation block diagram.

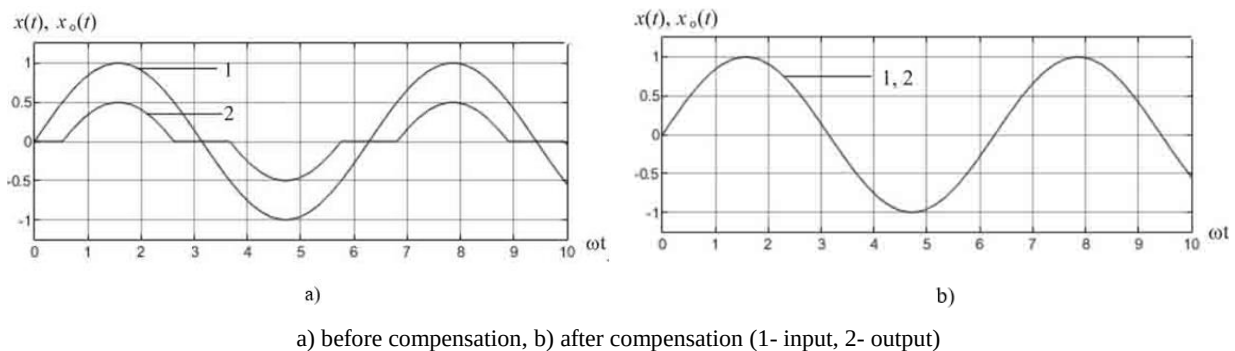


Figure 4. Simulation plot.

As can be seen from the figure, the output signal is distorted by insensitivity when the input is sinusoidal before compensation, but the output is still sinusoidal when compensated.

Thus, the dead-zone characteristics are completely compensated.

3. Feedback-based Compensation Method

The nonlinearity compensation can be realized through nonlinear feedback by $F_1(x)$ and $F_2(x)$.

The block diagram for compensation is shown in Figure 5.

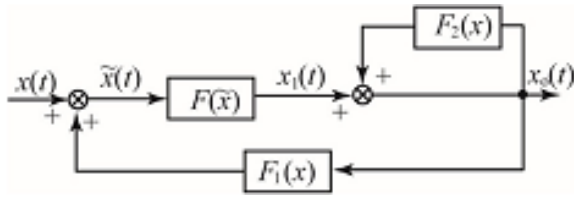


Figure 5. Compensation diagram by feedback.

This compensation method is convenient to use when the signal measurement at the input of a nonlinear element is difficult.

Consider the method of saturation nonlinearity compensation by feedback.

The elementary equation of the saturation nonlinearity compensation block diagram is

$$\begin{cases} x(t) = x_1(t) + F_2(x) \\ x_1(t) = F(\tilde{x}) = \begin{cases} k\tilde{x}(t); |\tilde{x}(t)| < \frac{b}{k} \\ b \text{sign} \tilde{x}(t); |\tilde{x}(t)| \geq \frac{b}{k} \end{cases} \\ \tilde{x}(t) = x(t) + F_1(x) \end{cases} \quad (9)$$

Let us derive $F_1(x)$ and $F_2(x)$ to extend the linear region of saturation nonlinearity.

For example, double linear:

$$x(t) = \begin{cases} kx(t); |x(t)| < \frac{2b}{k} \\ b \text{sign} x(t); |x(t)| \geq \frac{2b}{k} \end{cases} \quad (10)$$

