

Research Article

A Multi-agent Computational Model for the Transmission of Monetary Policy to the Intrinsic Value of Stocks

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Abstract

This paper presents a multi-agent computational framework that deploys heterogeneous agents to investigate how shifts in short-term interest rates and in the expected path of rates shape the intrinsic value of a broad stock market index. The model elucidates the transmission channels through which monetary policy propagates to market fundamentals, operationalized via the index's theoretical replicating portfolio. By distinguishing valuation changes rooted in fundamentals from those driven by sentiment or feedback dynamics, the framework enables the systematic identification and quantification of speculative expansions (bull markets) and contractions (bear markets), thereby advancing a more disciplined understanding of market cycles. A central innovation is an investment-oriented metric that produces a weekly time series of the Value Gap (VG), defined as the deviation between the model-implied intrinsic value and the observed index level. This measure supports continuous monitoring of mispricing, facilitates comparative analysis across monetary policy regimes, and offers practical signals for risk management and asset allocation. Empirical evaluation yields two principal findings. First, the adverse effect of tighter monetary policy on VG materializes only when the index constituents exhibit a negative aggregate net cash flow, indicating that balance-sheet conditions condition the pass-through from policy rates to valuation gaps. Second, symmetric adjustments in the policy rate—upward or downward—tend to induce correspondingly directional movements in the index's fundamental value (VB), highlighting a robust mapping from policy stance to market-implied fundamentals. Overall, the study contributes to the literature on monetary transmission and asset pricing by clarifying the interaction between policy rates, corporate cash-flow profiles, and valuation dispersion. It also delivers a transparent and implementable analytical tool for detecting market imbalances, guiding tactical positioning, and informing strategic investment decisions under evolving policy environments.

Keywords

Agent-based Computational Economics, Gap Value Index, Stock Markets, Monetary Policy Transmission

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1. Introduction

This paper presents a computer-based multi-agent financial model that uses stock valuation and computer simulation techniques to estimate two time series: one for the intrinsic value of a theoretical stock index portfolio, and another for a proposed value investing index, termed the 'Gap Value' (GV). This GV metric is designed to detect and quantify market imbalances—such as speculative bubbles (bull markets) or downturns (bear markets)—by measuring the divergence between observed market prices and calculated intrinsic values.

The model addresses the central research problem concerning the mechanisms by which monetary policy adjustments are transmitted to stock market fundamentals. The primary objective is to examine how variations in short-term and expected interest rates specifically impact the intrinsic value of the stock market index. The investigation seeks to answer key research questions, including: (i) How do adjustments in the central bank's policy rate influence the stock market's fundamental value (VB)? and (ii) Under what specific conditions, such as the aggregate net cash flow of firms, does monetary policy adversely affect the GV?

The methodology was based on the works of Lebaron, Testfason, and Judd, who themselves built on the seminal work of Testfason and Judd (2006) [19, 13, 24]. In addition to these texts, this paper also draws on insights from simulation and specificity techniques, as described in Law and Kelton, Werker and Brenner, Shannon, Menner, and Miranda [18, 26, 21, 20, 16]. Additionally, works are considered as Damodaran, Abrams and Stewart III [10, 1, 22], Damodaran, McKinsey & Co., and Stickney, Weil & Schipper [10, 15, 23], Damodaran [11] for adaptations and development of the model, and examples of Mathematica usage, such as Shingareva & Lizárraga-Celaya and Blachman [27, 6], Stinespring and Varian [17, 25], Kendrick, Mercado & Amman for examples applied to Economics and Finance [14]. Thus, it seeks to advance the understanding of the scientific literature on the

influences of monetary policy on the intrinsic value of stocks.

Section 2 outlines the methodology, which is centered on a computer-based model and its five constituent modules. This section also introduces the 'Gap Value' (GV) concept, a metric proposed for detecting and measuring financial market bubbles. The GV quantifies the discrepancy that arises when the market value of stocks exceeds their intrinsic value (indicative of a bull market) or, conversely, falls below it (indicative of a bear market). Section 3 presents the model's corollaries.

