

Research Article

Comparative Study of Wavelet Solutions for Forced Oscillatory Problems Arising with and Without Damping

Preeti* , Inderdeep Singh 

Department of Physical Sciences, Sant Baba Bhag Singh University, Jalandhar, India

Abstract

Oscillatory problems are a class of mathematical and physical phenomena in which the solutions display periodic or quasi-periodic variations with respect to time or space. Oscillatory systems subject to external forcing are central in many physical and engineering contexts, and the presence or absence of damping critically influences their behavior. In this comparative study, we develop and evaluate wavelet-based numerical schemes for forced oscillatory differential equations, considering both damped and undamped regimes. Specifically, we employ second-kind Chebyshev wavelets and Haar wavelets, together with their operational integration matrices, to discretize and approximate the solutions of second-order forced oscillators. The wavelet formulations transform the differential problems into systems of algebraic equations, which we then solve under a variety of forcing frequencies and damping parameters. Our numerical experiments demonstrate that Chebyshev wavelets, due to their higher smoothness and spectral accuracy, are particularly effective in capturing the transient decay and subtle features of damped oscillations. In contrast, Haar wavelets provide a computationally efficient and stable approximation in undamped systems, especially in scenarios prone to resonance. We compare these methods across key metrics such as convergence rate, error behavior, and computational cost. The numerical results confirm that Chebyshev wavelets are particularly effective in capturing the fine decay dynamics of damped oscillations, while Haar wavelets deliver fast, stable approximations for undamped systems, especially under resonant forcing. These findings are supported by several computational experiments, which validate both the accuracy and efficiency of the proposed wavelet schemes. These insights offer practical guidance for selecting suitable wavelet approaches tailored to the physical damping conditions of forced oscillatory problems.

Keywords

Operational Matrices, Chebyshev Wavelets of the Second Kind, Haar Wavelets, Forced Oscillatory Problems, Damped and Undamped Systems

1. Introduction

Oscillation refers to the repetitive movement between two positions or states. This can manifest as a periodic motion that repeats in a regular cycle as shown in [Figure 1](#), such as the

swinging of a pendulum from side to side or the vertical motion of a spring with a weight attached.

*Corresponding author: preetikainth15@gmail.com (Preeti)

Received: 29 October 2025; Accepted: 19 November 2025; Published: 17 December 2025



Copyright: © The Author(s), 2025. Published by Science Publishing Group. This is an **Open Access** article, distributed under the terms of the Creative Commons Attribution 4.0 License (<http://creativecommons.org/licenses/by/4.0/>), which permits unrestricted use, distribution and reproduction in any medium, provided the original work is properly cited.

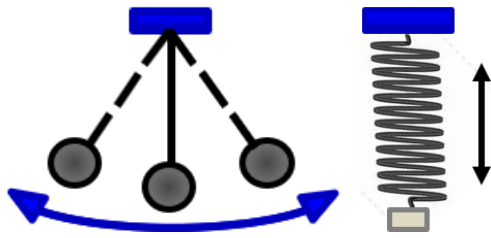


Figure 1. Oscillation.

Consider a spring OA hanging vertically from a fixed point O (see Figure 2). A body of mass m is attached to the lower end A of the spring. Since the mass of the body much greater than that of the spring, the spring's own weight can be ignored. Let e denote the extension of the spring due to the weight of the mass m . When the system reaches equilibrium, the point B corresponds to the equilibrium position, and the extension e represents the static elongation. Let k be the spring constant (or stiffness), which is the restoring force per unit extension caused by the spring's elasticity. At equilibrium, the downward gravitational force is balanced by the spring's restoring force, hence,

$$mg = T = ke \quad (1)$$

Let the mass m be displaced through a small distance x from its equilibrium position. The acceleration of the mass at this point is given by $\frac{d^2x}{dt^2}$. The force acting on the mass include its weight mg , acting downward, and the restoring force of the spring $k(e+x)$, acting upwards.

1.1. Forced Oscillation (Without Damping)

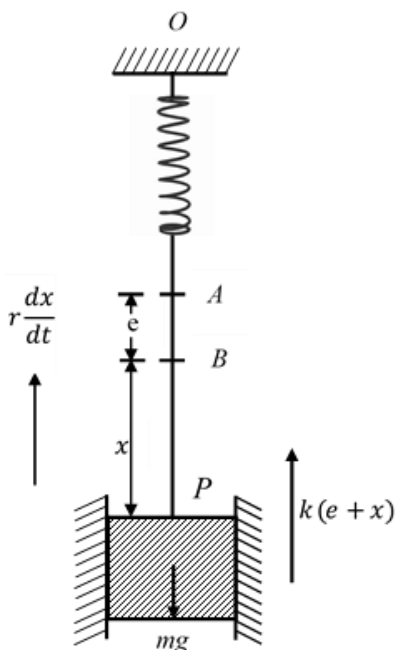


Figure 2. A suspended spring from fixed support.