Substituting first condition of (9) into second condition,

$$kF_1(x) + F_2(x) = 0, -\frac{2b}{k} = -\frac{b}{k} - F_1(x); x(t) > -\frac{b}{k} - F_1(x)$$

or

$$F_1(x) = \frac{b}{k}, F_2(x) = -b; x(t) > -\frac{2b}{k}$$

$$b + F_2(x) = 2b; \frac{2b}{k} = \frac{b}{k} \text{ In the third equation,}$$

$$-F_1(x); x(t) \geq \frac{b}{k} - F_1(x)$$

or

$$F_1(x) = -\frac{b}{k}, F_2(x) = b; x(t) \geq \frac{2b}{k}$$

In the fourth equation,

$$-b + F_2(x) = -b; -\frac{2b}{k} = -\frac{b}{k} - F_1(x); x(t) \leq -\frac{b}{k} - F_1(x)$$

or

we have

$$x(t) = \begin{cases} kx(t) + kF_1(x) + F_2(x); x(t) + F_1(x) < \frac{b}{k} \\ kx(t) + kF_1(x) + F_2(x); x(t) + F_1(x) > -\frac{b}{k} \\ b + F_2(x); x(t) + F_1(x) \geq \frac{b}{k} \\ -b + F_2(x); x(t) + F_1(x) \leq -\frac{b}{k} \end{cases}$$

or

$$x(t) = \begin{cases} kx(t) + kF_1(x) + F_2(x); \\ kx(t) + kF_1(x) + F_2(x); (t) > -\frac{b}{k} - F_1(x) \\ b + F_2(x); x(t) \geq \frac{b}{k} - F_1(x) \\ -b + F_2(x); x(t) \leq -\frac{b}{k} - F_1(x) \end{cases} \quad (11)$$

Comparing (10) with (11), the output signal is expressed as the form of (10), when the following conditions are satisfied.

In the first equation,

$$kF_1(x) + F_2(x) = 0, \frac{2b}{k} = \frac{b}{k} - F_1(x); x(t) < \frac{b}{k} - F_1(x)$$

or

$$F_1(x) = -\frac{b}{k}, F_2(x) = -kF_1(x) = b; x(t) < \frac{2b}{k}$$

In the second equation,

$$F_1(x) = \frac{b}{k}, F_2(x) = -b; x(t) \leq -\frac{2b}{k}$$

From the derived expression, the condition of double extension of the linear domain is given as

$$F_1(x) = -\frac{b}{k}, F_2(x) = b; x(t) \geq 0$$

$$F_1(x) = -\frac{b}{k}, F_2(x) = -b; x(t) < 0$$

or

$$\left. \begin{aligned} F_1(x) &= -\frac{b}{k} \text{sign } x(t) \\ F_2(x) &= b \text{sign } x(t) \end{aligned} \right\} \quad (12)$$

Finally, $F_1(x)$ and $F_2(x)$ must be relay characteristic.

Indeed, it is reasonable to assume that the state changes of $F_1(x)$, and $F_2(x)$ only occur when they are converted to

saturation.

Thus

$$F_1(x) = \begin{cases} 0; & |x| < b \\ -\frac{b}{k} \text{sign } x(t); & |x| \geq b \end{cases} \quad (13)$$

$$F_2(x) = \begin{cases} 0; & |x| < b \\ b \text{sign } x(t); & |x| \geq b \end{cases} \quad (14)$$

Substituting (13) and (14) into (11), and obtaining the expression in accordance with Equation (10), we see that the linear region of the saturation nonlinearity is doubled when $F_1(x)$ and $F_2(x)$ are introduced into the feedback loop.

Since the nonlinear functions $F_1(x)$ and $F_2(x)$ differ only in sign with the proportional factor, one relay characteristic element can be used instead of two elements in the linear domain expansion, as shown in Figure 6.

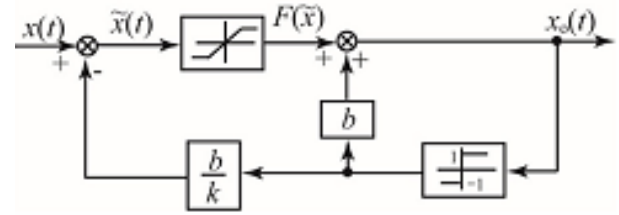


Figure 6. Schematic diagram for compensating saturation.

Thus, the linear domain extension of the saturation nonlinearity can be achieved by a feedback method.

The simulation circuit and simulation results for feedback compensation of saturation nonlinearity are given as follows.