Section 4 presents the results of the model's application and validation tests, including a Figureical analysis of the intrinsic value elasticity and the GV specification of the stock series compared to the benchmark and expected interest rates, based on a theoretical portfolio of the Ibovespa index. Special attention is paid to the events that significantly influenced the behavior of these series. Section 5 presents the concluding remarks.

2. Model Development

2.1. Module 1 – Stocks Companies' Cash Flow

The first step of the methodology model builds on the discounted cash flow technique (DCF). According to Damodaran, leveraged companies are best assessed based on their free cash flow to the firm (FCFF) [10]. FCFF is derived from EBIT (earnings before interest and taxes). Leveraged companies are those companies that use debt to finance their operations. They are not necessarily very debt-laden companies, but companies with any given burden of debt [10, 11, 5].

Table 1 shows how this process occurs, including their meaning and data source. The cash flows for the two subsequent years were estimated using the least squares method¹.

Table 1. Accounts used for cash flow calculation.

Symbol	FCFF item	Meaning	Data source in period t and previous periods
(=)	NS_t	Net Sales Revenue in period t	Income Statement for period t
(-)	SC_t	Sales costs in t	Income Statement for period
(-)	OE_t	Operating Expenses in t	Income Statement for period
(=)	$EBIT_t$	Earning before income and taxes in period t	
(+)	Depreciation/Amortization	Depreciation/Amortization in t	Cash Flow Statement for period t
(-)	Income tax (IT) + social charges	Income Tax (IT) + Social Charges in t	Income Statement for period t
(=)	Operational cash generation	Operational Cash Generation in t	
(-)	Permanent current investments	Permanent Current Investments	Income Statement and Balance Sheet for period

Symbol	FCFF item	Meaning	Data source in period t and previous periods
	(Working Capital)	(Working Capital) in t	t
=	FCFF	Free cash flow to the firm for period t	

Source: Adapted from Damodaran, Abrams and Stewart III [10, 1, 22]. The methodology was adapted according to current accounting standards in Brazil.

2.2. Module 2 – Inter-temporal Discounting Model

This model includes the time horizon of the cash flow projections to be discounted. Before projecting a company's cash flow, it is important to decide how many periods or stages will be included for cash flow in perpetuity. In accordance with Cuberthson & Nitzsche and Stewart III, a number of stages can be identified in a business growth to maturity², depending on the characteristics of each company [8, 22]. As one of the goals of the model is to build a series of weekly intrinsic values for the companies making up the stock index, they will be assumed to have two stages for the sake of simplicity. In this assumption, the first stage corresponds to the short term, while the second stage refers to an infinite time horizon (i.e., in perpetuity).

According to Bodie, Kane & Marcus and Damodaran, two-stage valuation models adopt the simplifying hypothesis that the companies will not experience any extraordinary changes in the upcoming years, yet they will experience some changes to their capital structure, growth, and productivity patterns [7, 10]. Therefore, the ideal is to project an adjustment period (the first stage). The dividend policy will not be definitive before the second stage, which includes the perpetuity. The first stage is expected to have a distinct level of growth with higher levels of net investments, both in tangible assets and working capital. The formula for the first stage³ is:

$$IV_A = \frac{FCFF_1}{(1+i)^1} \quad (1)$$

Where: IV_A stands for the intrinsic value in the first stage, $FCFF_1$ is the cash flow in the present period, and it refers to the benchmark/base interest rate as determined by the monetary authority. All these variables are calculated on a weekly basis. The intrinsic value in stage 1 is equal to the sum of results of a simple discount formula in a given period, where the basic interest rate i substitutes every company's weighted average cost of capital (WACC), a variable commonly adopted in finance and valuation techniques.

The underlying hypothesis is that the companies making up the stock theoretical portfolio have the Interest rate as a proxy of capital cost. This is a useful simplifying hypothesis, as it allows for aggregating all stock index companies without performing individual accounting calculations, which would

be laborious and make the model unfeasible.

