

Research Article

Measuring Inequality of Income Distributions in Egypt: An Empirical Study Using Weighted and Non-parametric Methods

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Abstract

One of the foundational tasks in statistical analysis is the design and implementation of sample surveys, which involve sampling error that affects the reliability and precision of the resulting estimates. Measures such as totals and means are insufficient on their own without corresponding indicators of statistical precision, such as confidence intervals. As such, any analytical method applied to survey data must accommodate data weighting, which is essential for producing valid and interpretable estimates. Within the field of economic research, income inequality represents a key application where the use of weighted data is critical. In this context, we introduce a weighted inequality index designed to improve the robustness of inequality measurement. To enhance its analytical rigor, the proposed index is accompanied by a non-parametric, bootstrap-based algorithm, designed to facilitate comparative assessments and statistical significance testing across various population subgroups (e.g., regions, countries, gender). A major advantage of this approach lies in its flexibility; it is suitable for both normally and non-normally distributed data, thereby broadening its applicability to real-world datasets that often deviate from standard distributional assumptions. To demonstrate the empirical utility and comparative performance of the proposed methodology, we applied it to household income data obtained from the Central Agency for Public Mobilization and Statistics (CAPMAS), based on nationally representative income and expenditure surveys conducted in 2015 and 2018. The empirical findings revealed a general decline in the values of the proposed inequality index across most Egyptian governorates between 2015 and 2018, indicating a modest trend toward greater income equality. This downward shift may be indicative of the effects of socioeconomic reform measures and targeted development policies aimed at reducing regional disparities. The results validate the practical relevance of the proposed index as a reliable tool for evaluating income inequality in diverse socioeconomic contexts.

Keywords

Weighted Data, Symmetry Index, Inequality Measurements

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Received: 23 June 2025; **Accepted:** 9 July 2025; **Published:** 4 August 2025



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1. Introduction

Examining economic inequality within a nation, analyses often focus on the distribution of income among individuals or across the country's regions. The latter approach introduces a spatial dimension to inequality studies and can uncover significant compelling evidence that regional inequality influences the likelihood of secessionist conflicts relationships that may be overlooked when considering the nation as a whole [4].

The concept of adjusting inequality indices by the population shares of a country's regions is widely recognized as an early development in the study of regional inequality [17]. This methodological approach has gained considerable recognition in regional economic research, as demonstrated by its widespread adoption in academic literature. For example, measures such as the Gini coefficient and the coefficient of variation have been used to assess regional inequality in countries like Russia, Ukraine, and Kazakhstan during the period from 1998 to 2009 [18]. Similarly, [5] utilize the Theil index to analyze inequality among 14 European countries during the period 1980-2009. Furthermore, [8] apply the Theil index, coefficient of variation, standard deviation of logarithms, and mean logarithmic deviation to compare inequality across 22 emerging countries between 1990 and 2006.

In the context of Egyptian data, [16] conduct a study that utilizes privileged access to detailed income records and applies newly developed statistical techniques to evaluate income inequality and assess potential biases associated with top income earners. Their findings indicate that measured income inequality in Egypt is relatively low by both regional and international standards. The distribution of top incomes follows the expected Pareto pattern and does not display significant deviations compared to global survey data. However, when adjustments are made to account for possible underreporting or exclusion of top incomes, the estimated Gini coefficient increases substantially, ranging from 1.1 to 4.1 percentage points, depending on the correction approach and the specific welfare measure applied.

Based on the importance of weighted inequality indices, Sebastian et al. (2024) introduce 'wINEQ' package in R-Program that enable us to compute inequality measures of a given variable taking into account weights and it is also suitable for ratio, interval and ordered scale [14]. This package includes Gini index, Generalized entropy index, Theil indices, Leti index, Palma index and Allison and Foster index. Also they propose to use Bootstrap distribution of inequality measures for significance tests.

Although these indicators are of significant importance and are widely utilized within the framework of weighted data, they have encountered numerous challenges. For instance, [2] noted that a lower Gini Index (GI) does not necessarily imply a more symmetrical distribution compared to one with a higher GI. This is due to the possibility that the Lorenz curves of the two datasets may intersect, reflecting underlying dif-

ferences in their distributional structures. Furthermore, the intersection of Lorenz curves corresponding to distinct datasets can yield identical GI values, thereby obscuring meaningful differences in distribution.

To establish a comprehensive ranking of data, such as income distributions, and to assess disparities in income inequality across countries, [2] proposes a social welfare-based inequality index. This index, which takes values between 0 and 1, is grounded in the concept of an equally distributed equivalent (EDE) level, as formulated in the Atkinson index. The EDE depends on a parameter known as inequality aversion (ϵ), which can range from 0 to infinity. As ϵ increases, the index assigns greater weight to income transfers at the lower end of the distribution and diminishing weight to those at the upper end. While the Atkinson index offers notable advantages, such as enabling a complete ordering of income distributions and explicitly revealing the underlying social welfare function, which is valuable for informing policy decisions, a critical limitation has been highlighted. Specifically, the ranking of income distributions can vary considerably based on the selected social welfare function and the degree of inequality aversion (ϵ), which may differ across countries due to varying societal preferences regarding inequality.

To avoid the need for explicit social welfare judgments, a family of Generalized Entropy ($GE(\alpha)$) indices offers an alternative approach for ranking inequality, particularly when the Lorenz curves of two or more distributions intersect [13]. The theoretical range of the $GE(\alpha)$ index extends from 0 to infinity, where a value of 0 signifies perfect equality, and higher values correspond to increasing levels of inequality. This family of indices is parameterized by α , which determines the sensitivity of the index to different parts of the income distribution. Specifically, $GE(0)$ corresponds to the mean logarithmic deviation, while $GE(1)$ represents the Theil index, originally introduced by Theil in 1967 [15]. Although $GE(\alpha)$ indices address some of the limitations inherent in the Gini and Atkinson indices-particularly in situations where Lorenz curves intersect-it is important to recognize that the specific form of the $GE(\alpha)$ index varies with the choice of α . This dependence on the parameter value can complicate cross-distribution comparisons, as different values of α emphasize different segments of the distribution, thereby affecting the interpretation and comparability of inequality measures across datasets."

Additionally, the Gini Index is known to be more sensitive to changes in the middle of the income distribution and less responsive to differences at the upper and lower ends [2]. Moreover, the GI is not well-suited for measuring inequality in distributions that include negative values. For example, in the case of wealth distributions, individuals may possess negative net wealth, where liabilities exceed assets. Although it is technically possible to compute the Gini coefficient for such distributions, the resulting values may exceed 1, thereby

complicating interpretation and diminishing the measure's practical utility.

To overcome these limitations, we introduce an extension of the symmetry-based inequality index, initially proposed by [7], which has been suitable to integrate survey weights for more accurate representation of population disparities. And then we introduce a non-parametric, bootstrap-based algorithm designed to facilitate comparative analysis and assess statistical significance across multiple groups. Accordingly, we organize the remaining of this paper as follows: Section 2 reviews inequality measures employed in the analysis of weighted data, highlighting their applications and limitations. Section 3 provides the development of the symmetry index, incorporating a non-parametric algorithm designed to assess statistical significance across multiple groups without assuming specific distributional forms. In Section 4, a comparative analysis is conducted, evaluating the performance of the proposed index against several established inequality measures using real data set from Egypt. The final Section 5 summarizes the findings and discusses the importance of the enhanced index for future research in regional economic disparities.

2. Inequality Indices Under Grouped Data

The concept of inequality takes a central place in the research of economists, sociologists, statisticians, and other social scientists. Within the field of economics, specific dimensions of inequality such as income inequality, lifetime inequality, wealth inequality, and inequality of opportunity are of particular analytical interest. In context of weighted data, numerous inequality measures have been developed. For the purposes of this comparative analysis in our study, we selected a set of well-established measures that provide a good precision under weighted data. These selected measures are outlined below, but first let us define $(x_1, x_2, \dots, x_n)$ as a random sample of size n from unknown continuous distribution, w_i is the weight of the i^{th} element, $\bar{x}$ is the arithmetic mean and $\bar{x}_w$ represents the weighted arithmetic mean that:

$$\bar{x}_w = \frac{\sum w_i x_i}{\sum w_i} \quad (1)$$

Now we can define these measures as follow:

1) Gini index (GI)

Gini index [9] is defined as the normalized area between the Lorenz curve of the distribution and the 45-degree line and it is bounded between (0) and (1), where it equals to zero in the case of complete equality and tends to one in the case of complete inequality. The mathematical formulation of GI as a weighted measure is

$$GI = \frac{\sum_{i=1}^n \sum_{j=1}^n |x_i - x_j| w_i w_j}{2(\sum w_i)^2 \bar{x}_w} \quad (2)$$

2) Lorenz curve

The Lorenz curve is defined as a continuous, increasing, and convex function that lies within the first quadrant of the Cartesian plane, assuming all values of the underlying variable are positive [11]. In the hypothetical case of perfect equality, where income or wealth is distributed uniformly across the population, the Lorenz curve coincides with the 45-degree line, also known as the line of equality. Conversely, in scenarios of income inequality, the Lorenz curve deviates below this line, reflecting the degree of disparity in the distribution. The further the curve is from the line of equality, the greater the observed inequality. Thus, the proximity of the Lorenz curve to the 45-degree line serves as an intuitive graphical indicator of the equity of the distribution (see Figure 1).

