



Research Article

Semi-analytical Solution of One-dimension Advection Diffusion Equation Coupled with Linear Partial Differential Equation with Constant Coefficient

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Abstract

This paper presents a semi-analytical solution for one-dimensional advection-diffusion equation coupled with a linear partial differential equation with constant coefficients. The mathematical model describes a grain-fumigant-air system during fumigation processes, where fumigant gas transports through a storage silo. The coupled system considers both diffusion and advection mechanisms with constant velocity and diffusivity parameters. The solution methodology employs the Laplace transformation technique to convert the partial differential equations into ordinary differential equations in the Laplace domain. The Stehfest numerical algorithm is subsequently applied to invert the Laplace transforms and obtain the time-domain solution. Numerical computations are performed using MATLAB software to simulate the fumigant concentration distributions. Graphical results illustrate the fumigant gas concentration in air versus vertical height within the silo for different time intervals. Additional plots demonstrate the fumigant concentration absorbed by grain particles over time. The analysis examines effects of varying initial gas concentration and flow velocity on the transport process. Results indicate that higher initial concentrations and increased velocities accelerate the fumigation process, requiring less time to fill the silo completely. The proposed solution provides a mathematical framework for optimizing fumigation parameters in agricultural storage applications.

Keywords

Advection, Advection-diffusion, Coupled Equation, Laplace Transformation Method, Linear Partial Differential Equations

1. Introduction

The advection diffusion equation plays an important role in describing solute transport due to the combined effect of diffusion and advection in a medium. These equations have drawn significant attention from hydrologists, civil engineers, and mathematical modelers. Many methods have been used to obtain the analytical, semi-analytical, and numerical solutions

of the one-dimensional, two-dimensional, and three-dimensional advection diffusion equations with different parameters, boundary conditions, and techniques. Kumar et al [1] and Kumar et al [2] solved analytically the one-dimensional advection diffusion equation with variable coefficients for a uniform input point source and of increasing nature by the Laplace

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transformation method in finite and semi-infinite media, respectively. Jaiswal et al [3] derived the solution of the one-dimensional advection diffusion equation with temporally dependent coefficients for a uniform input point source and input point of increasing nature by the Laplace transformation technique. Jaiswal and Kumar [4] obtained an analytical solution of the one-dimensional advection-dispersion equation with variable coefficients for a varying pulse-type input point source in a longitudinal domain for two problems: temporally dependent dispersion along uniform flow and spatially dependent dispersion along non-uniform flow by the Laplace transformation method. Kumar et al [5] developed an analytical solution for one-dimensional solute dispersion by the Laplace transformation method along unsteady flow through a heterogeneous medium; the dispersion parameter was considered proportional to the square of the velocity. Kumar et al [6] obtained an analytical solution for the one-dimensional advection diffusion equation for uniform flow in a longitudinal semi-infinite homogeneous porous medium by the Laplace method. The first-order decay, retardation factor, and zero-order production terms were also considered in this study. The first-order decay and zero-order production were considered inversely proportional to the dispersion coefficient. Chen and Liu [7] obtained a generalized analytical solution to the one-dimensional advection dispersion equation. The generalized analytical solution was obtained by using the Laplace transform with respect to time and the generalized integral transform technique with respect to the spatial coordinate. The solution was obtained in a finite spatial domain subject to an arbitrary time-dependent inlet boundary condition. Daga and Pradhan [8] obtained an analytical solution to the one-dimensional advection diffusion equation based on the mathematical combination between the variational iteration method and the homotopy perturbation method. Mazaheri [9] obtained an analytical solution for the one-dimensional advection diffusion equation with several point sources through arbitrary time-dependent emission rate patterns by the Laplace transform. Kumar and Yadav [10] obtained an analytical solution for conservative solute transport in homogeneous porous media for uniform and varying pulse-type input point sources in one dimension. Wadi et al [11] obtained an analytical solution of the one-dimensional advection dispersion equation by using the Laplace transformation technique. Sanskrityayn and Kumar [12] obtained an analytical solution of the advection-diffusion equation in a heterogeneous infinite medium by using Green's function method for two problems: continuous injection and instantaneous point injection. Kumar [13] obtained an analytical approach for the one-dimensional advection diffusion equation with temporally dependent variable coefficients of hyperbolic function in a semi-infinite porous domain by the Laplace transformation; the solute dispersion parameter and seepage velocity were considered as hyperbolically decreasing functions of time. Yadav and Kumar [14] derived an analytical solution for one-dimensional spatially dependent solute