The second stage assumes that the net cash flow result is in perpetuity. The inter-temporal discounting formula in the second stage is as follows:

$$IV_B = \frac{FCFF_3/(i_e - g_2)}{(1+i_e)^2} \quad (2)$$

Where: IV_B stands for the intrinsic value in the second stage, $FCFF_3$ refers to the free cash flow to the firm in the third period ($t+2$), i_e is the expected basic interest rate, g_2 corresponds to the growth rate in the second stage. All these variables are calculated on a weekly basis.

As shown, discount formula (2) includes an expected growth rate (g_2) and an expected interest rate (i_e). Rate g_2 is a geometric mean including the per share profit variation in period 2001-2010 plus an investors' confidence component (Ψ). An animal spirit variable (Ψ) stands for the stockholders' willingness to invest. According to Economatica data, this variable corresponds to an adjusted average rate of return on equity in the 1996-2010 period and a rate of variation obtained from the exam of the technical analysis trend graphics [4]. Each variable has a weighting of 0.5. It was also used to calibrate the model for the weekly changes in the investors' confidence about the company's stocks. The animal spirit (Ψ) is incorporated into the g_2 calculation.

By including inter-temporal discount rates (i_b and i_e), the model integrates the financial data provided by the cash flows, a variable of monetary policy, and the decision rules guiding the behavior of a representative agent. Indeed, this computer-based financial model is also multi-agent-based, and both inter-temporal discount rates can be understood as minimum attractive rates for the multiple agents making up in the model.

2.3. Module 3 – IV and GV in Stocks Theoretical Portfolio

This module targets the IV and GV of both companies and the Stocks theoretical portfolio. In general terms, the module is composed of the sum of intrinsic values in both stages. The weekly intrinsic value of a company is, therefore⁴:

$$IV_A + IV_B = \sum_{t=1}^n \frac{FCFF_t}{(1+i)^t} + \frac{FCFF_{n+1}}{(1+i_e)^2} \quad (3)$$

Following the model methodology, n (number of stocks) intrinsic values are generated (building on the discounted cash flows). The concept of GV, as introduced in this paper, is aimed as an index that predicts financial market bubbles as well as bull market and bear market conditions. The GV is determined using a weekly time series that comprises the difference between intrinsic values (as shown in Module 2) and market values (number of shares $\times$ share price on the cash market) that make up the Stocks theoretical portfolio. Equation (4) is a matrix equation that calculates the GV:

$$GV_{(n \times 1)} = \begin{bmatrix} GV_1^1 \\ GV_1^2 \\ \vdots \\ GV_1^n \end{bmatrix}_{(n \times 1)} = \begin{bmatrix} MV_1^1 \\ MV_1^2 \\ \vdots \\ MV_1^n \end{bmatrix}_{(n \times 1)} - \begin{bmatrix} IV_1^1 \\ IV_1^2 \\ \vdots \\ IV_1^n \end{bmatrix}_{(n \times 1)} \quad (4)$$

Where $GV_{(n \times W)}$ is the gap value, $MV_{(n \times W)}$ is the market value of the n shares in the Stocks theoretical portfolio, and $IV_{(n \times W)}$ is the intrinsic value. All these variables are calculated for 1 week, as represented in the vector columns. W is the number of weeks in the analysis.

The results are placed in a matrix, where the rows represent the weeks and the columns represent the weekly intrinsic values. Subtracting the values of every row vector gives the GV. The equation below shows the same in a more aggregate form:

$$GV_{(n \times W)} = \begin{bmatrix} GV_1^1 & GV_2^1 & \dots & GV_W^1 \\ GV_1^2 & GV_2^2 & \dots & GV_W^2 \\ \vdots & \vdots & \ddots & \vdots \\ GV_1^n & GV_2^n & \dots & GV_W^n \end{bmatrix}_{(n \times W)} = \begin{bmatrix} MV_1^1 & MV_2^1 & \dots & MV_W^1 \\ MV_1^2 & MV_2^2 & \dots & MV_W^2 \\ \vdots & \vdots & \ddots & \vdots \\ MV_1^n & MV_2^n & \dots & MV_W^n \end{bmatrix}_{(n \times W)} - \begin{bmatrix} IV_1^1 & IV_2^1 & \dots & IV_W^1 \\ IV_1^2 & IV_2^2 & \dots & IV_W^2 \\ \vdots & \vdots & \ddots & \vdots \\ IV_1^n & IV_2^n & \dots & IV_W^n \end{bmatrix}_{(n \times W)} \quad (5)$$

