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Research Article

Series and Parallel Connections in Intuitionistic Fuzzy Graphs

Anandkumar Narayanasamy Venkatesan[1], Nivethana Venkataswamy[2,*]

[1]Department of Electrical and Electronic Engineering, Sri Venkateswara Institute of Science and Technology, Thiruvallur, India

[2]Department of Mathematics, Sri Venkateswara Institute of Science and Technology, Thiruvallur, India

Abstract

In this paper, we introduce two new operations on Min-max Intuitionistic Fuzzy Graphs, namely Parallel and Series connections along with some of its basic properties. Parallel and Series connections between two Min-max Intuitionistic Fuzzy Graphs are constructed and illustrated with relevant examples. Parallel connections and Series connections are obtained by deleting any two edges and introducing a new edge. Definition of Max-max Intuitionistic Fuzzy Graph is also introduced. Construction of Parallel and Series connections between two Min-max Intuitionistic Fuzzy Graphs gives rise to Max-max IFG which is a new type of Intuitionistic Fuzzy Graph where both the membership and non-membership functions of some of the edges are less than or equal to maximum of membership and non-membership functions of their respective incident vertices. It is shown that the number of edges in a Parallel connection is twice the product of edges in the two Min-max Intuitionistic Fuzzy Graphs whereas in a Series connection it is four times the product of edges in the two Min-max Intuitionistic Fuzzy Graphs. The study reveals that any two electrical circuits with its enclosed components (resistors, capacitors etc.) can be represented as vertices of two Min-max Intuitionistic Fuzzy Graphs and their Parallel and Series connections can be generated.

Keywords

Fuzzy Graphs, Intuitionistic Fuzzy Graphs, Parallel Connections, Series Connections

1. Introduction

Introduction of Fuzzy Set Theory answered the problem of Set Theory about the extent of an element belonging to that set. The theory of fuzzy set was proposed by Zadeh [15] to handle the various uncertainties in many real applications. The theory of fuzzy sets is, basically, a theory of grade in which everything is a matter of degree, everything is expressed as functions and mapped to numerical values in closed interval from 0 to 1. Graph theory originated in 1735 with Konigsberg bridge problem. Later it found vast development and found applications in many fields. This led towards Fuzzy graph which was introduced by Rosenfeld [12] and later developed by Bhattacharya [2]. Fuzzy graphs is a nice tool for representing objects and their related information in graphical form on a numerical scale that vary between 0 to 1. The objects are represented by vertices and their relationship is represented by edges and the membership value of vertices and edges takes

*Correspondence: Nivethana Venkataswamy (vnivethana@gmail.com)

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any numerical value in [0, 1].

Intuitionistic fuzzy graphs on the other hand provides us information about both membership and non-membership values and finds application in fields such as networking, decision making, cluster analysis and many other real-life situations. Atanassov [1] introduced the concept of intuitionistic fuzzy relations and intuitionistic fuzzy graphs. Theory of intuitionistic fuzzy sets created an exponential growth in mathematics and its applications that ranges from traditional mathematics to information sciences. Karunambigai et.al. [3, 9] introduced the concept of IFG and its components and introduced various operations on IFG's in [10, 11]. Also they introduced different types of IFG such as Product IFG, min-max IFG, max-min IFG and Type 'n' IFG. Operations "$\cup$, + and Cartesian Product" are extended to IFG's with common vertices by Nivethana and Parvathi A in [8]. The study was also expended to complement and regular IFG's in [6, 7]. Nagoorgani [4, 5] discussed about isomorphic properties of fuzzy graphs with its properties. Sankar Sahoo and Madhumangal Pal [13], studied on types of Product IFG's. In [14], Talal Al-Hawary introduced parallel and series connections in fuzzy graphs and constructed fuzzy graphs through parallel connection. This was also defined as adding elements in parallel to some existing element.

2. Preliminaries and Definitions

Definition 2.1: A fuzzy graph with V as the underlying set is a pair G: (σ, μ) where

σ: V $\rightarrow$ [0, 1] is a fuzzy subset and μ: V $\times$V $\rightarrow$ [0, 1] is a fuzzy relation on σ such that $\mu(x, y) \leq \sigma(x) \wedge \sigma(y)$ for all x, y $\in$ V.