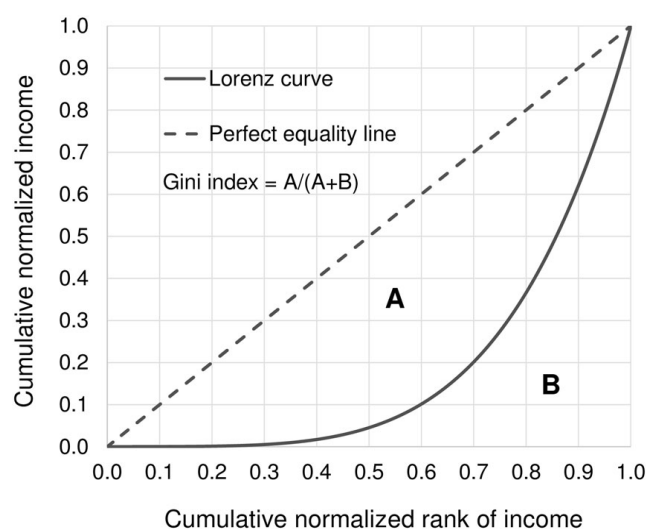


Figure 1. Illustrates the relation between Lorenz curve and GI.

3) Atkinson's index (AT)

We can define the Atkinson index [2] as the normalized ratio of the equally distributed equivalent level of observed data to the mean of the actual data distribution; it can be defined using the following form:

$$AT(\varepsilon) = \begin{cases} 1 - \left(\frac{1}{n} \sum_{i=1}^n \left(\frac{x_i}{\bar{x}} \right)^{1-\varepsilon} \right)^{\frac{1}{1-\varepsilon}}, & \varepsilon \neq 1 \\ 1 - \frac{\prod_{i=1}^n \left(\frac{x_i}{\bar{x}} \right)^{\frac{1}{n}}}{\bar{x}}, & \varepsilon = 1 \end{cases} \quad (3)$$

where the parameter $\varepsilon > 0$ is interpreted as the level of aversion of inequality. By using weighted mean we can redefine Atkinson index as follow:

$$AT(\varepsilon) = \begin{cases} 1 - \left(\frac{1}{\sum w_i} \sum_{i=1}^n \left(\frac{w_i x_i}{\bar{x}_w} \right)^{1-\varepsilon} \right)^{\frac{1}{1-\varepsilon}}, & \varepsilon \neq 1 \\ 1 - \frac{\prod_{i=1}^n \left(\frac{w_i x_i}{\bar{x}_w} \right)^{\frac{1}{n}}}{\bar{x}_w}, & \varepsilon = 1 \end{cases} \quad (4)$$

The index is bounded between 0 and 1 and increases with inequality, and it only can be computed for positive values of the variable.

4) Palma index (PI)

The Palma ratio [12] is an inequality measure defined as the ratio of the income share held by the top 10% (H) to that held by the bottom 40% (L), expressed as:

$$Palma = \frac{H}{L} \quad (5)$$

A higher Palma ratio indicates greater income inequality, as it reflects a larger concentration of income among the richest segment of the population. In this context, inequality is assessed based on income after accounting for taxes and social benefits.

5) Allison and Foster index (AF)

The Allison and Foster index measures inequality by calculating the difference between the mean score of individuals with values above the median and the mean score of those with values below the median. Originally proposed by [1], this index captures distributional disparities around the median. A more detailed examination and discussion of the index is provided by [6]. To define the mathematical formulation of this index let k is the median category derived from the relations of θ_k which denotes the median of the sample data and

$$\bar{x}^L = 2 \sum_{i=1}^{k-1} \theta_i (F_N^i - F_N^{i-1}) + \theta_k (0.5 - F_N^{k-1}) \quad (6)$$

is the mean value below the median, and

$$\bar{x}^U = 2 \sum_{i=k}^N \theta_i (F_N^i - F_N^{i-1}) + \theta_k (F_N^k - 0.5) \quad (7)$$

is the mean value above the median, then we can define AF index as:

$$AF = \bar{x}^U - \bar{x}^L \quad (8)$$

6) Generalized Entropy and the Theil's indices

The Generalized Entropy (GE) indices, introduced by [13], constitute a family of inequality measures that incorporate a sensitivity parameter α , which determines the weight assigned to different parts of the income distribution. By adjusting α , the GE measures can emphasize inequalities at the lower, middle, or upper ends of the distribution, where:

$$GE(\alpha) = \frac{1}{n(\alpha^2 - \alpha)} \sum_{i=1}^n \left[\left(\frac{x_i}{\bar{x}} \right)^\alpha - 1 \right] = \frac{1}{(\alpha^2 - \alpha) \sum w_i} \sum_{i=1}^n w_i \left[\left(\frac{x_i}{\bar{x}} \right)^\alpha - 1 \right] \quad (9)$$

in our study we assume $\alpha = 0.5$.

This study also incorporates the Theil indices (T_T and T_L) [15] which represent special cases of the Generalized Entropy (GE) class of inequality measures. Specifically, T_T corresponds to the GE index with $\alpha = 1$, and T_L corresponds to

$\alpha = 0$. These indices can be defined using the following functional forms:

$$T_T = \frac{1}{n} \sum_{i=1}^n \frac{x_i}{\bar{x}} \ln \left(\frac{x_i}{\bar{x}} \right) = \frac{1}{\sum w_i} \sum_{i=1}^n w_i \frac{x_i}{\bar{x}_w} \ln \left(\frac{x_i}{\bar{x}_w} \right) \quad (10)$$

$$T_L = \frac{1}{n} \sum_{i=1}^n \ln \left(\frac{\bar{x}}{x_i} \right) = \frac{1}{\sum w_i} \sum_{i=1}^n w_i \ln \left(\frac{\bar{x}_w}{x_i} \right) \quad (11)$$

7) Leti index (LI)

In order to define the mathematical formulation of Leti inequality measure (proposed by [10]) of a given variable taking into account weights, let n_j be the number of individuals in category j and let N be the total sample size. Cumulative distribution is given by

$$F_i = \frac{\sum_{j=1}^i n_j}{N} \quad (12)$$

then the Leti index can define as:

$$LI = 2 \sum_{i=1}^O F_i (1 - F_i), \quad O \text{ equals the number of categories} \quad (13)$$

3. The Proposed Index

We introduce an alternative formulation of the proposed symmetry index in [7] tailored for use with weighted data, making it suitable for applications involving data derived from sampling surveys. The central concept underlying this version of the index is the substitution of the arithmetic mean with its weighted counterpart, as elaborated in the following discussion.

3.1. The Derivation of Proposed Index and Its Properties

To define the symmetry weighted index (SWI_θ), assume y_i^θ where:

$$y_i^\theta = |x_i - \theta|, i = 1, 2, \dots, n \quad (14)$$

where θ represents the median of the underlying data. Hence the sample is partitioned into two sub-samples as follow

$$y_i^\theta = \begin{cases} x_i - \theta \rightarrow x_i > \theta \\ \theta - x_i \rightarrow x_i < \theta \end{cases} \quad (15)$$

$i = 1, 2, \dots, n$, (all transformed values are positive)

Given that the population is assumed to be continuous, the probability of observing a value exactly equal to θ is zero. Consequently, such observations should be excluded from the analysis, and the sample size should be adjusted accordingly. Now we can define the SWI_θ

$$SWI_\theta = \frac{GI_\theta^U}{GI_\theta^L} \quad (16)$$

where GI_+^θ presents the Gini index for subset 1 when $x_i > \theta$ and GI_-^θ presents the Gini index for subset 2 when $x_i < \theta$ on their weighted version that:

$$GI_+^\theta = \frac{\sum_{i=1}^{n_1} \sum_{j=1}^{n_1} |y_i - y_j| w_i w_j}{2(\sum w_i)^2 \bar{y}_{1w}}, \quad GI_-^\theta = \frac{\sum_{i=1}^{n_2} \sum_{j=1}^{n_2} |y_i - y_j| w_i w_j}{2(\sum w_i)^2 \bar{y}_{2w}} \quad (17)$$

where $\bar{y}_{1w}$ presents the sample weighted mean for subset 1 when $x_i > \theta$, $\bar{y}_{2w}$ presents the sample weighted mean for subset 2 when $x_i < \theta$. Also n_1 denotes the sample size for subset 1, and n_2 is the sample size for subset 2. And the properties of proposed index are:

- 1) The proposed index (SWI_θ) is bounded between 0 and ∞ . As the value of SWI_θ approaches 1, the data tends toward symmetry.
- 2) The index SWI_θ provides information about the asymmetry of the distribution. When $SWI_\theta > 1$, the distribution is considered right-skewed, whereas values of $SWI_\theta < 1$ (approaching 0) indicate a left-skewed distribution. The reason for using median-based skewness is that it provides a simple way to measure the symmetry of a distribution. This is based on the mathematical definition of the skewness parameter $\left(\frac{\text{Mean} - \text{Median}}{\text{Standard deviation}} \right)$ which reflects how much a distribution deviates from being symmetric.:
 - a. Positive value $\rightarrow$ right-skewed
 - b. Negative value $\rightarrow$ left-skewed
 - c. Near zero $\rightarrow$ symmetric
- 3) Besides, the reason of using median-based in SWI_θ instead of the traditional moment-based skewness, mean and standard deviation, is highly sensitive to outliers and can produce misleading results, particularly in small or non-normally distributed samples. In contrast in SWI_θ is more robust to extreme values and offer greater stability in distributional asymmetry estimates when sample sizes are limited. Furthermore, this nonparametric approach requires fewer assumptions about the underlying distribution, making it appropriate for exploratory analysis in small datasets.
- 4) SWI_θ does not change when all observations of the data change proportionally. That the change of location do alter (SWI_θ) where the mean for subset 1 when $x_i > \theta$ does not equal the mean for subset 2 when $x_i < \theta$ then $GI_+^\theta \neq GI_-^\theta$.