If the fixed support of the spring is subjected to an external periodic force, the motion of the system becomes a forced oscillation. Suppose the external periodic force acting on the system is represented by $P \cos(nt)$. Then, the equation of motion of the system is given by:

$$m \frac{d^2x}{dt^2} = mg - k(e+x) + P \cos nt$$

After simplification, we obtain

$$\frac{d^2x}{dt^2} + w^2x = S \cos nt,$$

where

$$w^2 = \frac{k}{m}, S = \frac{P}{m}$$

1.2. Forced Oscillation (With Damping)

If there is also a damping force proportional to the instantaneous velocity of the mass say $\Delta \frac{dx}{dt}$, then the equation of motion is:

$$m \frac{d^2x}{dt^2} = mg - k(e+x) + P \cos nt - \Delta \frac{dx}{dt}$$

After simplification, we obtain

$$\frac{d^2x}{dt^2} + A \frac{dx}{dt} + w^2x = S \cos nt,$$

where

$$w^2 = \frac{k}{m}, S = \frac{P}{m}, A = \frac{\Delta}{m}$$

Forced oscillations (both with and without damping) play a crucial role in physics and engineering. In mechanical engineering, for instance, analyzing how oscillations behave under external periodic forces (especially when damping is present) is vital for designing vibration-resistant systems such as vehicle suspensions or suspension bridges. In seismology, modeling damped forced oscillations helps in predicting how buildings respond to earthquake-induced ground motion. On the other hand, in applications like clocks and musical instruments, undamped forced oscillations are fundamental: they maintain continuous, periodic motion required for precise timekeeping or sustained sound production. Chebyshev wavelet methods are often superior for solving oscillatory problems because they combine the spectral accuracy of Chebyshev polynomials with the multi-resolution, localized nature of wavelets. This means that we can approximate highly oscillatory solutions very precisely using relatively few basis functions. The use of operational matrices of integration converts differential equations into a system of algebraic equations, making the numerical computation efficient and

stable. Studies show that Chebyshev wavelet collocation methods can achieve high accuracy even with a small number of collocation points, and they are effective for both linear and nonlinear oscillatory systems. The aim of this work is to develop and analyze a numerical framework based on Chebyshev wavelet methods for solving forced oscillatory systems, both with and without damping.

In this paper, the authors present methods utilizing the operational matrix of fractional-order integration to solve a typical n -term non-homogeneous fractional differential equation (FDE) in the study of Sheikhani and mashoof [1]. The authors introduced an iterative method for the solution of differential equations along with initial values on large intervals by Dizicheh et al. [2]. For the numerical solution of linear and nonlinear fractional differential equations has been reviewed in the study of Gupta and Ray [3]. The authors proposed a computational technique for solving Lane emden type equations using the Haar wavelets method combined with the Newton Raphson method Verma et al. [4]. In this research paper, the authors applied a computational approach that combines the Picard method with sine-cosine wavelets to solve fractional nonlinear differential oscillator equations Saeed and Saeed [5]. Legendre wavelet method has been discussed for fractional differential equations by Yuttanan and Razzaghi [6]. Here, the authors described a wavelet-based algorithm for nonlinear oscillator equations using operational matrices of derivatives by sathyaseelan and Hariharan [7]. The authors of this study present a novel to investigate the solutions of fractional oscillator-type equations Shah et al. [8, 24] by combining block-pulse functions with ultra-spherical wavelet operational matrix of general-order integration. Here in this work, the authors present a review of the wavelet method, demonstrating its power and efficiency in solving a wide range of linear and nonlinear reaction-diffusion equations Hariharan and Kannan [9]. The authors propose a numerical scheme that employs uniform Haar wavelet approximation combined with the quasilinearization process to solve certain nonlinear oscillator equations Kaur et al. [10]. Haar wavelet-based technique has been introduced to solve second order boundary problems Aziz and Sarler [11]. Haar and Laguerre wavelets-based algorithms have been developed for solving linear and nonlinear delay differential equations Aziz et al. [12, 17]. A numerical algorithm based on Chebyshev wavelets, Haar wavelets, Hermite wavelets and Adomian decomposition method have been presented for the solution of oscillation equations and LCR circuit equations in the study of Haq et al. [13, 20, 21, 25, 27]. Dremin et al. [14], the authors presented a thoroughly study of wavelets and their applications in practical computing. Haar wavelet-based approach has been developed Singh and Kumar [15] for solving damped oscillator problems and then comparison study has been made with Runge kutta fourth order as well as with Taylor series method. In the research work of Rashidinia et al. [16], an algorithm for solving the general form of distributed-order fractional differential equations in the time domain using Caputo fractional derivatives has been given. The methodology is