transport in semi-infinite porous media through Laplace transformation. Chaudhary et al [15] developed research titled "Analysis of 1-D pollutant transport in semi-infinite groundwater reservoir." Yadav and Roy [16]. Jaiswal et al [17] obtained an analytical solution for the transport of pollutant from time-dependent locations along groundwater. The Laplace integral transformation technique was applied to obtain the final solution. Yadav et al [18] obtained an analytical solution for conservative solute transport in time- and space-dependent flow in heterogeneously adsorbing porous media. The solute transport also involved first-order decay and zero-order production. Kumar et al [19] discussed the effect of sink/source on the transport of solutes for the 1-D ADE in porous media to derive a closed-form solution by the Laplace transform. Paudel et al [20] obtained an analytical solution for the advection diffusion equation of pollutant concentration by using the Laplace transformation technique. Sanskrityayn et al [21] proposed an analytical solution for the advection diffusion equation with variable flow and transport coefficients. Yadav et al [22] obtained an analytical solution to the one-dimensional advection diffusion equation in a heterogeneous aquifer system with space- and time-dependent groundwater velocity and dispersion coefficient in a semi-infinite medium by the Laplace transformation method. Yadav et al [23] obtained an analytical solution for the two-dimensional advection diffusion equation in semi-infinite homogeneous porous media by the Laplace transformation method. First-order decay proportional to the dispersion coefficient was also considered in the study. Mckaa et al [24] obtained an analytical solution of a simple advection-diffusion model of an oxygen transfer device. To get the analytical solution, the Laplace transformation and convolution were applied. Chatterjee and Singh [25] obtained a solution for the two-dimensional advection dispersion equation with depth-dependent variable source concentration in a semi-infinite medium. Thakur [26] conducted research titled "Two-dimensional solute transport with exponential concentration distribution and varying flow velocity." Yadav and Kumar [27] obtained an analytical solution of the advection diffusion equation for conservative solute transport in a semi-infinite heterogeneous porous medium with a pulse-type input point source of uniform nature in two dimensions by using the Laplace transformation method. First-order decay and zero-order production were also considered. Essa et al [28] developed research on the evaluation of the analytical solution of the advection diffusion equation in three dimensions. Ibrahim et al [29] used the Laplace transformation technique and explicit finite difference method to obtain the analytical and numerical solution of the two-dimensional advection-diffusion equation for pollutant concentration in a river with time-dependent dispersion coefficients. Chaudhary and Singh [30]. Chen et al [31]. Buske et al [32] carried out research on the solution of the coupled advection-diffusion and Navier-Stokes equations in the two-dimensional advection-diffusion equation. De Athayde et al [33] carried out research titled "Analytical solution of the coupled advection-diffusion and Navier-

Stokes equation for air pollutant emission simulation.”

The lack of an advection diffusion equation coupled with a linear partial differential equation is the motivation of this work to obtain a semi-analytical solution for the one-dimensional advection diffusion equation coupled with a linear partial differential equation with constant coefficients. The Laplace transformation technique and Stehfest algorithm [34] are applied to obtain the final solution to the system. The solution to the system is obtained in homogeneous and semi-infinite media. The computations are done with the help of MATLAB software. The solution to the coupled system is illustrated through graphs, presenting the effect of concentration and velocity.