3.2. Analysis of Symmetry Index (ANOSWI)

Testing for statistically significant differences in SWI_θ values across groups (such as gender, governorates, or countries) requires careful methodological consideration. This is due to the fact that the SWI_θ index is a bounded and non-linear measure, which renders standard parametric tests

(as ANOVA) inappropriate. Furthermore, the sampling distribution of the proposed index is almost non-normal, particularly in the context of small sample sizes or skewed data, characteristics commonly observed in income distributions, which are often positively skewed (see [7]). To address these challenges, this study adopts a non-parametric, bootstrap-based approach. Specifically, we assess group differences by evaluating the extent of overlap between confidence intervals. The following procedure is implemented:

- 1) In each group, we obtain the bootstrap samples by replacing the data with replacement (say 5000 iterations).
- 2) The SWI_θ index is computed for each bootstrap sample.
- 3) The mean and the 95% confidence interval of SWI_θ are calculated.
- 4) Comparisons are made across groups, where non-overlapping confidence intervals suggest statistically significant differences between groups.

4. Empirical Estimates of Inequality

In this section, we evaluate the performance of the proposed index (SWI_θ) by providing empirical estimates of SWI_θ with several inequality indices (previously presented in section 2) using real income weighted data from Egypt. The properties of the underlying data are provided next.

4.1. Data Description

This study considers a large set of economies during the years 2015 and 2018, utilizing income distribution data obtained from the Central Agency for Public Mobilization and Statistics (CAPMAS). A key advantage of using CAPMAS data lies in its provision of harmonized cross-city survey results, achieved through the integration of individual surveys conducted at the city level. The analysis in this paper focuses on disposable household income, defined as the sum of monetary and non-monetary income from various sources, which is widely regarded as the preferred metric for analyzing income distribution [3].

The decision to utilize income survey data from the years 2015 and 2018, rather than from 2019 or subsequent years is primarily due to the fact that these earlier surveys are disaggregated by the 23 governorate sectors, this is after merging the border governorates into one sector. This structure enables the data to be presented in a weighted and representative format that accurately reflects the distribution across the respective governorates. This would allow us to have more chance to find countries that may have the same value of the Gini index although they differ on their income distributions.

4.1.1. Descriptive Statistical Analysis of Household Income Data: 2015 Dataset

Table 1. The descriptive statistics of income distributions for 23 governorates at 2015.

Governorates	n	Min	Q1	Q2	Mean	Q3	Max	SD
Cairo	748	1336.1	7107.96	11280.12	19049.72	19866.25	916250.0	39696.90
Alexandria	492	2173.67	7361.35	10336.42	14005.88	15334.68	130797.8	12291.76
Port Said	495	3682.83	9608.19	14063.50	16980.28	19385.60	177975.0	13370.47
Suez	476	3142.22	7332.88	10843.09	13862.48	16857.52	105000.0	10575.04
Damietta	479	2120.35	6402.94	8711.80	10372.19	12112.13	52562.75	6111.36
Dakahlia	624	3292.17	7557.79	10203.10	12730.41	14737.00	92822.0	9017.83
Sharqia	610	2771.17	7586.75	9876.95	12377.54	14260.88	159408.0	9481.45
Qalyub	501	2827.50	6848.30	9102.10	15877.66	12872.75	2354545.0	105048.92
Kafr el-Sheikh	476	2628.57	6478.94	8797.78	10634.75	12274.15	68097.3	6993.60
Gharbia	486	2956.98	6572.58	8938.17	11499.33	12375.38	112229.4	10213.32
Monufia	502	3548.57	7518.55	10300.75	12354.97	14260.35	60910.5	7646.83
Beheira	535	2985.00	6209.13	8385.37	9957.76	11901.14	62520.0	6200.92
Ismailia	483	3081.17	6864.69	9449.08	12505.66	14469.63	306785.0	15541.47
Giza	687	1599.00	5849.38	8054.70	10019.97	11768.52	81000.0	7257.17
Beni Suef	495	2189.30	5286.45	7130.40	9076.13	10626.60	54267.1	6497.41
Fayyum	497	2735.17	6002.17	7985.50	10183.84	11536.50	59863.0	7196.12
Al-Minya	495	1925.57	4906.68	6684.17	8824.50	9706.48	162750.0	9745.50
Asyut	487	1773.40	4601.38	6600.50	8135.51	9227.39	46698.50	5852.31
Sohag	489	1280.41	3963.71	5666.38	6972.70	8127.59	33044.3	4754.03
Qena	491	2161.79	5284.44	7241.80	9000.85	10829.68	53000.0	6143.10
Aswan	474	2423.69	6188.77	8368.74	10206.74	12166.49	73900.0	7012.86
Luxor	500	2923.58	5511.27	7228.02	8329.90	10001.84	32536.3	4195.18
Border cities in Egypt	466	1794.77	7891.03	11295.66	13323.02	16036.86	61570.0	8220.54

n: denotes the sample size of each governorates

The sample sizes and corresponding descriptive statistics for the aforementioned data are presented in Table 1.

Based on the descriptive statistics presented in the preceding table, notable regional disparities in average household income levels across governorates in Egypt during 2015 are evident. The highest mean household income was recorded in Cairo Governorate, amounting to EGP 19049.72, followed by Port Said at EGP 16980.28, Qalyub at EGP 15877.66, and Alexandria at EGP 14005.88. These relatively elevated income levels in urbanized and economically developed governorates may reflect the concentration of economic activity, employment opportunities, and access to infrastructure and services. The corresponding standard deviations for these

governorates were EGP 39696.90, EGP 13370.47, EGP 105048.92, and EGP 12291.76, respectively, indicating varying degrees of income dispersion within each region. Notably, the extremely high standard deviation observed in Qalyub suggests the presence of substantial income inequality within the governorate.

Conversely, the lowest average household incomes were observed in several Upper Egypt governorates, with Sohag reporting the lowest mean income at EGP 6972.70, followed by Luxor at EGP 8329.90, and Al-Minya at EGP 8824.50. These governorates also reported relatively lower standard deviations-EGP 4754.03 for Sohag, EGP 4195.18 for Luxor, and EGP 9745.50 for Al-Minya-indicating a more com-

pressed, though overall lower, income distribution. These findings underscore persistent regional economic inequalities, particularly between the urban centers and less developed

southern governorates, which may be attributed to structural disparities in investment, labor market opportunities, and access to education and healthcare services.

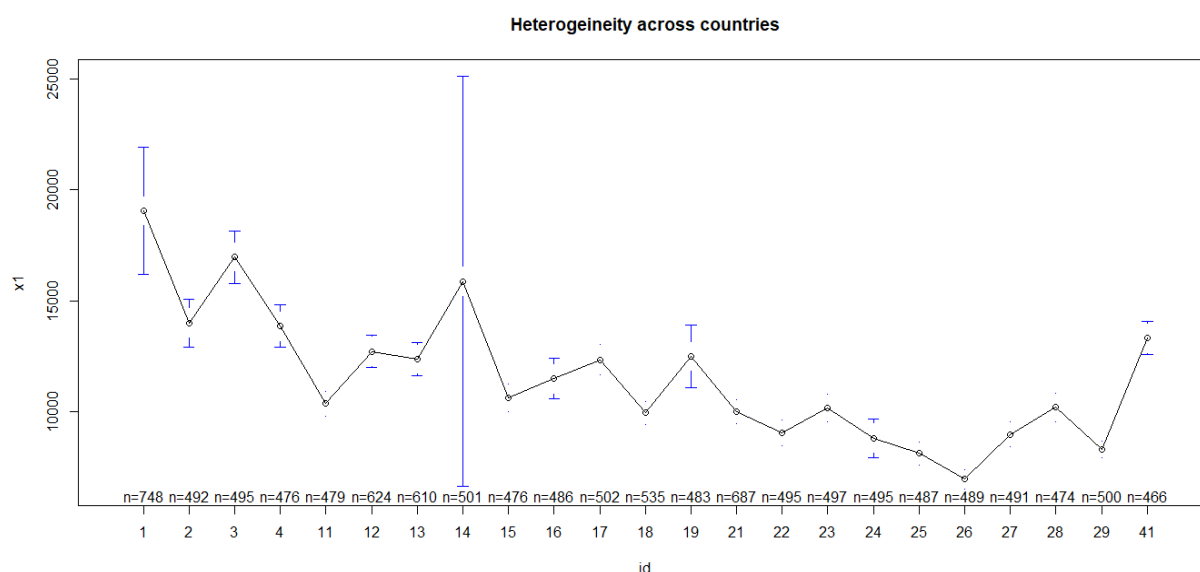


Figure 2. The 95% confidence interval around the means of income distributions for 23 governorates at 2015.