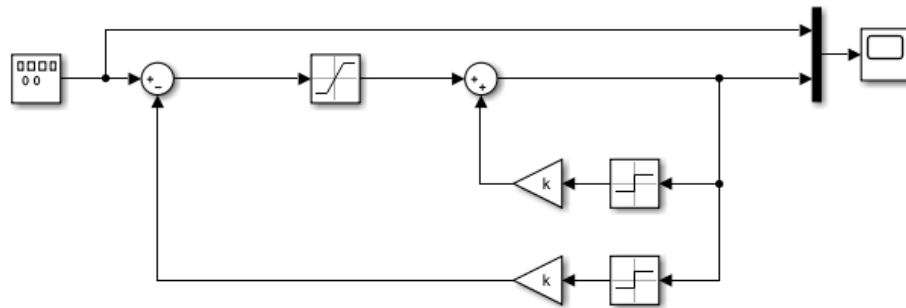
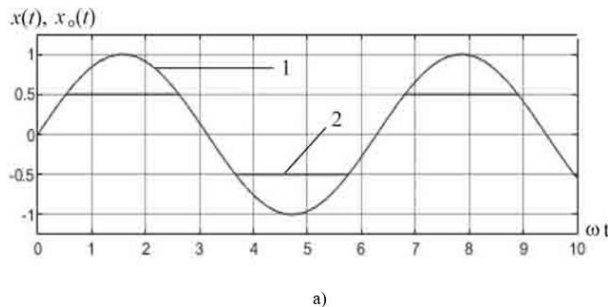
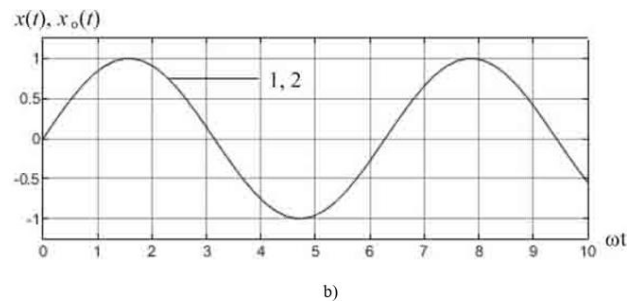


Figure 7. Simulation scheme.



a) before compensation



b) after-compensation (1-input, 2-output)

Figure 8. Simulation plot.

Also, the simulation results show that the linear region is doubled.

The required linear domain extension of the saturation nonlinearity can be achieved by feedback and direct coupling method.

4. Nonlinearity Compensation Method with Hysteresis

Here, a compensation method of relay characteristics with hysteresis is proposed.

In general, the presence of hysteresis in closed-loop automatic control system leads to a reduction of stability margin, in some cases, to self-oscillation and to an increase of static error.

The positive characteristic curves and analytical expressions of the relay characteristics with hysteresis are as follows.

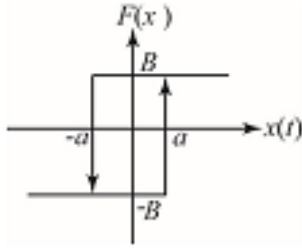


Figure 9. Static characteristics of relay with hysteresis.

$$F(x) = \begin{cases} B; & x(t) \geq a \\ -B; & x(t) \leq -a \\ B; & -a < x(t) < a \wedge \dot{x}(t) < 0 \\ -B; & -a < x(t) < a \wedge \dot{x}(t) > 0 \end{cases} \quad (15)$$

where a is hysteresis width.

Equation (15) can be rewritten as follows

$$F(x) = B \text{sign}[x(t) - a \text{sign}\dot{x}(t)] \quad (16)$$

From (16), the equivalent block diagram of the hysteresis nonlinearity can be proposed as follows:

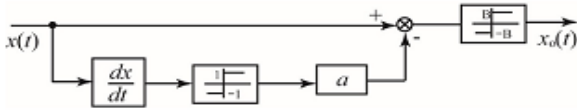


Figure 10. Equivalent block diagram of hysteresis nonlinearity.