Stressing the result of the GV matrix and using an alternative notation gives:

$$GV_{(n \times W)} = MV_{(n \times W)} - IV_{(n \times W)} \quad (6)$$

Where:

$$GV_{(n \times W)} = \begin{bmatrix} MV_1^1 - IV_1^1 & MV_2^1 - IV_2^1 & \dots & MV_W^1 - IV_W^1 \\ MV_1^2 - IV_1^2 & MV_2^2 - IV_2^2 & \dots & MV_W^2 - IV_W^2 \\ \vdots & \vdots & \ddots & \vdots \\ MV_1^n - IV_1^n & MV_2^n - IV_2^n & \dots & MV_W^n - IV_W^n \end{bmatrix}_{(n \times W)} \quad (7)$$

For the sake of easy reading of the matrix above, the second column is omitted in order to obtain:

$$GV_{(68 \times 156)} = \begin{bmatrix} MV_1^1 - \sum_{t=1}^1 \frac{FCFF_1^1}{(1+i_{b,1})^1} + \frac{FCFF_3^1/(i_{e,1}-g_{e,1,1})}{(1+i_{e,1})^2} & MV_2^1 - \sum_{t=1}^1 \frac{FCFF_1^1}{(1+i_{b,2})^1} + \frac{FCFF_3^1/(i_{e,2}-g_{e,2,1})}{(1+i_{e,2})^2} & \dots & MV_{156}^1 - \sum_{t=1}^1 \frac{FCFF_1^1}{(1+i_{b,156})^1} + \frac{FCFF_3^1/(i_{e,156}-g_{e,156,1})}{(1+i_{e,156})^2} \\ MV_1^2 - \sum_{t=1}^1 \frac{FCFF_1^2}{(1+i_{b,1})^1} + \frac{FCFF_3^2/(i_{e,1}-g_{e,1,2})}{(1+i_{e,1})^2} & MV_2^2 - \sum_{t=1}^1 \frac{FCFF_1^2}{(1+i_{b,2})^1} + \frac{FCFF_3^2/(i_{e,2}-g_{e,2,2})}{(1+i_{e,2})^2} & \dots & MV_{156}^2 - \sum_{t=1}^1 \frac{FCFF_1^2}{(1+i_{b,156})^1} + \frac{FCFF_3^2/(i_{e,156}-g_{e,156,2})}{(1+i_{e,156})^2} \\ \vdots & \vdots & \ddots & \vdots \\ MV_1^{68} - \sum_{t=1}^1 \frac{FCFF_1^{68}}{(1+i_{b,1})^1} + \frac{FCFF_3^{68}/(i_{e,1}-g_{e,1,68})}{(1+i_{e,1})^2} & MV_2^{68} - \sum_{t=1}^1 \frac{FCFF_1^{68}}{(1+i_{b,2})^1} + \frac{FCFF_3^{68}/(i_{e,2}-g_{e,2,68})}{(1+i_{e,2})^2} & \dots & MV_{156}^{68} - \sum_{t=1}^1 \frac{FCFF_1^{68}}{(1+i_{b,156})^1} + \frac{FCFF_3^{68}/(i_{e,156}-g_{e,156,68})}{(1+i_{e,156})^2} \end{bmatrix}_{(68 \times 156)} \quad (8)$$