based on Chebyshev wavelet of the second kind. Firstly, the operational matrix of fractional order of integration is derived followed by the introduction of a numerical technique based on Chebyshev wavelet for solving nonlinear multi order fractional differential equation Heydari et al. [18]. Chelyshkov polynomial method combined with picard iterative method used to solve nonlinear oscillator problems having arbitrary order Hamid et al. [19]. Haar wavelet method has been proposed for damped forced oscillator problems Murad and Husien [22]. To solve Duffing equation and painleve transcendents, the authors discussed a novel single term Haar wavelet series method Bujurke et al. [23]. The authors developed Chebyshev wavelet based numerical method for solving numerical differentiation by Singh and Preeti [26].

2. Chebyshev Wavelets of the Second Kind [26, 27]

In recent years, wavelet based numerical methods have gained significant attention for addressing a wide range of engineering and science problems. A single function, referred to as the mother wavelet, is scaled and translated to create a family of functions known as wavelets. The generic family of continuous wavelets can be expressed as follows: the translation and dilation parameters i.e. “ s ” and “ r ” respectively, can change continuously.

$$\varphi_{s,r}(t) = |s|^{-1/2} \varphi\left(\frac{t-r}{s}\right), s, r \in R, s \neq 0$$

The second kind Chebyshev wavelets, denoted as $\varphi_{n,m}$, are characterized by four parameters $\varphi_{n,m} = \varphi(k, n, m, t)$; k denotes a positive integer, $n = 1, 2, 3, 4, 5, \dots, 2^{k-1}$, m corresponds to the degree of Chebyshev polynomials and t represents normalized time. The wavelet is defined on the interval $[0, 1)$ as follows:

$$\varphi_{n,m}(t) = \begin{cases} 2^{\frac{k}{2}} \tilde{U}(2^k t - 2n + 1), & \frac{n-1}{2^{k-1}} \leq t \leq \frac{n}{2^{k-1}} \\ 0, & \text{elsewhere} \end{cases}$$

where

$$\tilde{U}_m(t) = \sqrt{\frac{2}{\pi}} U_m(t) \quad (2)$$

where $m = 0, 1, 2, 3, 4, \dots, M-1$ and M denotes fixed integer. In relation given by equation (2) is for orthonormality. Here $U_m(t)$ represents the Chebyshev polynomials which are orthogonal to the weight function $\omega(t) = \sqrt{1-t^2}$ over the interval $[-1, 1]$, and the following recurrence formula has been satisfied:

$$U_0(t) = 1, U_1(t) = 2t, U_{m+1}(t) = 2tU_m(t) - U_{m-1}(t), m = 1, 2, 3, 4, \dots$$

To apply Chebyshev wavelets over the normalized interval, the weight function is suitably translated and scaled as $\omega_n(t) = \omega(2^k t - 2n + 1)$.

Any square-integrable function $f(x) \in L^2(\mathbb{R})$ can be expressed as a finite series expansion using Chebyshev wavelets of the second kind as:

$$f(x) = \sum_{n=1}^{\infty} \sum_{m=0}^{\infty} c_{n,m} \varphi_{n,m}(x), \quad (3)$$

where

$$c_{n,m} = \langle f(x), \varphi_{n,m}(x) \rangle_{L^2_{\omega}[0,1]} = \int_0^1 f(x) \varphi_{n,m}(x) \omega_n(x) dx,$$

in which the inner product denoted by $\langle \cdot, \cdot \rangle_{L^2_{\omega}[0,1]}$ in $L^2_{\omega}[0,1]$. The infinite series given by eqn. (3) can be truncated and we write it as

$$f(x) \cong \sum_{n=1}^{\infty} \sum_{m=0}^{\infty} c_{n,m} \varphi_{n,m}(x) = C^T \varphi(x)$$

where C and $\varphi(x)$ are $2^{k-1}M \times 1$ matrices are given as

$$C = \begin{pmatrix} c_{1,0}, c_{1,1}, \dots, c_{1,M-1}, c_{2,0}, c_{2,1}, \dots, c_{2,M-1}, \dots, c_{2^{k-1},0}, \\ c_{2^{k-1},1}, \dots, c_{2^{k-1},M-1} \end{pmatrix}^T,$$

$$\varphi(x) = \begin{pmatrix} \varphi_{1,0}, \varphi_{1,1}, \dots, \varphi_{1,M-1}, \varphi_{2,0}, \varphi_{2,1}, \dots, \varphi_{2,M-1}, \dots, \varphi_{2^{k-1},0}, \\ \varphi_{2^{k-1},1}, \dots, \varphi_{2^{k-1},M-1} \end{pmatrix}^T$$