Figure 2 offers a visual representation of the results presented in Table 1 by illustrating the 95% confidence intervals around the mean household incomes for 23 governorates in the year 2015. The figure clearly indicates that Cairo, Port Said, and Qalyubia exhibit the highest average household incomes. In contrast, the lowest averages are observed in Sohag, Luxor, and Al-Minya. Furthermore, the figure highlights notable heterogeneity in income distribution across the

different governorates.

4.1.2. Descriptive Statistical Analysis of Household Income Data: 2018 Dataset

The above analysis is repeated for the data of the year 2018. The results are obtained and provided in the following table (Table 2).

Table 2. The descriptive statistics of income distributions for 23 governorates at 2018.

Governorates	n	Min	Q1	Q2	Mean	Q3	Max	SD
Cairo	1233	2771.88	9787.50	15249.75	23989.78	25225.50	1068571.0	43218.30
Alexandria	523	3303.75	10861.52	15750.00	21921.01	23675.50	444000.0	26473.185
Port Said	490	4535.83	14438.50	19958.63	26137.79	30291.12	675682.5	34286.308
Suez	455	2337.88	9658.00	15626.00	20420.03	23996.67	148500.0	17739.371
Damietta	482	4323.00	9588.75	12679.00	15191.59	17948.62	62633.0	8489.200
Dakahlia	632	5328.75	11051.76	14435.63	17701.50	20195.80	204325.0	13121.02
Sharqia	613	2982.50	10039.20	13087.50	17706.96	19287.50	435961.7	24826.65
Qalyub	504	3508.00	9216.46	12713.00	15198.60	18172.87	122125.0	9763.45
Kafr el-Sheikh	490	3409.57	10058.42	13804.35	17413.03	20183.50	267600.0	15758.17
Gharbia	490	2469.00	11280.45	15368.45	18914.00	21979.00	104379.0	12828.95
Monufia	513	3570.60	7898.75	10908.00	13454.73	16083.40	125896.0	10036.98
Beheira	552	3604.20	7535.04	10082.78	13495.46	14824.95	491875.0	22109.64

Governorates	n	Min	Q1	Q2	Mean	Q3	Max	SD
Ismailia	490	3457.60	8828.30	12050.00	14922.82	18096.65	83877.0	9455.73
Giza	682	3974.17	9481.44	12853.30	15911.40	18176.50	167430.0	11916.45
Beni Suef	497	3543.50	8603.71	11137.00	13792.62	15991.60	95562.5	9168.16
Fayyum	484	2658.00	8695.70	11685.10	14137.78	16591.50	60989.0	8619.12
Al-Minya	494	2567.11	7060.75	9728.50	12335.97	14318.70	107237.0	9437.94
Asyut	492	1907.71	6007.25	8398.84	10860.15	13408.47	55215.0	7780.99
Sohag	494	3308.33	6897.06	9493.75	11402.59	13248.38	86468.0	7404.11
Qena	493	2735.78	7677.00	11131.20	13757.27	16425.80	229234.0	12863.13
Aswan	492	2790.13	7496.40	10412.05	12617.66	15023.50	60602.0	8137.25
Luxor	498	3218.71	7599.33	9739.85	11595.78	14237.00	65017.0	6031.05
Border cities in Egypt	392	3063.09	8848.63	11965.57	15582.25	17533.75	117494.0	12762.65

n: denotes the sample size of each governorates

As indicated by the data presented in the preceding table, substantial regional variation in household income levels persists across Egyptian governorates in 2018. The highest average household income was observed in Port Said Governorate, with a mean income of EGP 26137.79. This was followed by Cairo at EGP 23989.78 and Alexandria at EGP 21921.01. These urban and economically strategic governorates benefit from concentrated economic infrastructure, diversified employment sectors, and relatively higher living standards, which may explain their elevated income averages. The associated standard deviations-EGP 34286.31 for Port Said, EGP 43218.30 for Cairo, and EGP 26473.19 for Alexandria-suggest considerable income dispersion within these regions, especially in Cairo, reflecting underlying income inequality.

In contrast, the lowest average household incomes were

recorded in several Upper Egypt governorates, which have historically faced structural development challenges. Asyut registered the lowest mean household income at EGP 10860.15, followed by Sohag at EGP 11402.59 and Luxor at EGP 11595.78. The corresponding standard deviations-EGP 7780.99 for Asyut, EGP 7404.11 for Sohag, and EGP 6031.05 for Luxor-indicate more compressed income distributions but also reflect overall lower economic activity and opportunity in these areas. These disparities reinforce the persistent spatial inequalities in income distribution across the country, highlighting the need for geographically targeted socioeconomic policies aimed at fostering inclusive development and reducing inter-governorate income gaps. The graphically presentation of these results are obtained and provided in Figure 3.

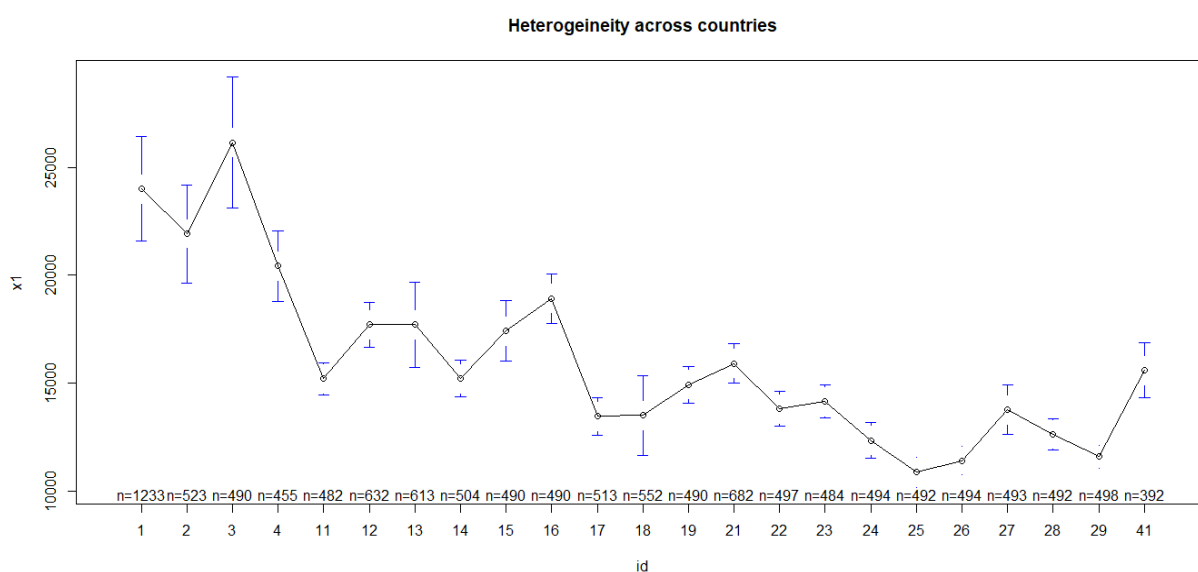


Figure 3. The 95% confidence interval around the means of income distributions for 23 governorates at 2018.