Matrix (8) can also be partitioned in the following row-vectors:

$$GV_{(1 \times W)}^1 = \left[MV_1^1 - \sum_{t=1}^1 \frac{FCFF_1^1}{(1+i_{b,1})^1} + \frac{FCFF_3^1/(i_{e,1}-g_{e,1,1})}{(1+i_{e,1})^2} \quad \dots \quad MV_W^1 - \sum_{t=1}^1 \frac{FCFF_1^1}{(1+i_{b,W})^1} + \frac{FCFF_3^1/(i_{e,W}-g_{e,W,n})}{(1+i_{e,W})^2} \right]_{(1 \times W)}$$

$$GV_{(1 \times W)}^2 = \left[MV_1^2 - \sum_{t=1}^1 \frac{FCFF_1^2}{(1+i_{b,1})^1} + \frac{FCFF_3^2/(i_{e,1}-g_{e,1,2})}{(1+i_{e,1})^2} \quad \dots \quad MV_W^2 - \sum_{t=1}^1 \frac{FCFF_1^2}{(1+i_{b,W})^1} + \frac{FCFF_3^2/(i_{e,W}-g_{e,W,n})}{(1+i_{e,W})^2} \right]_{(1 \times W)} \quad (9)$$

$$\vdots$$

$$GV_{(1 \times W)}^n = \left[MV_1^n - \sum_{t=1}^1 \frac{FCFF_1^n}{(1+i_{b,1})^1} + \frac{FCFF_3^n/(i_{e,1}-g_{e,1,n})}{(1+i_{e,1})^2} \quad \dots \quad MV_W^n - \sum_{t=1}^1 \frac{FCFF_1^n}{(1+i_{b,W})^1} + \frac{FCFF_3^n/(i_{e,W}-g_{e,W,n})}{(1+i_{e,W})^2} \right]_{(1 \times W)}$$

Summing up the row vectors in the matrix system (9) – which represent the n weekly time series of GVs of all stocks in the Stocks theoretical portfolio – gives the aggregate weekly time series of GVs as follows:

$$\sum_{t=1}^n GV_{(1 \times W)} = [GV_1^1 + \dots + GV_1^n \quad GV_2^1 + \dots + GV_2^n \quad \dots \quad GV_W^1 + \dots + GV_W^n]_{(1 \times W)} \quad (10)$$

Returning to the notion of GV and considering the system of row vector in (9) gives:

$$\sum_{t=1}^n GV_{(1 \times W)} = [(MV_1^1 - IV_1^1) + \dots + (MV_1^n - IV_1^n) \quad \dots \quad (MV_W^1 - IV_W^1) + \dots + (MV_W^n - IV_W^n)] \quad (11)$$

2.4. Module 4 – Gap Value Elasticity in Relation to Benchmark and Expected Interest Rates

GV elasticities are deduced from equations (8) using the software package Mathematica 7⁵ as follows. Differentiating (8) in relation to i_b , which is a weekly benchmark interest rate,

$$\frac{\partial GV_{(n \times W)}}{\partial i_b} = \begin{bmatrix} MV_1^1 + \frac{FCFF_1^1}{(1+i_{b,1})^2} & \dots & \dots & MV_W^1 + \frac{FCFF_1^1}{(1+i_{b,W})^2} \\ MV_1^2 + \frac{FCFF_1^2}{(1+i_{b,1})^2} & \dots & \dots & MV_W^2 + \frac{FCFF_1^2}{(1+i_{b,W})^2} \\ \vdots & \vdots & \ddots & \vdots \\ MV_1^n + \frac{FCFF_1^n}{(1+i_{b,1})^2} & \dots & \dots & MV_W^n + \frac{FCFF_1^n}{(1+i_{b,W})^2} \end{bmatrix}_{(n \times W)} \quad (12)$$