As in insensitive nonlinearity compensation, the hysteresis can be compensated by a nonlinear direct coupling method.

The block diagram for compensation is shown in Figure

$$B \text{sign}x(t) = B \text{sign}\{x(t) + F_1(x) - a \text{sign}[x(t) + F_1(x)]'\} + F_2(x) \quad (21)$$

Thus, the compensation condition is defined as follows.

$$\begin{cases} F_1(x) = a \text{sign}\dot{x}(t) \\ F_2(x) = 0 \end{cases} \quad (22)$$

From the above considerations, the compensation scheme can be proposed as follows:

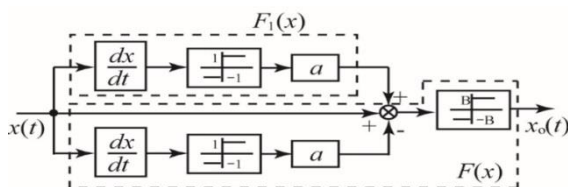


Figure 12. Hysteresis compensation scheme.

11.

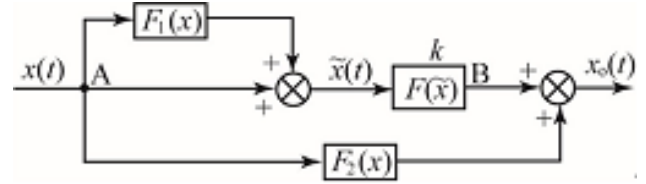


Figure 11. Compensation block diagram of hysteresis nonlinearity.

When the hysteresis is completely compensated, we have

$$F(x) = \begin{cases} B; & x(t) > 0 \\ 0; & x(t) = 0 \\ -B; & x(t) < 0 \end{cases} \quad (17)$$

Rewriting it,

$$F(x) = B \text{sign}x(t) \quad (18)$$

The elementary equation of the block diagram is

$$\begin{cases} x(t) = F(\tilde{x}) + F_2(x) \\ \tilde{x}(t) = x(t) + F_1(x) \end{cases} \quad (19)$$

From (19), we can obtain

$$x(t) = F\{x(t) + F_1(x) - a \text{sign}[x(t) + F_1(x)]'\} + F_2(x) \quad (20)$$

If we have completely compensated the hysteresis from (18) and (20), then we have

The followings are the nonlinear compensation simulation circuit and simulation results of relay characteristics with hysteresis.

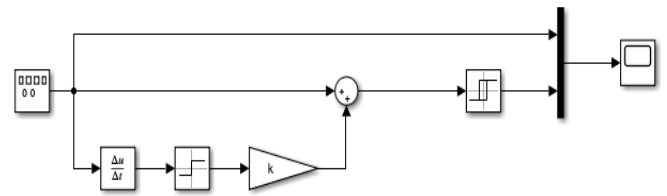


Figure 13. Simulation scheme.

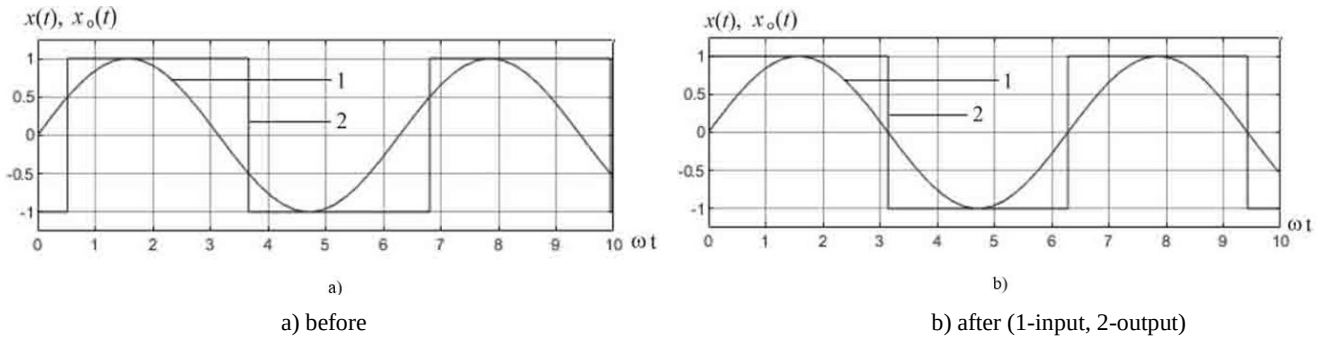


Figure 14. Simulation plot.