2. Mathematical Formulas

The advection-diffusion equation coupled with a linear partial differential equation is modeled from an application of grain fumigation in a grain store, which is the grain-fumigant-air system. During fumigation, fumigant gas is pumped into the storage with constant velocity. Then, the gas mixes with the air in the silo. The flow of the gas into the silo can be modeled as a transport process where the processes of diffusion and advection occur. The fumigant transfers from the inlet into the storage and accumulates on the grain solid. Work by Darby [35] has proven that some amount of the fumigant will absorb into the grain, and at the same time some amount also desorbs back to the air. Therefore, Darby [35] introduced a coupled advection-diffusion equation and linear partial differential equation as follows:

$$\frac{\partial c(x,t)}{\partial t} + V \frac{\partial c(x,t)}{\partial x} = D \frac{\partial^2 c(x,t)}{\partial x^2} - A_1 c(x,t) + A_2 q(x,t) \quad (1)$$

$$\frac{\partial q(x,t)}{\partial t} = A_3 c(x,t) - A_4 q(x,t) \quad (2)$$

Where x is the direction of flow (m), t is time(s), $c(x,t)$ the concentration of the fumigant in air, $q(x,t)$ is the concentration of fumigant in grain, V is the velocity that the quantity is moving, and D is the diffusivity and the additional parameter A_1, A_2, A_3 and A_4 .

Based on James Darby (2011) which is about the mathematical model defining the kinetics of the sorption and application to predict concentrations and grain residues for practical fumigation scenarios, the formula of the constant A_1, A_2, A_3 and A_4 are given by:

$$w = \left(k_{fA} + \frac{K_{fG} F}{\rho_g} \right)$$

$$A_1 = \left(r_{fA} + \frac{s_{sorp} (k_{fA})^2}{\varepsilon w} - \frac{s_{sorp} k_{fA}}{\varepsilon} \right)$$

$$A_2 = \left(\frac{s_{sorp} k_{fA} k_{fG}}{\varepsilon w} \right)$$

$$A_3 = \left(\frac{s_{sorp} k_{fG}}{(1-\varepsilon)\rho_g} - \frac{s_{sorp} (k_{fG})^2 F}{w(1-\varepsilon)(\rho_g)^2} - r_{fG} \right)$$

$$A_4 = \left(\frac{s_{sorp} k_{fG} k_{fA} F}{w(1-\varepsilon)(\rho_g)^2} \right)$$

K_{fA} : The mass transfer coefficient of gaseous fumigant from air to grain.

K_{fG} : The mass transfer coefficient of adsorbed fumigant in grain.

r_{fG} : The rate of the first-order reaction of adsorbed fumigant in the grain.

r_{fA} : Rate of first order “reaction” of gaseous fumigant in air.

S_{sorp} : Specific surface area for sorption.

F : partition factor.

ρ_g : The true density of the grain kernels.

ε : porosity of the bulk grain.

D_{sorp} : The diffusivity of fumigant into grain.

3. Methodology

$$\frac{\partial c(x,t)}{\partial t} + V \frac{\partial c(x,t)}{\partial x} = D \frac{\partial^2 c(x,t)}{\partial x^2} - A_1 c(x,t) + A_2 q(x,t) \quad (3)$$

$$\frac{\partial q(x,t)}{\partial t} = A_3 c(x,t) - A_4 q(x,t) \quad (4)$$

With initial and boundary conditions:

Initial Condition: $c(x, 0) = 0, q(x, 0) = 0$

Boundary condition: $c(0, t) = c_0, c(\infty, t) = 0,$

$q(\infty, t) = 0$

Applying the Laplace transform:

Then (3) becomes:

$$sC(x, s) - c(x, 0) + V \frac{dC(x, s)}{dx} = D \frac{d^2 C(x, s)}{dx^2} - A_1 C(x, s) + A_2 Q(x, s) \quad (5)$$

and (4) becomes:

$$sQ(x, s) - q(x, 0) = A_3 C(x, s) - A_4 Q(x, s) \quad (6)$$

Applying the initial condition, for equations (5) and (6) become:

$$Q(x, s) = \frac{A_3 C(x, s)}{(s + A_4)} \quad (8)$$

Substitute (8) into equation (7), we get.

$$D \frac{dC^2(x, s)}{dx^2} - V \frac{dC(x, s)}{dx} + \left(-(s + A_1) + \frac{A_2 A_3}{(s + A_4)} \right) C(x, s) = 0 \quad (9)$$

Which is the homogenous second order linear ODE.

$$m = \frac{V}{2D} \pm \frac{\sqrt{(s + A_4)^2 V^2 - 4D(s + A_4)[-(s + A_1)(s + A_4) + A_2 A_3]}}{2D(s + A_4)}$$