For $k = 1, M = 6$, the six second kind Chebyshev wavelet basis functions over $[0,1]$ are:

$$\varphi_{1,0}(t) = \frac{2}{\sqrt{\pi}}$$

$$\varphi_{1,1}(t) = \frac{2}{\sqrt{\pi}}(4t - 2),$$

$$\varphi_{1,2}(t) = \frac{2}{\sqrt{\pi}}(16t^2 - 16t + 3),$$

$$\varphi_{1,3}(t) = \frac{2}{\sqrt{\pi}}(64t^3 - 96t^2 + 40t - 4),$$

$$\varphi_{1,4}(t) = \frac{2}{\sqrt{\pi}}(256t^4 - 512t^3 + 336t^2 - 80t + 5),$$

$$\varphi_{1,5}(t) = \frac{2}{\sqrt{\pi}}(1024t^5 - 2560t^4 + 2304t^3 - 896t^2 + 140t - 6)$$

3. Haar Wavelets and Its Properties

The amplitude of the switched rectangular waveforms that and

$$x(t) = x(0) + t \cdot x'(0) + \sum_{n=1}^{2^{k-1}} \sum_{m=0}^{M-1} c_{n,m} \int_0^t \int_0^t \varphi_{n,m}(t) dt dt \quad (11)$$

Applying ICs $x(0) = x_0$, $x'(0) = x_{00}$, in (10) and (11), we obtain

make up the orthogonal family of Haar functions can vary from one function to the next. A wavelet family, or basis, is comprised up of a series of square-shaped functions that have been rescaled. This is known as Haar wavelet. In the interval $[\alpha \ \gamma]$, the Haar wavelet function $h_i(x)$ is defined as:

$$h_i(x) = \begin{cases} 1, & \alpha \leq x < \beta \\ -1, & \beta \leq x < \gamma \\ 0, & \text{elsewhere,} \end{cases} \quad (4)$$

where $\alpha = \frac{k}{m}, \beta = \frac{k+0.5}{m}, \gamma = \frac{k+1}{m}$, $m = 2^j$ and $j = 0, 1, 2, 3, \dots, J$. The level of the resolution denoted by J . The integer $k = 0, 1, 2, \dots, m-1$ is the translation parameter. The formula to find index i is $i = m + k + 1$. The values of $i = 2$ at the minimum and $i = 2^{j+1}$ at the maximum.

The collocation points are calculated as:

$$x_l = \frac{(l-0.5)}{2M}, \quad l = 1, 2, 3, 4, \dots, 2M. \quad (5)$$

The operational matrix Q , which is $2M \times 2M$, is calculated as below

$$Q_{1,i}(x) = \int_0^{x_l} h_i(x) dx \quad (6)$$

$$Q_{n+1,i}(x) = \int_0^x Q_{n,i}(x) dx, \quad n = 1, 2, 3, 4, \quad (7)$$

From (6), we have:

$$Q_{i,1}(x) = \begin{cases} x - \alpha, & \alpha \leq x < \beta \\ \gamma - x, & \beta \leq x < \gamma \\ 0, & \text{elsewhere.} \end{cases}$$

4. Proposed Technique Using Chebyshev Wavelets of the Second Kind

Consider the differential equation

$$x'' + ax' + bx = W(t) \quad (8)$$

with ICs $x(0) = x_0$, $x'(0) = x_{00}$.

Assume that

$$x''(t) = \sum_{n=1}^{2^{k-1}} \sum_{m=0}^{M-1} c_{n,m} \varphi_{n,m}(t) \quad (9)$$

Integrating (9) twice with respect to t , we obtain

$$x'(t) = x'(0) + \sum_{n=1}^{2^{k-1}} \sum_{m=0}^{M-1} c_{n,m} \int_0^t \varphi_{n,m}(t) dt \quad (10)$$

$$\kappa'(t) = \kappa_0 + \sum_{n=1}^{2^{k-1}} \sum_{m=0}^{M-1} c_{n,m} \int_0^t \varphi_{n,m}(t) dt \quad (12)$$

and

$$\kappa(t) = \kappa_{00} + t \cdot \kappa_0 + \sum_{n=1}^{2^{k-1}} \sum_{m=0}^{M-1} c_{n,m} \int_0^t \int_0^t \varphi_{n,m}(t) dt dt \quad (13)$$

Inserting the values from (9), (12) and (13) in (8), we obtain

$$\sum_{n=1}^{2^{k-1}} \sum_{m=0}^{M-1} c_{n,m} \left\{ \varphi_{n,m}(t) + a \cdot \int_0^t \varphi_{n,m}(t) dt + b \cdot \int_0^t \int_0^t \varphi_{n,m}(t) dt dt \right\} = F(t) \quad (14)$$

where

$$F(t) = W(t) - a \cdot \kappa_0 - b \cdot \{\kappa + t \cdot \kappa_0\}$$

This formulation provides a systematic approach for approximating solutions to differential equations using second-kind Chebyshev wavelets.