As can be seen from the simulation results, the input of the compensated system is fed with a sinusoidal signal, which results in a hysteresis-free relay.

Finally, let us consider the compensation of more complex nonlinear characteristics, which are combined with insensitivity and hysteresis.

This typical non-shaping feature allows three-position relays, whose positive characteristics and analytical expressions are as follows:

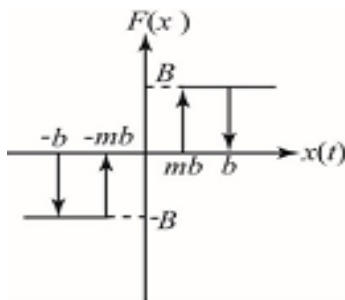


Figure 15. Static characteristics of three position relay.

$$F(x) = \begin{cases} 0; & -mb < x(t) < mb \\ B; & x(t) \geq b \text{ or } mb \leq x(t) < b \wedge \dot{x}(t) < 0 \\ -B; & x(t) \leq -b \text{ or } -b < x(t) \leq -mb \wedge \dot{x}(t) > 0 \end{cases} \quad (23)$$

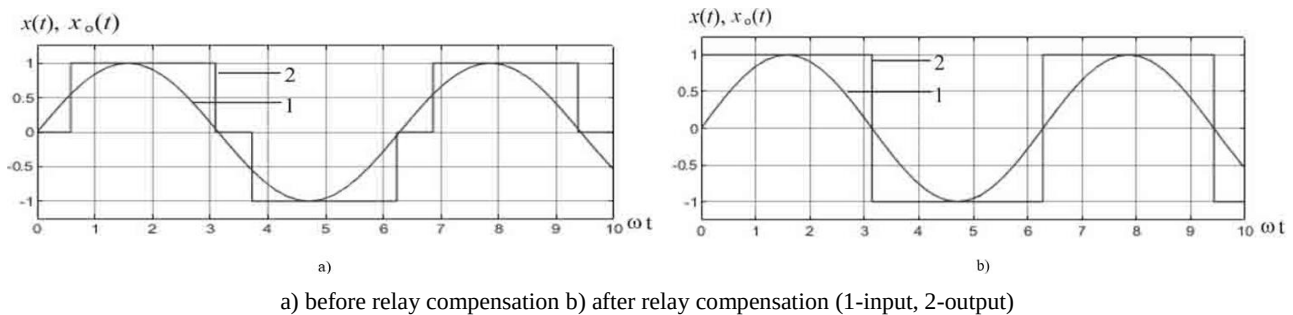


Figure 18. Simulation plot.

Then, it is necessary to compensate the insensitivity and the hysteresis.

The compensation scheme is shown in Figure 16.

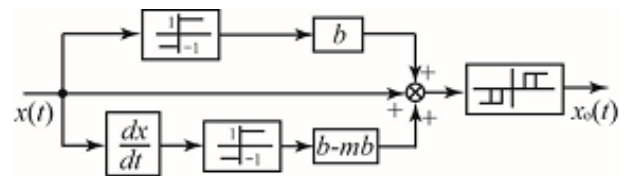


Figure 16. Schematic diagram of three position relay characteristics compensation.

The followings are the simulation circuit and simulation results for the nonlinear compensation with dead and hysteresis.

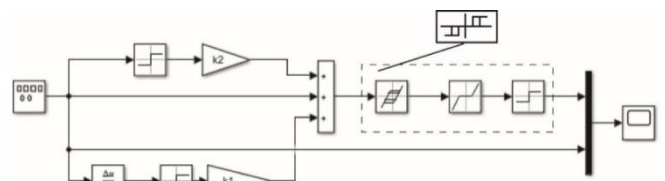


Figure 17. Simulation scheme.