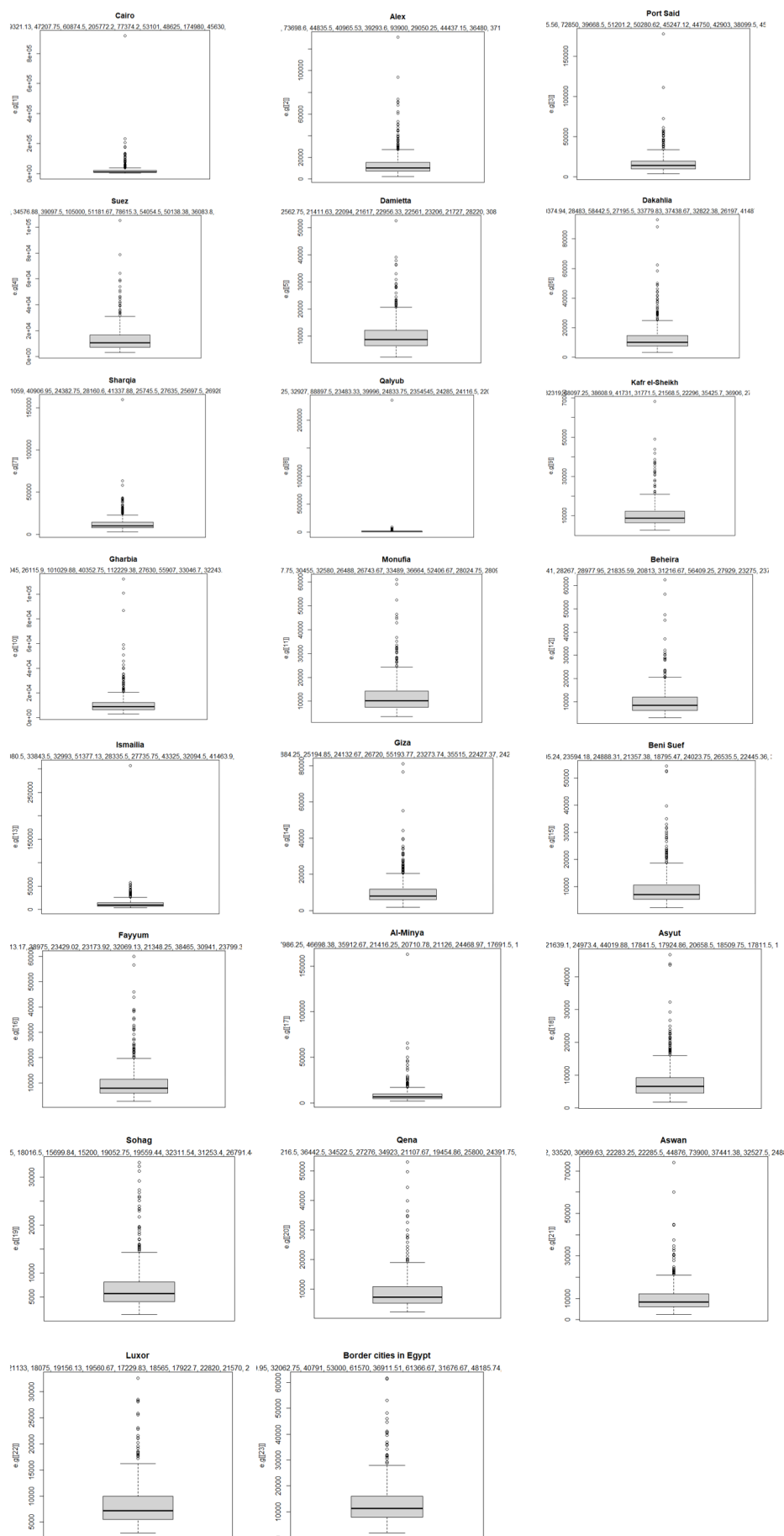


Figure 4. Boxplots of income distributions for 23 governorates at 2018.

Figure 3 illustrates that the highest average household incomes in 2018 were recorded in Port Said, Cairo, and Alexandria, reaffirming the economic prominence of these urban and industrialized governorates. In contrast, the lowest average household incomes are observed in Asyut, Sohag, and Luxor, reflecting ongoing regional disparities between Upper Egypt and the more developed urban centers. Additionally, the figure reveals substantial heterogeneity in income distribution across governorates, suggesting marked differences not only in central tendency but also in the spread and variability of income levels.

To further explore and visually summarize the distributional characteristics of household income data across the country, the adjacent figure presents a boxplot disaggregated by governorate. This representation enables a clearer understanding of the degree of skewness, presence of outliers, and interquartile range within each governorate's income distribution, offering deeper insights into the inequality patterns and variability inherent in the 2018 dataset. Such graphical tools complement the descriptive statistics by highlighting intra-regional disparities that may not be immediately apparent through tabular summaries alone.

Figure 4 reveals considerable variation in individual income levels across governorates, with numerous observations significantly exceeding the corresponding average income within their respective regions. This pattern is indicative of the presence of statistical outliers and underscores the internal disparities in income distribution at the subnational

level. The existence of such extreme values suggests a high degree of income concentration among certain individuals or households, which may distort mean-based indicators and warrant further examination using robust measures of inequality. The visual evidence provided by the figure reinforces the need to account for within-governorate heterogeneity when assessing overall income inequality, as substantial intra-regional disparities may persist even in governorates with relatively high average income.

4.2. Comparison Results of Income Inequality Measures

Now we proceed to compute the values of the proposed index with a set of recognized comparative inequality measures that widely used under weighted data. These include the Gini Index (GI), Atkinson's Index (AT), Palma Index (PI), the Allison and Foster Index (AF), the Generalized Entropy Index (GE), both variants of Theil's Index (TT and TL), and the Leti Index (LI). Each of these indices offers a distinct perspective on income inequality, capturing different aspects of income dispersion and sensitivity to various segments of the distribution. The computed values for all indices are presented in Table 3 for the 2015 income data and Table 5 for the 2018 income data. This comparative assessment allows for a more comprehensive understanding of the evolution and characteristics of income inequality across the two time periods.

4.2.1. Comparative Analysis of Income Inequality Indicators: 2015 Dataset

Table 3. Income inequality indicators for 23 governorates in Egypt in 2015.

Governorates	GI	SWI_{θ}	AT	PI	AF	TT	TL	GE	LI
Cairo	0.496	2.243	0.339	3.043	0.026	0.588	0.414	0.456	0.667
Alexandria	0.368	1.835	0.196	1.586	0.105	0.254	0.218	0.228	0.667
Port Said	0.329	1.957	0.162	1.323	0.085	0.203	0.176	0.184	0.668
Suez	0.346	1.696	0.176	1.403	0.126	0.212	0.193	0.197	0.668
Damietta	0.288	1.484	0.125	1.045	0.161	0.142	0.134	0.136	0.667
Dakahlia	0.311	1.634	0.144	1.195	0.117	0.178	0.155	0.163	0.668
Sharqia	0.300	1.677	0.136	1.137	0.063	0.176	0.146	0.156	0.668
Qalyub	0.508	2.388	0.388	3.039	0.008	1.365	0.490	0.687	0.667
Kafr el-Sheikh	0.296	1.810	0.131	1.118	0.129	0.160	0.140	0.147	0.668
Gharbia	0.339	1.802	0.171	1.402	0.092	0.233	0.188	0.203	0.667
Monufia	0.294	1.698	0.129	1.094	0.172	0.151	0.138	0.142	0.668
Beheira	0.281	1.587	0.119	1.019	0.126	0.143	0.127	0.132	0.668
Ismailia	0.347	1.673	0.182	1.438	0.037	0.277	0.200	0.223	0.669

Governorates	GI	SWI_{θ}	AT	PI	AF	TT	TL	GE	LI
Giza	0.322	1.629	0.156	1.245	0.109	0.187	0.169	0.174	0.667
Beni Suef	0.320	1.694	0.151	1.258	0.148	0.186	0.164	0.171	0.668
Fayyum	0.322	1.630	0.153	1.245	0.151	0.186	0.165	0.171	0.668
Al-Minya	0.354	1.826	0.188	1.498	0.050	0.279	0.209	0.231	0.668
Asyut	0.330	1.843	0.160	1.308	0.159	0.193	0.175	0.179	0.668
Sohag	0.319	1.628	0.151	1.230	0.188	0.179	0.164	0.168	0.668
Qena	0.310	1.524	0.144	1.174	0.148	0.173	0.155	0.160	0.667
Aswan	0.310	1.593	0.144	1.170	0.120	0.172	0.156	0.160	0.668
Luxor	0.251	1.582	0.094	0.872	0.194	0.106	0.099	0.101	0.669
Border cities in Egypt	0.306	1.422	0.144	1.127	0.186	0.158	0.155	0.153	0.668

As we can see from the above table, the proposed index SWI_{θ} incorporating additional distributional features, including the skewness parameter, thus providing a more nuanced and comprehensive measure of inequality. This capability allows it to distinguish between distributions that may appear similar under traditional indices but are in fact structurally different. This advantage can be illustrated by discuss the following two examples.

Example 1: we can demonstrate the advantages of the proposed index by examine three income distributions corresponding to the governorates of Qena, Aswan, and the Border Cities in Egypt. These governorates were selected based on the observation that, when evaluated using several conventional inequality measures including: GI, AT, TL and GE, they appear to exhibit comparable levels of income inequality. That GI equals 0.310, 0.310 and 0.306 for Qena, Aswan, and the Border Cities in Egypt with respectively, and based on AT values we obtain 0.144, 0.144 and 0.144 for

Qena, Aswan, and the Border Cities in Egypt with respectively. Also we can observe the same conclusion from TL results which are 0.155, 0.156 and 0.155 for Qena, Aswan, and the Border Cities in Egypt with respectively, and from GE results we obtain 0.160, 0.160 and 0.153 for Qena, Aswan, and the Border Cities in Egypt with respectively.

However from Figure 5, a closer inspection of the actual income distributions, as seen in Figure 5-b along with their corresponding boxplot representations in Figure 5-a, reveals notable differences in the shape and spread of the distributions across these governorates. These visual discrepancies indicate that the standard indices may overlook important distributional characteristics, such as asymmetry and tail behavior, that can substantially affect interpretations of inequality. In contrast, our index SWI_{θ} addresses this limitation by incorporating additional distributional features including the skewness parameter, where the results of SWI_{θ} are 1.524, 1.593 and 1.422 for Qena, Aswan, and the Border Cities in Egypt with respectively.