5. Test Experiment

In this Section, we have performed some numerical examples for solving forced oscillatory problems arising in many applications of sciences and engineering.

Case I: If $A = 0, w^2 = 4, W(t) = \cos 2t$

This happened when forced oscillations occurs without damping. The equation of motion becomes

$$\kappa'' + 4\kappa = \cos 2t,$$

with ICs $\kappa(0) = \kappa'(0) = 0$.

The analytical or exact solution is:

$$\kappa(t) = \frac{1}{3}(\cos t - \cos 2t)$$

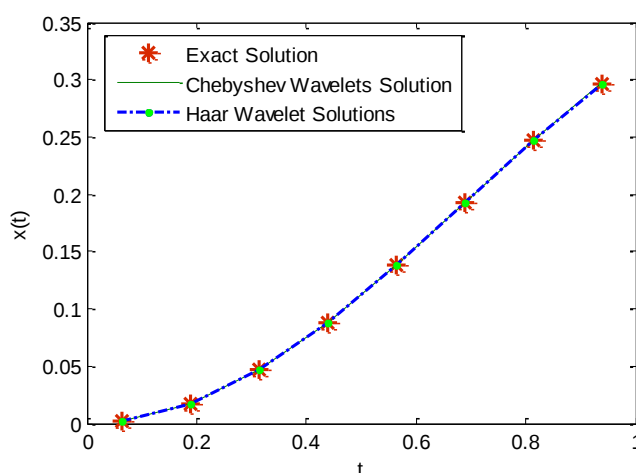


Figure 3. Comparison of analytical, Haar wavelet and Chebyshev wavelets solutions.

Table 1. Comparison of absolute errors achieved using Chebyshev wavelets and Haar wavelets.

$\kappa/16$	Exact Solutions or Analytical solutions	Absolute Errors via Chebyshev Wavelets	Absolute Errors via Haar Wavelets
1	0.0019499478	2.5041×10^{-9}	1.5747×10^{-5}
3	0.0173218970	8.2011×10^{-9}	6.4225×10^{-5}
5	0.0468682761	1.2870×10^{-8}	1.5494×10^{-4}
7	0.0882722750	1.6820×10^{-8}	2.7602×10^{-4}
9	0.1382493274	1.9709×10^{-8}	4.1105×10^{-4}
11	0.1927624127	2.1361×10^{-8}	5.4069×10^{-4}
13	0.2472875657	2.1749×10^{-8}	6.4457×10^{-4}
15	0.2971128604	2.0397×10^{-8}	7.0328×10^{-4}

Case II: If $A = 3, w^2 = 2, W(t) = \cos 2t$

This happened when forced oscillations occurs with damping. The equation of motion becomes

$$\kappa'' + 3\kappa' + 2\kappa = \cos 2t,$$

with ICs $\kappa(0) = \kappa'(0) = 0$.

The analytical solutions is:

$$\kappa(t) = -\frac{1}{5}e^{-t} + \frac{1}{4}e^{-2t} + \frac{1}{20}(3 \sin 2t - \cos 2t)$$

Table 2. Comparison of absolute errors achieved using Chebyshev wavelets and Haar wavelets of Case II.

$x/16$	Exact Solutions or Analytical solutions	Absolute Errors via Chebyshev Wavelets	Absolute Errors via Haar Wavelets
1	0.0018329397	2.1506×10^{-10}	2.0639×10^{-4}
3	0.0144319943	6.0044×10^{-10}	4.4877×10^{-4}
5	0.0347086663	8.1980×10^{-10}	5.1632×10^{-4}
7	0.0581674820	9.5328×10^{-10}	4.7085×10^{-4}
9	0.0809878651	1.0187×10^{-9}	3.5719×10^{-4}
11	0.1000501565	1.0396×10^{-9}	2.0809×10^{-4}
13	0.1129670146	1.0182×10^{-9}	4.7622×10^{-5}
15	0.1181071589	1.0377×10^{-9}	1.0660×10^{-4}