5. Simulink Example

As an example of simulation, consider a furnace temperature control system with three-position relaying which has

ON/OFF control characteristic.

Figure 19 shows the simulation scheme of the nonlinear compensation controller of the furnace temperature control system.

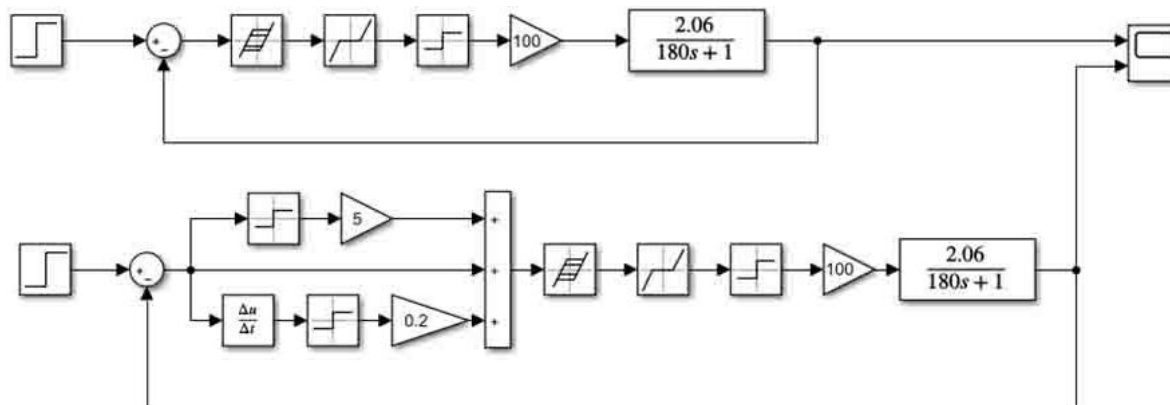


Figure 19. Simulink block diagram.

The time constant of the plant is 180(s). Nonlinear characteristics include a gap of 0.3 °C, a dead-zone of ± 5 V, and a proportional stability factor of $K = 100$.

To compensate for the insensitivity and clearance effects, the direct coupling compensation factors are set as $K_p = 5$,

$K_d = 0.2$.

The reference temperature is 100 °C.

Figures 20 and 21 show the response and error characteristic.