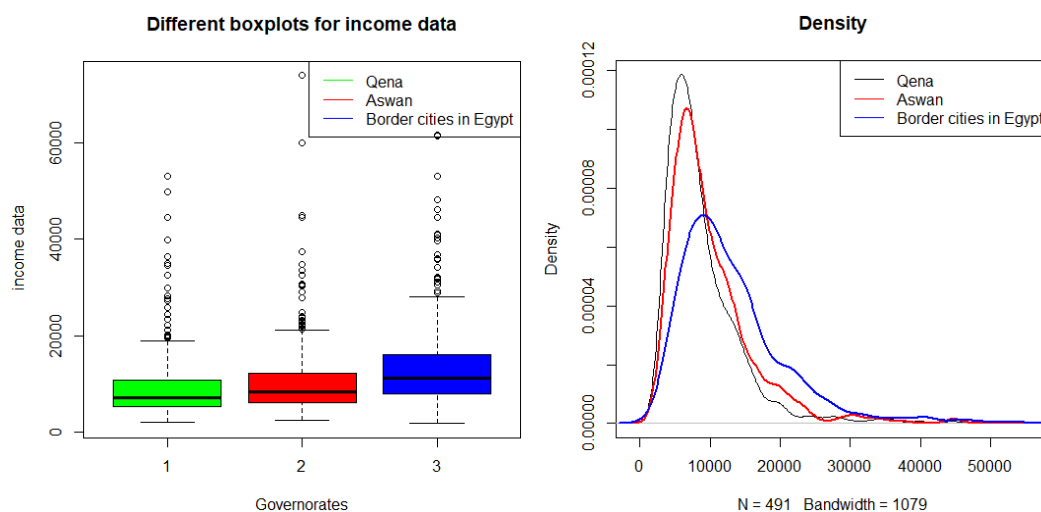


Figure 5. Provide the visual description for the characteristics of the three distributions, that: **Figure 5-a.** Presents the boxplots of them. **Figure 5-b.** Provides their density curves.

Example 2: A similar observation can be made when comparing the governorates of Beni Suef and Sohag. When assessed using the Atkinson Index (AT), Theil's Index (TL), and the Leti Index (LI), both governorates appear to exhibit nearly identical levels of income inequality, with values of 0.151, 0.164, and 0.668, respectively. However, the proposed index SWI_{θ} is able to discern subtle differences between the two distributions. Specifically, Beni Suef records a value of $SWI_{\theta} = 1.694$, while Sohag has a slightly lower value of $SWI_{\theta} = 1.628$, indicating that income inequality in Beni Suef is marginally higher than in Sohag. This further highlights the sensitivity of the proposed index to distributional characteristics that may not be captured by conventional measures.

The observed disparities in income inequality, particularly the elevated levels recorded in Qalyub ($SWI_{\theta} = 2.388$) and Cairo Governorates ($SWI_{\theta} = 2.243$) (see next figure) in 2015, underscore the urgent need for regionally tailored policy interventions. These figures, as indicated by the proposed symmetry Weighted Inequality Index (SWI_{θ}), suggest spatial concentrations of income disparity that may be linked to structural factors such as uneven access to economic opportunities, disparities in public service delivery, and varying levels of urbanization. Conversely, the significantly lower inequality observed in the Border Cities ($SWI_{\theta} = 1.422$) implies a more equitable distribution of income, potentially due to smaller population size, targeted development projects, or lower economic diversification. These findings should inform policy frameworks aimed at promoting inclusive growth, such as fiscal decentralization, investment in under-resourced governorates, and equitable distribution of infrastructure and services.

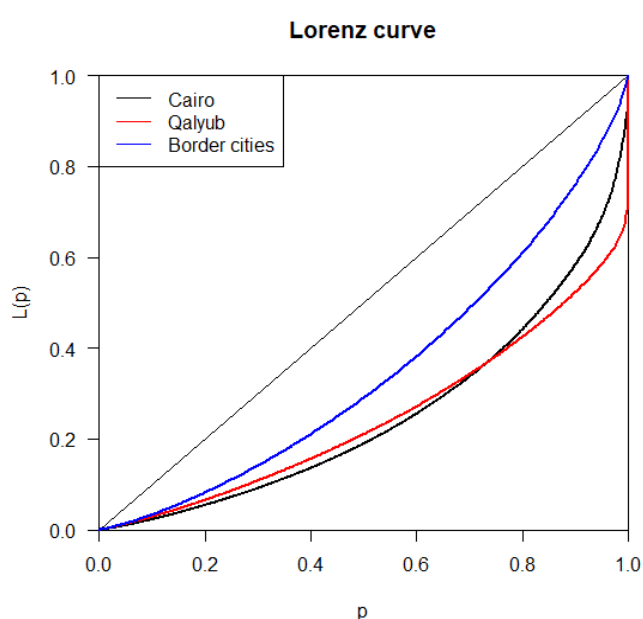


Figure 6. Presents the Lorenz curve for income distribution of Qalyub, Cairo and Border Cities.

To assess whether the differences in SWI_{θ} values across governorates are statistically significant, we employed a bootstrap-based approach using the following steps: for each governorate, the income data were resampled with replacement 5,000 times. The SWI_{θ} index was then computed for each resample, and the resulting values were used to estimate the mean and the corresponding 95% confidence interval for each governorate. The presence or absence of overlap between these confidence intervals indicates statistical significance, with non-overlapping intervals suggesting meaningful differences in income inequality across governorates. The outcomes of this analysis are presented in Table 4 and visually summarized in Figure 7.

Table 4. Mean values and 95% confidence intervals of SWI_{θ} for data of 2015.

Governorates	Mean (SWI_{θ})	Lower CI	Upper CI
Cairo	2.23	1.89	2.60
Alexandria	1.85	1.62	2.10
Port Said	1.91	1.66	2.17
Suez	1.69	1.49	1.92
Damietta	1.48	1.32	1.66
Dakahlia	1.63	1.46	1.82
Sharqia	1.67	1.46	1.90
Qalyub	2.15	1.45	2.74
Kafr el-Sheikh	1.78	1.57	2.02
Gharbia	1.84	1.60	2.08
Monufia	1.70	1.46	1.91
Beheira	1.59	1.40	1.79
Ismailia	1.71	1.40	2.06
Giza	1.61	1.43	1.80
Beni Suef	1.68	1.48	1.90
Fayyum	1.65	1.45	1.87
Al-Minya	1.79	1.53	2.10
Asyut	1.77	1.57	2.00
Sohag	1.69	1.48	1.94
Qena	1.56	1.36	1.77
Aswan	1.57	1.36	1.79
Luxor	1.55	1.37	1.78
Border cities in Egypt	1.46	1.26	1.67

Based on the above results, we can see that the highest

values of SWI_{θ} appears at Cairo and Qalyubia with coefficients of 2.23 and 2.14, respectively, reflecting a relatively high level of income inequality in these governorates. In

contrast, the Border Cities recorded the lowest SWI_{θ} value of 1.46, suggesting a more equitable income distribution.

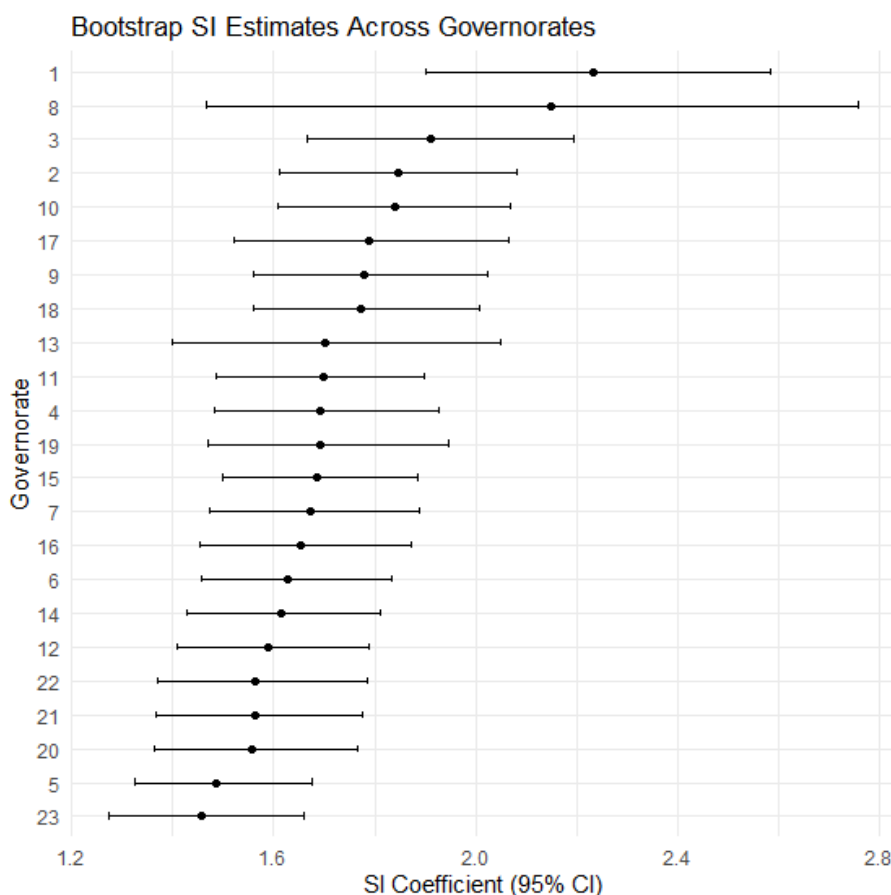


Figure 7. Presents the mean values and 95% confidence intervals of SWI_{θ} for data of 2015.

From Figure 7 we noted that, the confidence intervals associated with these estimates do not entirely overlap, implying statistically significant differences in inequality levels across the governorates. To formally assess whether income distributions differ significantly across governorates, the Kruskal-Wallis test, which is a non-parametric rank-based method that extends the two-sample Wilcoxon test to multiple groups, was employed. The test yielded a statistic of 1480 with a p-value less than $2.2e-16$, which is less than the significance level of 0.05. This result provides strong statistical evidence that income distributions vary significantly among the governorates under study.