Therefore the general solution can be written as:

$$C(x, s) = A e^{(\alpha + \beta)x} + B e^{(\alpha - \beta)x}$$

$$\beta = \frac{\sqrt{(s + A_4)^2 V^2 - 4D(s + A_4)[-(s + A_1)(s + A_4) + A_2 A_3]}}{2D(s + A_4)}$$

Applying the boundary conditions, we find: A=0 Hence B = $\frac{C_0}{s}$. Thus:

$$C(x, s) = \frac{C_0}{s} \exp \left(\frac{V}{2D} - \frac{\sqrt{(s + A_4)^2 V^2 - 4D(s + A_4)[-(s + A_1)(s + A_4) + A_2 A_3]}}{2D(s + A_4)} x \right)$$

After simplification,

$$C(x, s) = \frac{C_0}{s} \exp \left(\frac{V - \sqrt{V^2 + 4D[(s + A_1) + \frac{(A_2 A_3)}{(s + A_4)}]}}{2D} x \right)$$

4. Results

Concentration of Phosphine Gas at Different Times
(V = 0.002 m/s, C₀ = 10 mg/L)

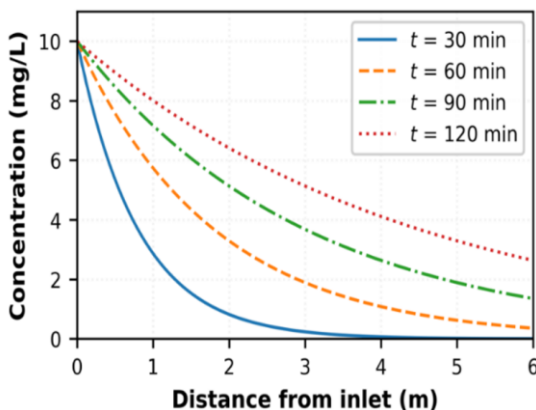


Figure 1. Concentration of fumigant gas in air vs. height for V=0.002 m/s, C₀=10 mg/L.

Concentration of Phosphine Gas in Grain at Different Times
(V = 0.002 m/s, C₀ = 10 mg/L)

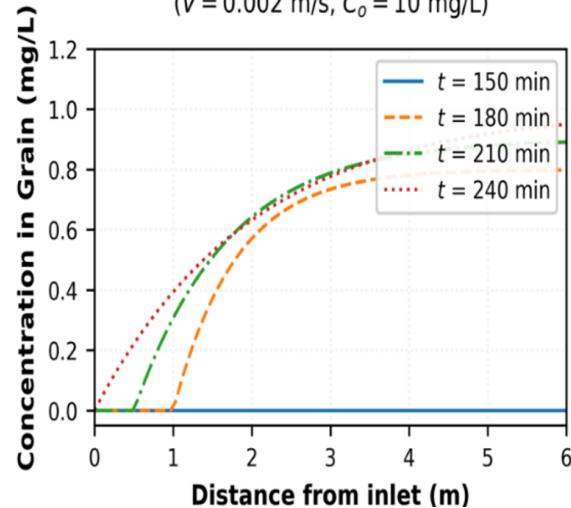


Figure 2. Concentration of fumigant gas in grain vs. height for V=0.002 m/s, C₀=10 mg/L.

Figure 1 shows the concentration of the phosphine gas flow in air versus the vertical height of the silo. The initial concentration of the phosphine gas is 10 mg/L with constant velocity at V=0.002 m/s. The concentration of phosphine gas is plotted for four intervals of time to illustrate the gas-movement behavior until the gas completely fills the silo.

Based on the graph, the fumigation-gas process needs more than 120 minutes to ensure the phosphine gas completely fills

the silo of 6 m height. At $t=30$ minutes, fumigation gas of 10 mg/L has reached a height of more than 1 m and continues moving until the entire silo space is occupied. Therefore, it can be concluded that fumigation with constant velocity $V=0.002$ m/s takes at least 120 minutes to fill a 6 m silo, and the gas will be absorbed by the grain after 120 minutes.