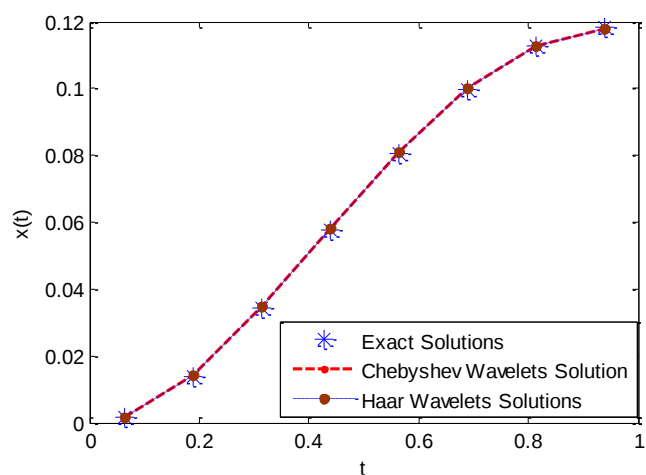


Figure 4. Comparison of Analytical, Haar wavelet and Chebyshev wavelets solutions.

Case III: If $A = 2$, $w^2 = 1$, $W(t) = 1$

This happened when forced oscillations occurs with damping. The equation of motion becomes

$$\kappa'' + 2\kappa' + \kappa = 1,$$

Table 3. Comparison of absolute errors achieved using Chebyshev wavelets and Haar wavelets.

$x/16$	Exact Solutions or Analytical solutions	Absolute Errors via Chebyshev Wavelets	Absolute Errors via Haar Wavelets
1	0.0018736207	5.6272×10^{-11}	1.4052×10^{-4}
3	0.0155279221	1.6385×10^{-10}	3.3850×10^{-4}
5	0.0397544870	2.3538×10^{-10}	4.3963×10^{-4}

with ICs $\kappa(0) = \kappa'(0) = 0$.

The analytical solutions is:

$$\kappa(t) = 1 - (1 + t)e^{-t}$$

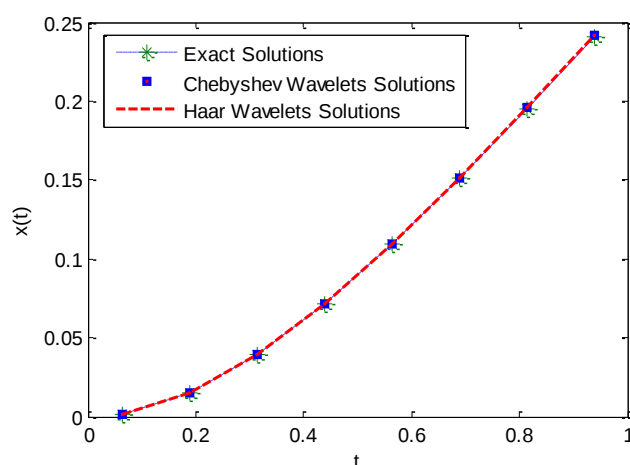


Figure 5. Comparison of Analytical, Haar wavelet and Chebyshev wavelets solutions.

$x/16$	Exact Solutions or Analytical solutions	Absolute Errors via Chebyshev Wavelets	Absolute Errors via Haar Wavelets
7	0.0718802432	2.8932×10^{-10}	4.7215×10^{-4}
9	0.1097143363	3.2719×10^{-10}	4.5771×10^{-4}
11	0.1514717121	3.5163×10^{-10}	4.1278×10^{-4}
13	0.1957080004	3.6804×10^{-10}	3.4964×10^{-4}
15	0.2412640983	3.6242×10^{-10}	2.7736×10^{-4}

6. Conclusions

In this work, we introduced and applied operational integration matrices based on second-kind Chebyshev wavelets to address oscillatory problems frequently encountered in science and engineering. The numerical experiments demonstrate that this method offers both simplicity and high precision. For performance comparison, we also use the operational matrices based on Haar wavelets. The numerical outcomes achieved with second-kind Chebyshev wavelets are significantly closer to the analytical or exact solutions than those obtained from Haar wavelets, indicating that the Chebyshev-based approach is more efficient for this class of problems. Looking ahead, this method could be extended to address two- and three-dimensional nonlinear fractional equations.

Author Contributions

Preeti: Conceptualization, Data curation, Methodology, Software, Writing – original draft, Writing – review & editing

Inderdeep Singh: Formal Analysis, Investigation, Supervision

Conflicts of Interest

The authors declare no conflicts of interest.