Definition 2.2: A fuzzy graph G: (σ, μ) with underlying graph G: (V, E) is complete if $\mu(x,y) = \sigma(x) \wedge \sigma(y)$ for all x, y $\in$ V.

Definition 2.3: A fuzzy graph G: (σ, μ) with underlying graph G: (V, E) is strong if $\mu(x,y) = \sigma(x) \wedge \sigma(y)$ for all (x, y) $\in$ E

Definition 2.4: A Minmax intuitionistic fuzzy graph is of the form G: (V, E) where

(a). V = $\{v_1, v_2,..., v_n\}$ such that μ_A: V $\rightarrow$ [0, 1] and ν_A: V$\rightarrow$ [0, 1] denotes the degree of membership and non-membership of the elements $v_i \in V$ respectively and $0 \leq \mu_A(v_i) + \nu_A(v_i) \leq 1$ for every $v_i \in V$; (i = 1, 2, …, n).

(b). E $\subseteq$ V$\times$V where μ_B: V$\times$V$\rightarrow$ [0, 1] and ν_B: V$\times$V$\rightarrow$ [0, 1] are such that $\mu_B(v_i, v_j) \leq \min\{\mu_A(v_i), \mu_A(v_j)\}$, and $\nu_B(v_i, v_j) \leq \max\{\nu_A(v_i), \nu_A(v_j)\}$ where $0 \leq \mu_B(v_i, v_j) + \nu_B(v_i, v_j) \leq 1$ for every $(v_i, v_j) \in E$.

Here the triple $(v_i, \mu_A(v_i), \nu_A(v_i))$ denotes the degree of membership and non-membership of the vertex vi $\in$ V. The triple $((v_i, v_j), \mu_B(v_i, v_j), \nu_B(v_i, v_j))$ denotes the degree of membership and degree of non- membership of the edge $(v_i, v_j) \in$ E.

Definition 2.5: A Maxmin intuitionistic fuzzy graph is of the form G: (V, E) where

(a). V = $\{v_1, v_2,..., v_n\}$ such that μ_A: V $\rightarrow$ [0, 1] and ν_A: V$\rightarrow$ [0, 1] denotes the degree of membership and non-membership of the elements $v_i \in V$ respectively and $0 \leq \mu_A(v_i) + \nu_A(v_i) \leq 1$ for every $v_i \in V$; (i = 1, 2, …, n).

(b). E $\subseteq$ V$\times$V where μ_B: V$\times$V$\rightarrow$ [0, 1] and ν_B: V$\times$V$\rightarrow$ [0, 1] are such that $\mu_B(v_i, v_j) \leq \max\{\mu_A(v_i), \mu_A(v_j)\}$, and $\nu_B(v_i, v_j) \leq \min\{\nu_A(v_i), \nu_A(v_j)\}$ where $0 \leq \mu_B(v_i, v_j) + \nu_B(v_i, v_j) \leq 1$ for every $(v_i, v_j) \in E$.

Definition 2.6: An intuitionistic fuzzy graph G: (V, E) is said to be complete intuitionistic fuzzy graph if $\mu_B(v_i, v_j) = \min\{\mu_A(v_i), \mu_A(v_j)\}$and $\nu_B(v_i, v_j) = \max\{\nu_A(v_i), \nu_A(v_j)$ for all $v_i, v_j \in V$.

3. Series and Parallel Connections in Intuitionistic Fuzzy Graphs

Let $G_1(V, E)$ and $G_2(V', E')$ be two Intuitionistic fuzzy graphs where V= $\{v_1, v_2, v_3, ..., v_n\}$ and V′= $\{v_1', v_2', v_3', ..., v_n'\}$, E $\subseteq$ V $\times$ V and E′ $\subseteq$ V′ $\times$ V′ with membership and non-membership functions as follows:

μ_A: V $\rightarrow$ [0, 1], ν_A: V $\rightarrow$ [0, 1], μ_B: V $\times$ V $\rightarrow$ [0, 1], ν_B: V $\times$ V $\rightarrow$ [0, 1], edge e

μ_A': V′ $\rightarrow$ [0, 1], ν_A': V′ $\rightarrow$ [0, 1], μ_B': V′ $\times$ V′ $\rightarrow$ [0, 1], ν_B': V′ $\times$ V′ $\rightarrow$ [0, 1].