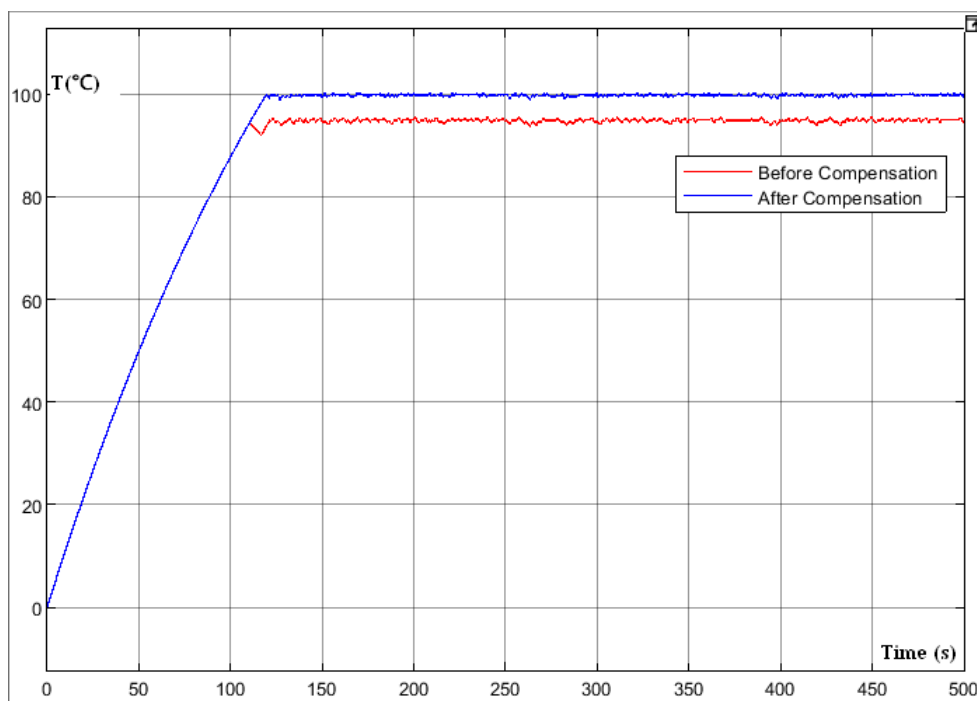


Figure 20. Response characteristic.

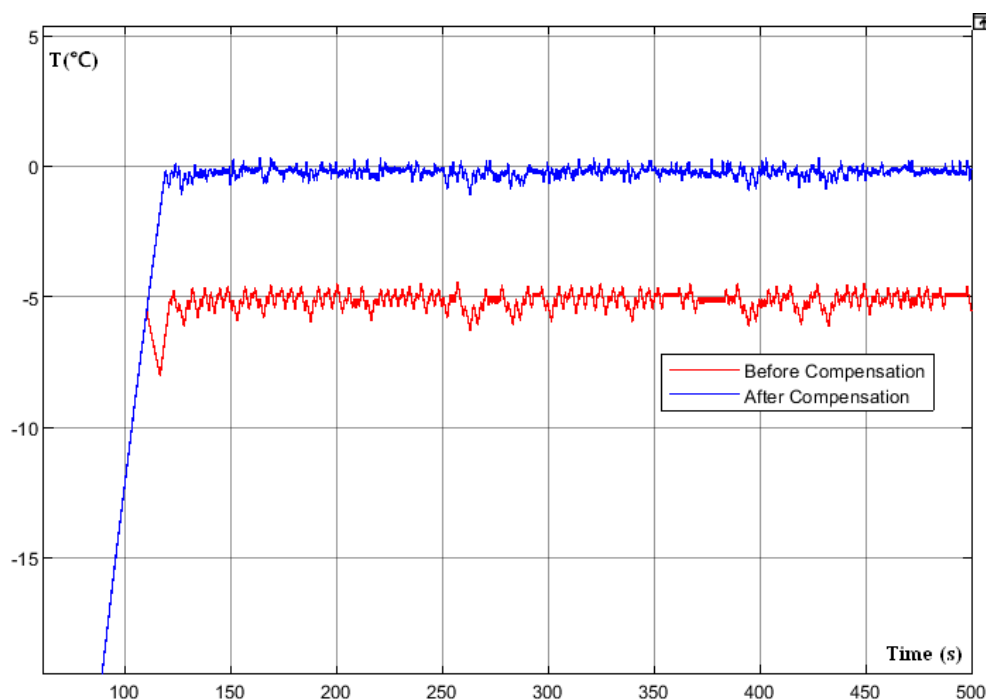


Figure 21. Error characteristic.

From Figures 20 and 21, the static error is significantly reduced by designing a compensation controller to improve the nonlinearity with ideal relay characteristics.

Thus, nonlinearities such as saturation, clearance and insensitivity can be linearized. The two-position and three-position relaying characteristics can be improved by an ideal signifier, so we can improve the system quality.

6. Conclusion

In this paper, we propose a direct coupling and feedback compensation method compensating for the inherent nonlinearities such as insensitivity and hysteresis, and validate it through simulations.

Thus, using the compensation method proposed in this paper, it is found that the controller structure is simple and easy to implement, unlike the previous literature.

It is also possible to fully compensate the effect of nonlinear elements on the control system by implementing full compensation method.

Abbreviations

MDTN	Multi-Dimensional Taylor Networks
ADRC	Active-Disturbance Rejection Control
LOO	Low-Order Observer

Conflicts of Interest

The authors declare no conflicts of interest.

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