References

- [1] Sheikhan, A. H. R., & Mashoof, M. Numerical solution of fractional differential equation by wavelets and hybrid functions. *Boletim da sociedade paranaense de matemática*, 2018, 36(2), 231-244. <https://doi.org/10.5269/bspm.v36i2.30904>
- [2] Dizicheh, A. K., Ismail, F., Kajani, M. T., & Maleki, M. A Legendre wavelet spectral collocation method for solving oscillatory initial value problems. *Journal of Applied Mathematics*, 2013, 2013, 1-5. <https://doi.org/10.1155/2013/591636>
- [3] Gupta, A. K., & Ray, S. S. Wavelet methods for solving fractional order differential equations. *Mathematical Problems in Engineering*, 2014, 2014. <https://doi.org/10.1155/2014/140453>
- [4] Verma, A. K., Kumar, N., & Tiwari, D. Haar wavelets collocation method for a system of nonlinear singular differential equations. *Engineering Computations*, 2021, 38(2), 659-698. <https://doi.org/10.1108/EC-04-2020-0181>
- [5] Saeed, A., & Saeed, U. Sine-cosine wavelet method for fractional oscillator equations. *Mathematical Methods in the Applied Sciences*, 2019, 42(18), 6960-6971. <https://doi.org/10.1002/mma.5802>
- [6] Yuttanan, B., & Razzaghi, M. Legendre wavelets approach for numerical solutions of distributed order fractional differential equations. *Applied Mathematical Modelling*, 2019, 70, 350-364. <https://doi.org/10.1016/j.apm.2019.01.013>
- [7] Sathyaseelan, D., & Hariharan, G. Wavelet-Based Approximation Algorithms for Some Nonlinear Oscillator Equations Arising in Engineering. *Journal of The Institution of Engineers (India): Series C*, 2020, 101(1), 185-192. <https://link.springer.com/article/10.1007/s40032-019-00517-x>
- [8] Shah, F. A., & Abass, R. Solution of fractional oscillator equations using ultraspherical wavelets. *International Journal of Geometric Methods in Modern Physics*, 2019, 16(05), 1950075. <https://doi.org/10.1142/S0219887819500750>
- [9] Hariharan, G., & Kannan, K. Review of wavelet methods for the solution of reaction–diffusion problems in science and engineering. *Applied Mathematical Modelling*, 2014, 38(3), 799-813. <https://doi.org/10.1142/S0219887819500750>
- [10] Kaur, H., Mittal, R. C., & Mishra, V. Haar wavelet solutions of nonlinear oscillator equations. *Applied Mathematical Modelling*, 2014, 38(21-22), 4958-4971. <https://doi.org/10.1016/j.apm.2014.03.019>
- [11] Aziz, I., & Šarler, B. The numerical solution of second-order boundary-value problems by collocation method with the Haar wavelets. *Mathematical and Computer Modelling*, 2010, 52(9-10), 1577-1590. <https://doi.org/10.1016/j.mcm.2010.06.023>
- [12] Aziz, I., & Amin, R. Numerical solution of a class of delay differential and delay partial differential equations via Haar wavelet. *Applied Mathematical Modelling*, 2016, 40(23-24), 10286-10299. <https://doi.org/10.1016/j.apm.2016.07.018>
- [13] Haq, I., & Singh, I. Solving Some Oscillatory Problems using Adomian Decomposition Method and Haar Wavelet Method. *Journal of Scientific Research*, 2020, 12(3). <https://doi.org/10.3329/jsr.v12i3.44287>