Definition 3.1: The parallel connection of G_1 and G_2 with respect to the edge e_1 and e_2 in the intuitionistic fuzzy graphs represented by $P((G_1, e_1): (G_2, e_2))$ is obtained by deleting the edge $e_1 \in G_1$, $e_2 \in G_2$ and identifying the end vertices $v_m \in G_1$ of e_1 and $v_n' \in G_2$ of e_2 as v, such that $\mu_A(v) = \mu_A(v_m) \wedge \mu_A(v_n')$, $\nu_A(v) = \nu_A(v_m) \vee \nu_A(v_n')$ and similarly identifying the other two end vertices $v_n \in G_1$ of e_1 and $v_m' \in G_2$ of e_2 as v′, such that $\mu_A(v') = \mu_A(v_n) \wedge \mu_A(v_m')$, $\nu_A(v') = \nu_A(v_n) \vee \nu_A(v_m')$ for any m and n and finally adding a new edge vv′, such that $\mu_B(vv') = \mu_B(e_1) \wedge \mu_B(e_2)$ and $\nu_B(vv') = \nu_B(e_1) \vee \nu_B(e_2)$.

Definition 3.2: The series connection of G_1 and G_2 with respect to the edge e_1 and e_2 in the intuitionistic fuzzy graphs represented by $S((G_1, e_1): (G_2, e_2))$ is obtained by deleting the edge $e_1 \in G_1$, $e_2 \in G_2$ and identifying one of the end vertex $v_m \in G_1$ of e_1 and $v_n' \in G_2$ of e_2 as v, such that $\mu_A(v) = \mu_A(v_m) \wedge \mu_A(v_n')$, $\nu_A(v) = \nu_A(v_m) \vee \nu_A(v_n')$ and finally adding a new edge $v_n v_m'$ where $v_n \in G_1$ of e_1 and $v_m' \in G_2$ of e_2 with $\mu_B(v_n v_m') = \mu_B(e_1) \wedge \mu_B(e_2)$ and $\nu_B(v_n v_m') = \nu_B(e_1) \vee \nu_B(e_2)$ for any m and n.

Example 3.3: Consider the following graphs G_1 and G_2

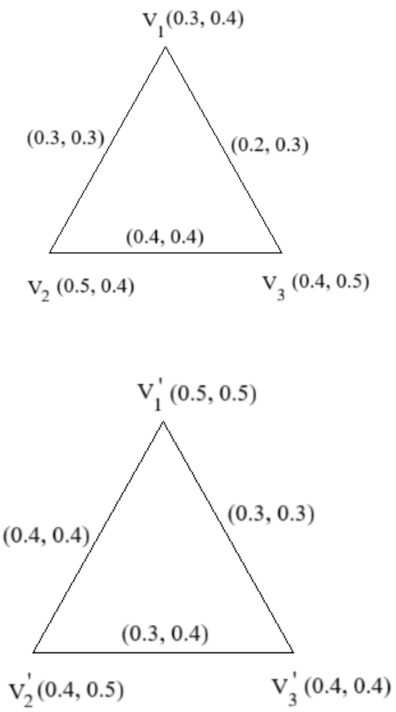

Figure 1. *$G_1(V, E)$ and $G_2(V', E')$.*

A parallel connection v_1v_2): ($G_{2,}$ $v_2'v_3'$)) is obtained by deleting the edges $v_1v_2 \in G_1$ and $v_2'v_3' \in G_2$ and identifying $v_1 \in G_1$ and $v_3' \in G_2$ as v and $v_2 \in G_1$ and $v_2' \in G_2$ as v′ and introducing a new edge vv′ as per the definition. The resultant intuitionistic fuzzy graph is as follows.