4.2.2. Comparative Analysis of Income Inequality Indicators: 2018 Dataset

Following the same methodological framework applied to the 2015 income survey data, the analysis is now extended to the 2018 survey data. Table 5 presents the comparative results of the various inequality measures under consideration, this comparison enables a comprehensive assessment of the consistency and sensitivity of different inequality measures over time.

Table 5. Income inequality indicators for 23 governorates in Egypt in 2018.

Governorates	GI	SWI_{θ}	AT	PI	AF	TT	TL	GE	LI
Cairo	0.469	2.104	0.309	2.674	0.027	0.510	0.370	0.404	0.664
Alexandria	0.396	1.972	0.228	1.843	0.050	0.334	0.259	0.280	0.667

Governorates	GI	SWI_{θ}	AT	PI	AF	TT	TL	GE	LI
Port Said	0.352	1.712	0.190	1.466	0.036	0.295	0.210	0.234	0.668
Suez	0.378	1.789	0.209	1.670	0.140	0.259	0.234	0.239	0.667
Damietta	0.276	1.541	0.113	0.993	0.196	0.129	0.120	0.123	0.668
Dakahlia	0.291	1.671	0.129	1.089	0.068	0.168	0.138	0.148	0.668
Sharqia	0.348	1.809	0.186	1.439	0.036	0.312	0.206	0.238	0.666
Qalyub	0.295	1.621	0.130	1.082	0.103	0.154	0.140	0.143	0.667
Kafr el-Sheikh	0.332	1.681	0.166	1.308	0.058	0.219	0.181	0.192	0.667
Gharbia	0.315	1.652	0.147	1.216	0.156	0.175	0.160	0.164	0.668
Monufia	0.319	1.508	0.153	1.214	0.094	0.187	0.166	0.171	0.669
Beheira	0.348	1.893	0.187	1.463	0.025	0.332	0.207	0.243	0.667
Ismailia	0.304	1.577	0.136	1.123	0.154	0.158	0.146	0.150	0.666
Giza	0.310	1.748	0.144	1.202	0.080	0.183	0.155	0.164	0.666
Beni Suef	0.297	1.587	0.132	1.120	0.119	0.161	0.141	0.148	0.668
Fayyum	0.301	1.555	0.136	1.126	0.197	0.153	0.146	0.147	0.668
Al-Minya	0.330	1.656	0.162	1.314	0.103	0.202	0.177	0.184	0.668
Asyut	0.339	1.586	0.169	1.361	0.186	0.199	0.185	0.188	0.667
Sohag	0.293	1.634	0.128	1.089	0.108	0.154	0.137	0.142	0.669
Qena	0.329	1.720	0.164	1.278	0.054	0.218	0.179	0.190	0.667
Aswan	0.312	1.625	0.144	1.195	0.184	0.166	0.156	0.158	0.668
Luxor	0.261	1.310	0.102	0.887	0.137	0.114	0.108	0.109	0.667
Border cities in Egypt	0.342	1.584	0.174	1.426	0.122	0.223	0.192	0.201	0.668

Consistent with the findings from the previous analysis, it was observed that, excluding the proposed index, multiple governorates frequently exhibited identical values for the conventional inequality measures. This issue reemerged in the analysis of the 2018 data, highlighting a limitation in the discriminatory power of traditional indices when assessing governorate disparities in income inequality. See for example Sharqia and Beheira when assessed using Gini index (that $GI = 0.348$ for both cities), however the proposed index SWI_{θ} is able to discern subtle differences between the two distributions. That Sharqia records a value of $SWI_{\theta} = 1.809$, while Beheira has a slightly lower value of $SWI_{\theta} = 1.893$, indicating that income inequality in Beheira is higher than in Sharqia.

Besides, the same conclusion can be obtained when comparing between Dakahlia and Sohag using PI index (that reveals $PI = 1.089$ for both cities), however by using the proposed index SWI_{θ} , Dakahlia records a value of $SWI_{\theta} = 1.671$, while Sohag has a slightly lower value of $SWI_{\theta} = 1.634$, indicating that income inequality in Dakahlia is higher than in Sohag. On the same sequence see both comparisons of (Ismailia and Fayyum) and (Giza and Aswan) when assessed

using the AT.

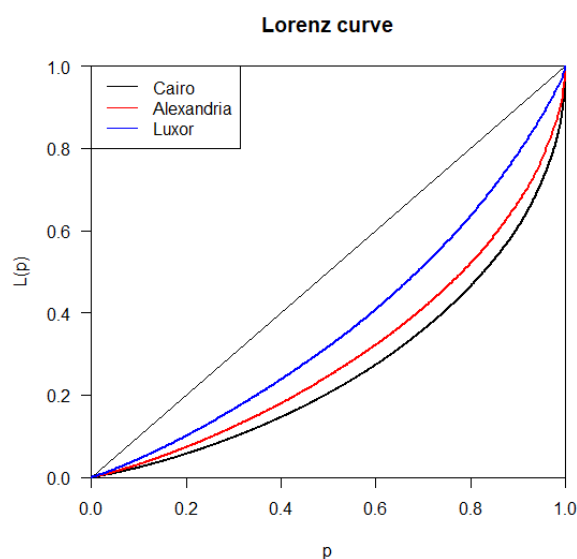


Figure 8. Presents the Lorenz curve for income distribution of Cairo, Alexandria and Luxor.

The findings from Table 5 reveal that in 2018, Cairo Governorate exhibited the highest level of income inequality ($SWI_{\theta} = 2.104$), followed closely by Alexandria Governorate ($SWI_{\theta} = 1.972$). In contrast, Luxor Governorate demonstrated the lowest inequality level, with SWI_{θ} value of 1.310, according to the proposed symmetry Weighted Inequality Index. These results, which are further corroborated by the graphical representation of the Lorenz curve in the subsequent figure, highlight the persistent and geographically uneven nature of income distribution across Egyptian governorates. The concentration of inequality in highly urbanized and economically central regions such as Cairo and Alexan-

dria may reflect deeper structural imbalances, including labor market segmentation, spatial segregation, and unequal access to high-income opportunities. Policymakers should prioritize the design and implementation of region-specific redistributive strategies, such as urban subsidy reform, inclusive employment schemes, and strengthened social safety nets targeted at urban poor populations.

By assessing whether the differences in SWI_{θ} values across governorates are statistically significant, a bootstrap-based approach is employed and the outcomes of this analysis are presented in Table 6 and visually summarized in Figure 9.

Table 6. Mean values and 95% confidence intervals of SWI_{θ} for data of 2018.

Governorates	Mean (SWI_{θ})	Lower CI	Upper CI
Cairo	2.07	1.86	2.32
Alexandria	1.93	1.67	2.26
Port Said	1.70	1.36	2.09
Suez	1.77	1.55	2.05
Damietta	1.58	1.41	1.79
Dakahlia	1.69	1.47	1.91
Sharqia	1.76	1.45	2.11
Qalyub	1.61	1.41	1.84
Kafr el-Sheikh	1.71	1.43	1.96
Gharbia	1.62	1.43	1.85
Monufia	1.52	1.33	1.72
Beheira	1.87	1.53	2.30
Ismailia	1.57	1.37	1.78
Giza	1.73	1.53	1.95
Beni Suef	1.63	1.39	1.93
Fayyum	1.56	1.39	1.75
Al-Minya	1.67	1.47	1.90
Asyut	1.62	1.44	1.84
Sohag	1.60	1.41	1.81
Qena	1.73	1.42	2.11
Aswan	1.62	1.41	1.90
Luxor	1.32	1.16	1.50
Border cities in Egypt	1.63	1.39	1.93

The analysis reveals that Cairo and Alexandria exhibit the highest values of the proposed inequality index SWI_{θ} , with estimated coefficients of 2.07 and 1.93, respectively. These values indicate a relatively high degree of income inequality within these governorates. In contrast, Luxor displays the lowest SWI_{θ} value at 1.32, suggesting a more equitable income distribution.