Figure 2 shows that at $t=150$ minutes, the concentration of the fumigation gas is zero because the gas is still moving to fill the entire space of the silo. The grain at a height of more than 1 m starts to absorb the gas after the gas has been flowing for 180 minutes through the silo. The gas continues to diffuse into the grain at higher parts until the grain at all heights contains 1 mg/L of gas concentration after 240 minutes of gas flow.

Figure 3 represents the concentration of the phosphine gas in air when the constant velocity is increased to 0.004 m/s. At $t=30$ minutes, the fumigation gas of 10 mg/L has reached a height of more than 4 m, which is better than the result in Figure 2.

Concentration of Phosphine Gas in Air at Different Times
($V = 0.004$ m/s, $C_0 = 10$ mg/L)

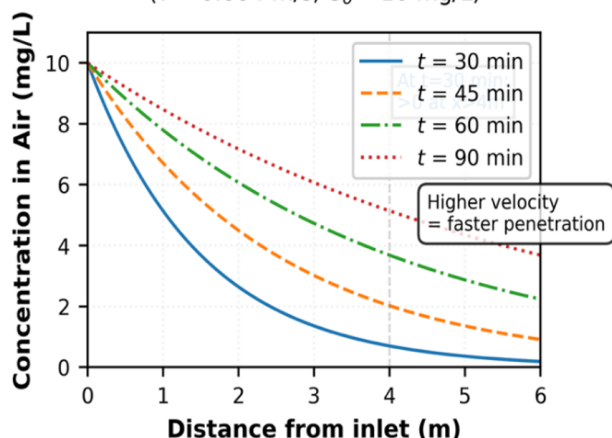


Figure 3. Concentration of fumigant gas in air vs. height for $V=0.004$ m/s, $C_0=10$ mg/L.

Concentration of Phosphine Gas in Grain at Different Times
($V = 0.004$ m/s, $C_0 = 10$ mg/L)

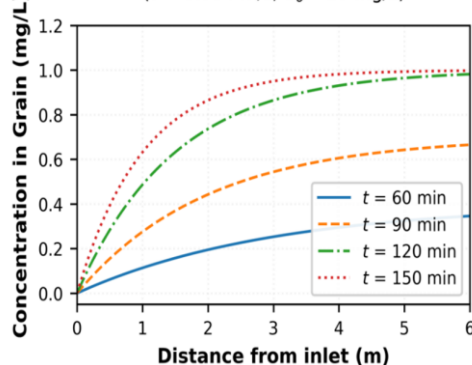


Figure 4. Concentration of fumigant gas in grain vs. height for $V=0.004$ m/s, $C_0=10$ mg/L.

Moreover, the maximum time for the gas to completely fill the silo is $t=60$ minutes, whereas in Figure 1 it is $t=120$ minutes. It can be concluded that changing the velocity from 0.002 m/s to 0.004 m/s affects the fumigation process: less time is needed for the gas to completely fill the 6 m silo when the velocity is increased.

Figure 4 shows that when the velocity is increased, the gas absorption by the grain becomes faster; the grain in all parts takes only 120 minutes to absorb 1 mg/L of phosphine gas, compared with Figure 2, which required 240 minutes.

Figure 5 illustrates the behavior of phosphine gas concentration when the initial concentration of fumigation gas is decreased from 10 mg/L to 5 mg/L at constant velocity $V=0.002$ m/s. The gas with an initial concentration of 5 mg/L needs at least 90 minutes to reach the maximum height of the silo, which is faster than the gas with an initial concentration of 10 mg/L.

Concentration of Phosphine Gas in Air at Different Times
($V = 0.002$ m/s, $C_0 = 5$ mg/L)

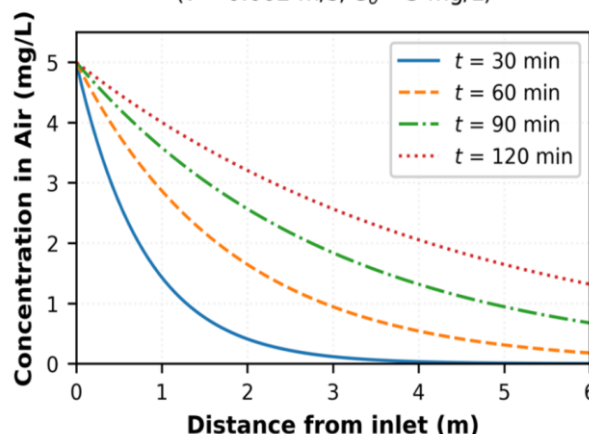


Figure 5. Concentration of fumigant gas in air vs. height for $V=0.002$ m/s, $C_0=5$ mg/L.