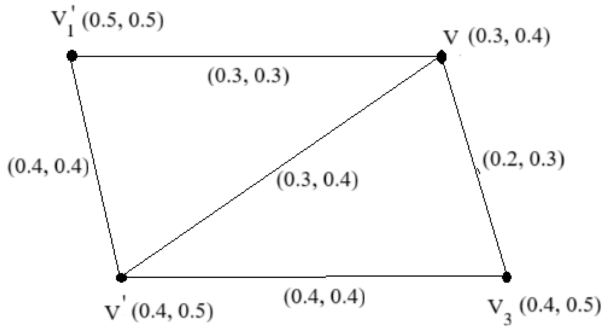

Figure 2. *$P((G_1, v_1v_2)$: $(G_2, v_2'v_3'))$.*

A series connection S((G1, v1v2): (G2, v2′v3′)) is obtained by deleting the edges v1v2 ∈ G1 and v2′v3′∈ G2 and identifying v2 ∈ G1 and v2′∈ G2 as v and introducing a new edge v1v3′ as per the definition. The resultant intuitionistic fuzzy graph is as follows.

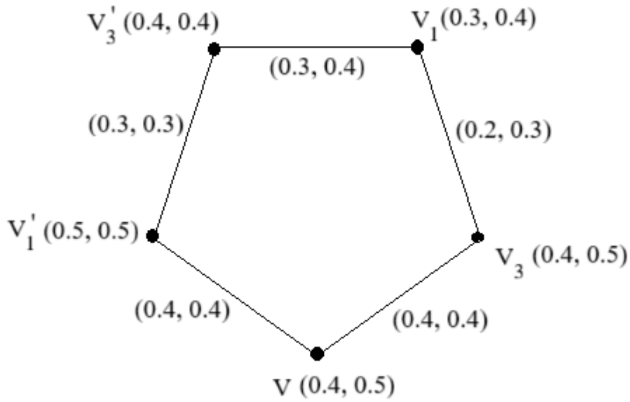

Figure 3. *$S((G_1, v_1v_2)$: $(G_2, v_2'v_3'))$.*

Theorem 3.4: Let $G_1(V, E)$ and $G_2(V', E')$ be two complete min-max intuitionistic fuzzy graphs then $P(G_1, G_2)$ and $S(G_1, G_2)$ need not be complete intuitionistic fuzzy graphs and not necessarily a min-max intuitionistic fuzzy graph.

Proof: The proof of the theorem follows from the below example.

Example 3.5: Consider the following graphs G_1 and G_2

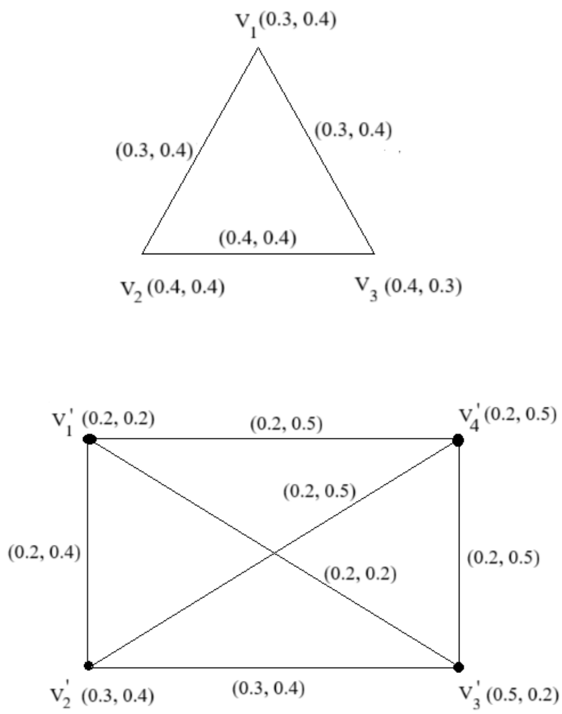

Figure 4. *$G_1(V, E)$ and $G_2(V', E')$.*

Consider the parallel connection $P((G_1, v_1v_2)$: $(G_2, v_1'v_2'))$ by identifying $v_1 \in G_1$ and $v_1' \in G_2$ as v and $v_2 \in G_1$ and $v_2' \in G_2$ as v′, the resultant intuitionistic fuzzy graph is given below.