- [14] Dremine, I. M., Ivanov, O. V., & Nechitailo, V. A. Wavelets and their uses. *Physics-Uspekhi*, 2001, 44(5), 447.
<https://doi.org/10.1070/PU2001v044n05ABEH000918>
- [15] Singh, I., & Kumar, S. Numerical solution of damped forced oscillator problem using Haar wavelets. *Iranian Journal of Numerical Analysis and Optimization*, 2015, 5(1), 73-83.
<https://doi.org/10.22067/IJNAO.V5I1.45153>
- [16] Rashidinia, J., Eftekhari, T., & Maleknejad, K. A novel operational vector for solving the general form of distributed order fractional differential equations in the time domain based on the second kind Chebyshev wavelets. *Numerical Algorithms*, 2021, 88(4), 1617-1639.
<https://doi.org/10.1007/s11075-021-01088-8>
- [17] Srinivasa, K., & Mundewadi, R. A. Wavelets approach for the solution of nonlinear variable delay differential equations. *International Journal of Mathematics and Computer in Engineering*, 2023, 1(2), 139-148.
<https://doi.org/10.2478/ijmce-2023-0011>
- [18] Heydari, M. H., Hooshmandasl, M. R., & Cattani, C. A new operational matrix of fractional order integration for the Chebyshev wavelets and its application for nonlinear fractional Van der Pol oscillator equation. *Proceedings-Mathematical Sciences*, 2018, 128, 1-26.
<https://doi.org/10.1007/s12044-018-0393-4>
- [19] Hamid, M., Usman, M., Haq, R. U., & Tian, Z. A spectral approach to analyze the nonlinear oscillatory fractional-order differential equations. *Chaos, Solitons & Fractals*, 2021, 146, 110921. <https://doi.org/10.1016/j.chaos.2021.110921>
- [20] Kaur, M., & Singh, I. Hermite wavelet method for solving oscillatory electrical circuit equations. *J. Math. Comput. Sci.*, 2021, 11(5), 6266-6278. <https://doi.org/10.28919/jmcs/6186>
- [21] Selvi, M. S. M., & Rajendran, L. Application of modified wavelet and homotopy perturbation methods to nonlinear oscillation problems. *Applied Mathematics and Nonlinear Sciences*, 2019, 4(2), 351-364.
<https://doi.org/10.2478/AMNS.2019.2.00030>
- [22] Murad, M. S., & Hussien, A. M. Numerical Solution of System of Damped Forced Oscillator Ordinary Differential Equations. *Cihan International Journal of Social Science*, 2017, 1(1), 22-29.
https://scholar.google.com/scholar?hl=en&as_sdt=0%2C5&q=Numerical+Solution+of+System+of+Damped+Forced+Oscillator+Ordinary+Differential+Equations+by+22.%09Murad%2C+M.+S.%2C+%26+Hussien%2C+A.+M&btnG=
- [23] Bujurke, N. M., Shiralashetti, S. C., & Salimath, C. S. An application of single-term Haar wavelet series in the solution of nonlinear oscillator equations. *Journal of computational and applied mathematics*, 2009, 227(2), 234-244.
<https://doi.org/10.1016/j.cam.2008.03.012>
- [24] Hesameddini, E., Shekarpaz, S., & H. Latifzadeh, The Chebyshev wavelet method for numerical solutions of a fractional oscillator. *Int. J. Appl. Math. Res*, 1, 2012, 493-509.
<https://citeseerx.ist.psu.edu/document?repid=rep1&type=pdf&doi=212265df448ac0db771454add6133cb3b0f8135c>
- [25] Singh, I. & Preeti. Analysis of Charge and current flow in the LCR series Circuit via Chebyshev wavelets. *Contemporary Mathematics*, 2024, 5(1), 886-896.
<https://doi.org/10.37256/cm.5120242822>
- [26] Singh, I. & Preeti. Chebyshev wavelet based technique for numerical differentiation. *Advances in Differential equations and Control Processes*, 2023, 30(1), 15-25.
<http://dx.doi.org/10.17654/0974324323002>
- [27] Singh, I. & Preeti. Analysis of Charge and Current flow in LCR series circuit via Chebyshev wavelets. *Contemporary Mathematics*, 2024, 5(1), 887-896.
<https://doi.org/10.37256/cm.5120242822>

Biography



Preeti is currently pursuing her PhD in Applied Mathematics and holds a Master's degree in Mathematics. Her research primarily focuses on solving differential equations i.e. ordinary as well as partial differential equations and integral equations using Chebyshev wavelets techniques. She has published five

research papers in Scopus-indexed journals and one in an Web of Science-indexed journal. In addition, she has contributed two book chapter and published a research paper in peer-reviewed journal. Her work emphasizes the development of Chebyshev wavelets of different kinds for first order, second order and higher-dimensional models arising in applied sciences.



Inderdeep Singh is an Associate Professor in the Department of Physical Sciences, Mathematics at Sant Baba Bhag Singh University, Jalandhar. He obtained his PhD in Mathematics from NIT Jalandhar and has published extensively in the field of differential equations i.e. ordinary and partial differential

equations and computational methods. His research interests include numerical analysis, wavelet methods, semi-analytical methods, hybrid transform methods, fractional differential equations, and homotopy-based analytical techniques. Dr. Singh has been actively involved in international research collaborations and has contributed to various interdisciplinary applications. He has authored over 55 research articles in peer-reviewed journals and serves as a reviewer for several reputed international publications. Dr. Singh has also participated in numerous national and international conferences as a keynote speaker, technical committee member, and session chair. His work is widely recognized in the area of semi-analytical methods for solving higher-dimensional linear and nonlinear models.

Research Field

Preeti: Differential equation i.e. first and second order ordinary differential equations, partial differential equations like Schrodinger equations, Burger's Huxley equation, Fisher's equations, and integral equations, Chebyshev wavelet methods of first kind, second kind etc.

Inderdeep Singh: Numerical analysis, computational mathe-

mathematics, wavelet numerical methods, numerical analysis techniques, semi analytical methods, transform based solution methods, homotopy analysis method applications, hybrid

transform methods, fractional order differential equations, linear and nonlinear partial differential equations.