Concentration of Phosphine Gas in Grain at Different Times
($V = 0.002$ m/s, $C_0 = 5$ mg/L)

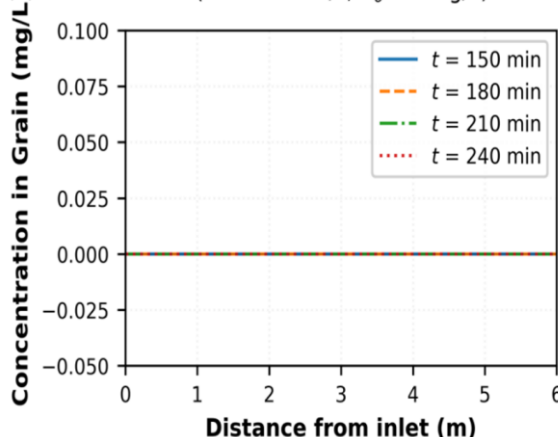


Figure 6. Concentration of fumigant gas in grain vs. height for $V=0.002$ m/s, $C_0=5$ mg/L.

Since the value of constant velocity is the same in both observations, it is clear that the initial concentration of the fumigation gas also affects the rate at which the gas moves.

Figure 6 shows that the concentration of the gas in the grain remains zero as time increases when the applied initial concentration is 5 mg/L, meaning that there is no gas absorbed by the grain.

Figure 7 represents the behavior of phosphine gas concentration when the initial concentration is 5 mg/L and the constant velocity is increased from 0.002 m/s to $V=0.004$ m/s. Based on the result in Figure 7, the gas moves faster to reach the top of the silo at 6 m.

Concentration of Phosphine Gas in Air at Different Times
($V = 0.004$ m/s, $C_0 = 5$ mg/L)

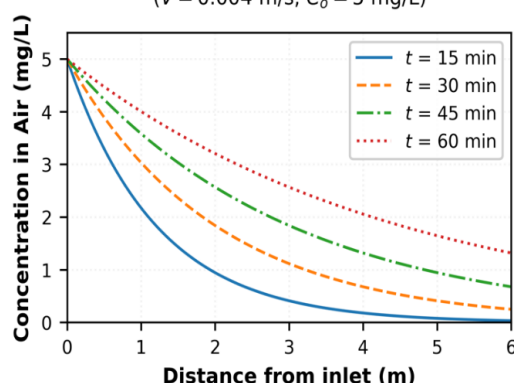


Figure 7. Concentration of fumigant gas in air vs. height for $V=0.004$ m/s, $C_0=5$ mg/L.

At $t=30$ minutes, the concentration of 5 mg/L has achieved a height of more than 3 m, which is better than the result with constant velocity 0.002 m/s. However, as time increases, the fumigation gas still does not appear in the grain.

5. Conclusions

An analytical solution to the one-dimensional advection-diffusion equation coupled with a linear partial differential equation with constant coefficients is obtained by the Laplace transformation method. The Stehfest algorithm is used to obtain the inverse Laplace transformation. Based on the results, it can be concluded that the initial concentration and velocity of the fumigation gas affect the rate of gas absorption. When the initial concentration is lower, no gas is absorbed by the grain. Higher values of initial concentration and velocity lead to a faster gas-absorption process, since less time is needed for the gas to fill the top surface of the silo and reach the grain.

Author Contributions

Mohammad Jawad Qasimi: Conceptualization, Formal

Analysis, investigation, Methodology, Software, visualization, Writing – original draft, Writing – review & editing

Norma Alias: Supervision, Writing – review & editing

Conflicts of Interest

The authors declare no conflicts of interest.