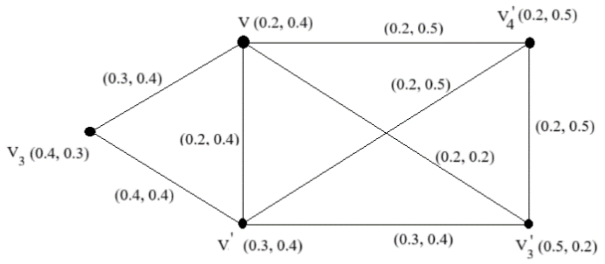

Figure 5. *P(G1, G2).*

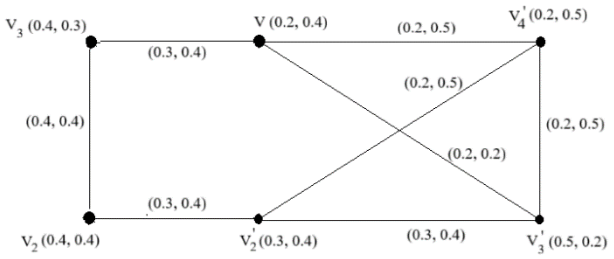

Figure 6. *S(G1, G2).*

$P(G_1, G_2)$ and $S(G_1, G_2)$ is neither a Minmax intuitionistic fuzzy graph nor complete which is obvious from membership functions of $vv_3 \in P(G_1, G_2)$, $v_3v' \in P(G_1, G_2)$ and $vv_3 \in S(G_1, G_2)$. Thus the IFG's $P(G_1, G_2)$ and $S(G_1, G_2)$ obtained above is named as Maxmax intuitionistic fuzzy graph and defined as follows.

Definition 3.6: A Maxmax intuitionistic fuzzy graph is of the form G: (V, E) where

(a). $V = \{v_1, v_2, \ldots, v_n\}$ such that $\mu_A: V \to [0, 1]$ and $\nu_A: V \to [0, 1]$ denotes the degree of membership and non-membership of the elements $v_i \in V$ respectively and $0 \leq \mu_A(v_i) + \nu_A(v_i) \leq 1$ for every $v_i \in V$; ($i = 1, 2, \ldots, n$).

(b). $E \subseteq V \times V$ where $\mu_B: V \times V \to [0, 1]$ and $\nu_B: V \times V \to [0, 1]$ are such that $\mu_B(v_i, v_j) \leq \max\{\mu_A(v_i), \mu_A(v_j)\}$, and $\nu_B(v_i, v_j) \leq \max\{\nu_A(v_i), \nu_A(v_j)\}$ where $0 \leq \mu_B(v_i, v_j) + \nu_B(v_i, v_j) \leq 1$ for every $(v_i, v_j) \in E$.`

Theorem 3.7: Two intuitionistic fuzzy graphs G_1 and G_2 with 'm' and 'n' number of edges respectively has 2mn parallel connections.

Proof: The number of ways of deleting one edge from each group containing 'm' and 'n' number of edges respectively is 'mn'. By definition of parallel connection, combining two end vertices of deleted edges can be done in two ways. Hence the total number of possible parallel connections are 2mn.

Theorem 3.8: Two intuitionistic fuzzy graphs G_1 and G_2 with 'm' and 'n' number of edges respectively has 4mn series connections.

Proof: The number of ways of deleting one edge from each group containing 'm' and 'n' number of edges respectively is 'mn'. By definition of series connection, identifying a pair of vertices from the end vertices of the deleted edges can be done in 4 different ways. Hence the total number of possible series connections are 4mn.

4. Conclusion

This study reveals that the Parallel and Series connections on two Minmax intuitionistic fuzzy graphs generates IFG which need not be necessarily Minmax IFG. The same study can be carried out on product IFG's also. This article can be extended to electrical circuit diagrams by representing them using IFG and parallel, series connections of those circuit diagrams can be generated using the above definitions 3.1 and 3.2.

Abbreviations

IFG's	Intuitionistic Fuzzy Graphs

Author Contributions

Anandkumar Narayanasamy Venkatesan: Conceptualization, Resources

Nivethana Venkataswamy: Methodology, Writing – review & editing

Conflicts of Interest

The authors declare no conflicts of interest.

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