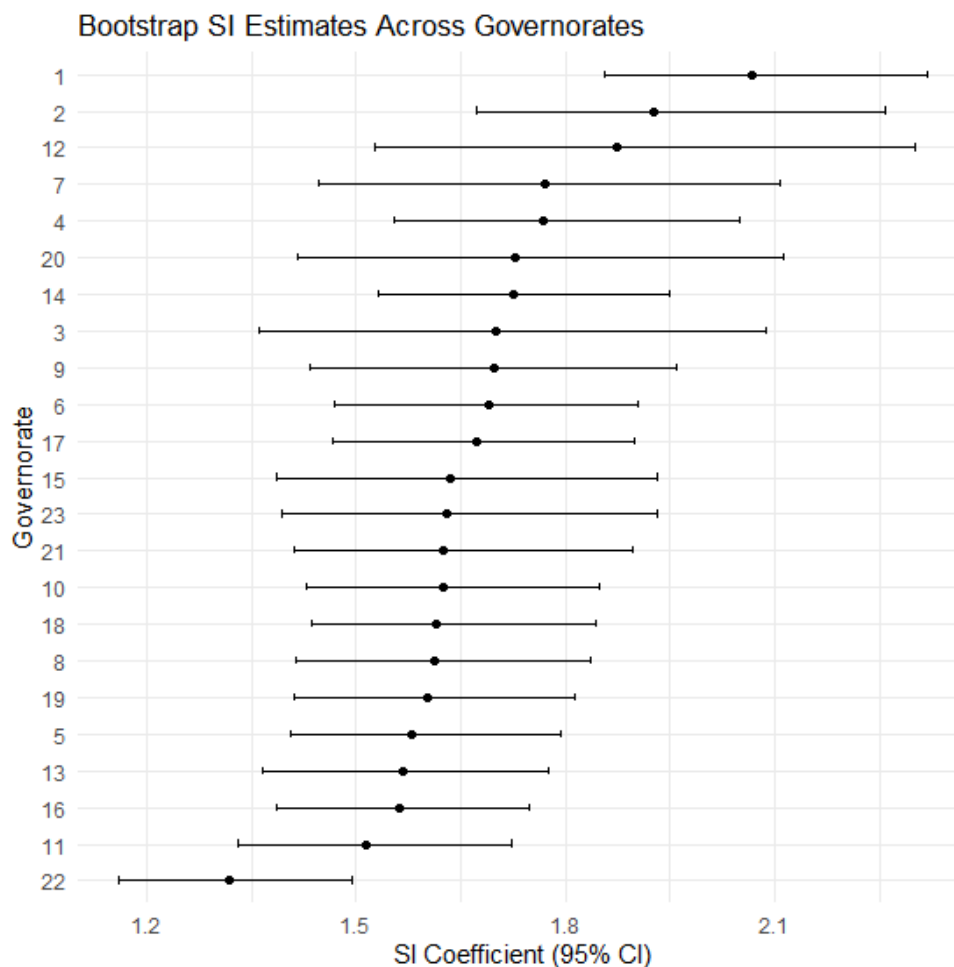


Figure 9. Presents the mean values and 95% confidence intervals of SWI_{θ} for data of 2018.

Importantly, the confidence intervals for these estimates do not fully overlap, which implies statistically significant differences in income inequality across the governorates. To further validate these findings, the Kruskal-Wallis test was applied to assess whether the income distributions differ significantly among the governorates. The test produced a statistic of 1409.2 with a p-value less than $2.2e-16$, which is substantially below the conventional significance level of 0.05. This outcome provides robust evidence that the income distributions are not homogeneous across governorates, thereby confirming significant regional disparities in income inequality.

4.2.3. A Two-period Comparison of Income Inequality Patterns

Furthermore, conducting a comparative assessment across the two time periods depicted in Figure 10 facilitates a more nuanced and comprehensive understanding of the evolution, structure, and regional characteristics of income inequality. By examining changes in the distributional patterns between 2015 and 2018, this analysis allows for the identification of

temporal shifts in inequality levels and highlights the persistence or emergence of spatial disparities among governorates. Such a longitudinal perspective is essential for evaluating the potential impact of socioeconomic policies implemented during this period and for uncovering underlying structural factors influencing income distribution dynamics over time.

The comparative analysis of the proposed Symmetry Weighted Inequality Index (SWI_{θ}) between 2015 and 2018 reveals a general downward trend in income inequality across the majority of Egyptian governorates. This observed decline, as depicted in the preceding figure, indicates a potential improvement in income distribution and suggests modest progress toward reducing regional inequality during the period under review. Although the magnitude of change varies by governorate, this overall pattern may be attributed to the implementation of targeted socioeconomic reforms, the expansion of social protection programs, regional development initiatives, or evolving household income dynamics, including increased labor force participation or remittance flows. If substantiated, such a trend could signal early signs of policy effectiveness in addressing spatial income disparities.

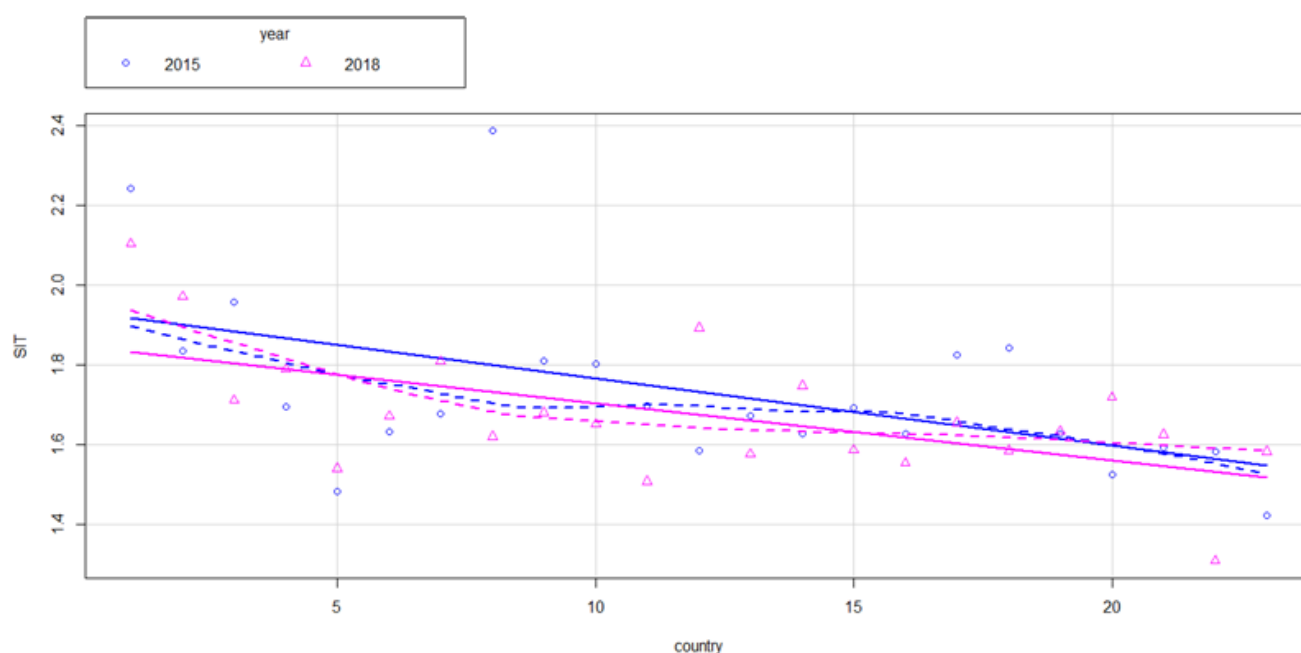


Figure 10. Presents a comparison between the values of SWI_{θ} in 2015 and its corresponding values in 2018.

However, several restrictions must be noted when interpreting these findings. First, while the decrease in SWI_{θ} values is suggestive, causality cannot be inferred without more granular econometric analysis or the integration of policy evaluation frameworks. Second, potential variations in data collection methodologies, sample composition, or reporting standards between the two survey waves may influence the comparability of results. Furthermore, the index captures only relative income inequality and may not reflect absolute changes in living standards or poverty levels. Additional research, including multivariate regression analysis and disaggregated demographic profiling, is necessary to identify the key drivers of change and to evaluate the equity and inclusiveness of observed development outcomes.

5. Conclusion

In this study, the proposed symmetry-based inequality index has been generalized to accommodate weighted data, thereby enhancing its applicability and improving the accuracy of results in contexts where data weighting is essential, such as nationally representative surveys. This generalization ensures that the index remains robust and reflective of population level dynamics. The proposed index appears as an effective tool for evaluating income inequality. Unlike traditional measures such as the Gini index, which may assign identical values to different distributions, the proposed index is capable of distinguishing between distributions that vary in terms of asymmetry or disparity between the upper and lower ends of the Lorenz curve. Moreover, it captures the directional nature of skewness, whether positive or negative, which is often overlooked by conventional inequality measures.

To validate its practical utility, we conducted a comparative empirical analysis assessing the income inequality across governorates in Egypt, using data from the nationally representative income surveys of 2015 and 2018. The findings revealed a general decline in the values of the proposed index over this period, suggesting progress toward more equity in income distributions. This trend may reflect the impact of socioeconomic reforms and targeted policy interventions aimed at reducing disparities among governorates. Finally, we propose that this index, due to its simplicity and interpretability, holds promise not only within the field of socioeconomics but also as a broader statistical tool. It may serve as a valuable measure of heterogeneity or dispersion across a range of disciplines dealing with positively or negatively skewed quantitative variables extending its relevance to environmental studies, health sciences, finance, and beyond.

Abbreviations

GI	Gini Index
AT	Atkinson Index
EDE	An Equivalent Level of Equal Distribution
E	The Parameter of Inequality Degree
GE	The Class of Generalized Entropy Indices
T_T and T_L	Theil Indices
PI	Palma Index
AF	Allison and Foster index
LI	Leti Index
SWI_{θ}	The Proposed Index (Symmetry Weighted Index)

Author Contributions

Eman Mohamed: Data curation, Formal Analysis, Investigation, Methodology, Software, Writing – original draft

Hend Abdulghaffar Auda: Investigation, Methodology, Supervision, Validation

Ibrahim Hassan Ibrahim: Investigation, Supervision, Validation, Writing – review & editing

Data Availability Statement

All data analyzed during this study are obtained from the Central Agency for Public Mobilization and Statistics (CAPMAS). The website is listed as a reference.

Conflicts of Interest

The authors declare no conflicts of interest.

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