References

- [1] Kumar, A., Jaiswal, D. K. and Kumar, N. (2009). Analytical solutions of one-dimensional advection-diffusion equation with variable coefficients in a finite domain. *Journal of Earth System Science*, 118(5), 539-549.
- [2] Kumar, A., Jaiswal, D. K. and Kumar, N. (2010). Analytical solutions to one-dimensional advection-diffusion equation with variable coefficients in semi-infinite media. *Journal of Hydrology*, 380(3-4), 330-337.
- [3] Jaiswal, D. K., Kumar, A. and Yadav, R. R. (2011). Analytical solution to the one-dimensional advection-diffusion equation with temporally dependent coefficients. *Journal of Water Resource and Protection*, 3(1), 76-84.
- [4] Jaiswal, D. K. and Kumar, A. (2011). Analytical solution of advection-dispersion equation for varying pulse type input point source in one-dimension. *International Journal of Engineering, Science and Technology*, 3(1), 22-29.
- [5] Kumar, A., Jaiswal, D. K. and Kumar, N. (2012). One-dimensional solute dispersion along unsteady flow through a heterogeneous medium, dispersion being proportional to the square of velocity. *Hydrological Sciences Journal*, 57(6), 1218-1232.
- [6] Kumar, A., Jaiswal, D. K. and Yadav, R. R. (2012). Analytical solutions of one-dimensional temporally dependent advection-diffusion equation along longitudinal semi-infinite homogeneous porous domain for uniform flow. *IOSR Journal of Mathematics*, 2(1), 1-11.
- [7] Chen, J. S. and Liu, C. W. (2011). Generalized analytical solution for advection-dispersion equation in finite spatial domain with arbitrary time-dependent inlet boundary condition. *Hydrology and Earth System Sciences*, 15(8), 2471-2479.
- [8] Daga, A. and Pradhan, V. H. (2013). Analytical solution of advection-diffusion equation in homogeneous medium. *International Journal of Science, Spirituality, Business and Technology (IJSSBT)*, 2(1), 1-8.
- [9] Mazaheri, M., Samani, J. M. V. and Samani, H. M. V. (2013). Analytical solution to one-dimensional advection-diffusion equation with several point sources through arbitrary time-dependent emission rate patterns. *Journal of Agricultural Science and Technology*, 15(6), 1231-1245.
- [10] Kumar, A. and Yadav, R. R. (2014). One-dimensional solute transport for uniform and varying pulse type input point source through heterogeneous medium. *Environmental Technology*, 36(4), 487-495.

- [11] Wadi, A. S., Dimian, M. F. and Ibrahim, F. N. (2014). Analytical solutions for one-dimensional advection–dispersion equation of the pollutant concentration. *Journal of Earth System Science*, 123(6), 1317-1324.
- [12] Sanskritayn, A. and Kumar, N. (2016). Analytical solution of advection diffusion equation in heterogeneous infinite medium using Green's function method. *Journal of Earth System Science*, 125(8), 1713-1723.
- [13] Kumar, L. K. (2017). An analytical approach for one-dimensional advection diffusion equation with temporally dependent variable coefficients of hyperbolic function in semi-infinite porous domain. *International Research Journal of Engineering and Technology*, 4(9), 1587-1592.
- [14] Yadav, R. R. and Kumar, L. K. (2017). One-dimensional spatially dependent solute transport in semi-infinite porous media: an analytical solution. *International Journal of Engineering, Science and Technology*, 9(4), 20-27.
- [15] Chaudhary, M., Thakur, C. K. and Singh, M. K. (2020). Analysis of 1-D pollutant transport in semi-infinite groundwater reservoir. *Environmental Earth Sciences*, 79(1), 24.
- [16] Yadav, R. R. and Roy, J. (2022). Analytical solutions of one-dimensional scale dependent advection-dispersion equations for finite domain solute transport. *Groundwater for Sustainable Development*, 16, 100712.
- [17] Jaiswal, D. K., Kumar, N. and Yadav, R. R. (2022). Analytical solution for transport of pollutant from time-dependent locations along groundwater. *Journal of Hydrology*, 610, 127826.
- [18] Yadav, R. R., Kushwaha, S., Roy, J. and Kumar, L. K. (2023). Analytical solutions for scale and time dependent solute transport in heterogeneous porous medium. *Journal of Water Resources and Ocean Science*, 12(1), 1-11.
- [19] Kumar, R., Chatterjee, A., Singh, M. K. and Singh, V. P. (2019). Study of solute dispersion with source/sink impact in semi-infinite porous medium. *Pollution*, 6(1), 87-98.
- [20] Paudel, K., Bahandari, P. S. and Kafe, J. (2021). Analytical solution for advection-dispersion equation of the pollutant concentration using Laplace transformation. *Journal of Nepal Mathematical Society (JNMS)*, 4(1), 1-12.
- [21] Sanskritayn, A., Suk, H., Chen, J. S. and Park, E. (2021). Generalized analytical solutions of the advection dispersion equation with variable flow and transport coefficients. *Sustainability*, 13(14), 7796.
- [22] Yadav, R. R., Kushwaha, S., Kumar, L. K. and Roy, J. (2023). An analytical approach to contaminant transport with spatially and temporally dependent dispersion in a heterogeneous porous medium. *Cumhuriyet Science Journal*, 44(3), 538-546.
- [23] Yadav, R. R., Kumar, L. K. and Kushwaha, S. (2020). Analytical solution for two-dimensional advection diffusion equation in semi-infinite homogeneous porous media by Laplace transformation method. *Pollution*, 6(4), 901-915.
- [24] Mckaa, S., Dougall, E. A. and Mottram, N. J. (2016). Analytical solutions of a simple advection-diffusion model of an oxygen transfer device. *Journal of Mathematics in Industry*, 6(1), 3.
- [25] Chatterjee, A. and Singh, M. K. (2018). Two-dimensional advection-dispersion equation with depth-dependent variable source concentration. *Pollution*, 4(1), 1-8.
- [26] Thakur, C. K., Chaudhary, M., van der Zee, S. E. A. T. M. and Singh, M. K. (2019). Two-dimensional solute transport with exponential concentration distribution and varying flow velocity. *Pollution*, 5(4), 721-737.
- [27] Yadav, R. R. and Kumar, L. K. (2019). Solute transport for pulse type input point source along temporally and spatially dependent flow. *Pollution*, 5(1), 53-70.
- [28] Essa, K. S. M., Mosallem, A. M. and Shalaby, A. S. (2021). Evaluation of analytical solution of advection diffusion equation in three dimensions. *Atmospheric Science Letters*, 22(11), e1043.
- [29] Ibrahim, F., Saleh, A., Wadi, A. and Hadhouda, M. (2022). Remediation of pollution in a river by releasing clean water using the solution of advection-diffusion equation in two-dimension. *Information Sciences Letters*, 11(5), 1385-1392.
- [30] Chaudhary, M. and Singh, M. K. (2020). Study of multispecies convection-dispersion transport equation with variable parameters. *Journal of Hydrology*, 591, 125562.
- [31] Chen, J. S., Ho, Y. C., Liang, C. P., Wang, S. W. and Liu, C. W. (2019). Semi-analytical model for coupled multispecies advective-dispersive transport subject to rate-limited sorption. *Journal of Hydrology*, 579, 124164.
- [32] Buske, D., Bodmann, B. and Vilhena, M. T. (2015). On the solution of the coupled advection-diffusion and Navier-Stokes equations. *American Journal of Environmental Engineering*, 5(1A), 1-8.
- [33] De Athayde, A. S., Piovesan, L. R., Bodmann, B. E. J. and De Vilhena, M. T. M. B. (2018). Analytical solution of the coupled advection diffusion and Navier-Stokes equation for air pollutant emission simulation. *American Journal of Environmental Engineering*, 8(4), 150-153.
- [34] Stehfest, H. (1970). Algorithm 368: Numerical inversion of Laplace transform [D5]. *Communications of the ACM*, 13(1), 47-49.
- [35] Darby, J., Willis, T. and Damcevski, K. (2009). Modelling the kinetics of ethyl formate sorption by wheat using batch experiments. *Pest Management Science*, 65(9), 982-990.