Partitioning matrix (12) gives the row values that represent the elasticity curve of all Stocks's stocks in relation to the several levels of interest rates determined by Monetary Authority. Summing up these row vectors gives the series of

$$\frac{\partial GV_{(1 \times W)}}{\partial i_b} = \left[\left(MV_1^1 + \frac{FCFF_1^1}{(1+i_{b,1})^2} + \dots + MV_1^n + \frac{FCFF_1^n}{(1+i_{b,1})^2} \right) \dots \left(MV_W^1 + \frac{FCFF_1^1}{(1+i_{b,W})^2} + \dots + MV_W^n + \frac{FCFF_1^n}{(1+i_{b,W})^2} \right) \right] \quad (13)$$

After that, matrix (8) is differentiated in relation to the expected interest rate. The calculations, similar to those provided before, give the series of GV for every Stocks's stock in relation to the expected interest rate, as well as the series of

gives the matrix equation (12), which shows the several elasticity values of the intrinsic values of every share in the Stocks theoretical portfolio in relation to the Monetary Authority's benchmark interest rate. Elasticity (12) herein determines the effect of, for instance, a 1-percentage-point variation on the intrinsic value of a share in the Bovespa Index.

Stocks's GV elasticity in relation to the Interest rate rate, the first objective of this paper. The vector calculations are as follows:

GV of the Stocks's theoretical portfolio as a whole in relation to this very variable (expected interest rate). Therefore, result (14) below attains the second objective of this paper.

$$\frac{\partial GV_{(n \times W)}}{\partial i_e} = \begin{bmatrix} MV_1^1 + \frac{FCFF_1^1}{(1+i_{e,1})^2 \cdot (i_{e,1} - g_{e,1,1})^2} + \frac{2FCFF_1^1}{(1+i_{e,1})^3 \cdot (i_{e,1} - g_{e,1,1})} & \dots & \dots & MV_W^1 + \frac{FCFF_1^1}{(1+i_{e,W})^2 \cdot (i_{e,W} - g_{e,W,1})^2} + \frac{2FCFF_1^1}{(1+i_{e,W})^3 \cdot (i_{e,W} - g_{e,W,1})} \\ MV_1^2 + \frac{FCFF_1^2}{(1+i_{e,1})^2 \cdot (i_{e,1} - g_{e,1,2})^2} + \frac{2FCFF_1^2}{(1+i_{e,1})^3 \cdot (i_{e,1} - g_{e,1,2})} & \dots & \dots & MV_W^2 + \frac{FCFF_1^2}{(1+i_{e,W})^2 \cdot (i_{e,W} - g_{e,W,2})^2} + \frac{2FCFF_1^2}{(1+i_{e,W})^3 \cdot (i_{e,W} - g_{e,W,2})} \\ \vdots & \vdots & \ddots & \vdots \\ MV_1^n + \frac{FCFF_1^n}{(1+i_{e,1})^2 \cdot (i_{e,1} - g_{e,1,n})^2} + \frac{2FCFF_1^n}{(1+i_{e,1})^3 \cdot (i_{e,1} - g_{e,1,n})} & \dots & \dots & MV_W^n + \frac{FCFF_1^n}{(1+i_{e,W})^2 \cdot (i_{e,W} - g_{e,W,n})^2} + \frac{2FCFF_1^n}{(1+i_{e,W})^3 \cdot (i_{e,W} - g_{e,W,n})} \end{bmatrix}_{(n \times W)} \quad (14)$$

Partitioning matrix (12) gives the row values that represent the elasticity curve of all Stocks stocks in relation to the several levels of interest rates determined by Monetary Authority thought W weeks. Now let each expression contained in the matrix (14) cells be GV_w^s , where s stands for the number of stocks in the Stocks's theoretical portfolio (listed in alphabetical order), and w stands for the week to which a given share's market value and intrinsic value estimation correspond. From that follows:

$$\begin{aligned} \frac{\partial GV_{(1 \times W)}}{\partial i_{b,w}} &= \begin{bmatrix} \frac{\partial GV_1^1}{\partial i_{b,1}} & \frac{\partial GV_2^1}{\partial i_{b,2}} & \dots & \frac{\partial GV_W^1}{\partial i_{b,W}} \end{bmatrix} \\ \frac{\partial GV_{(1 \times W)}}{\partial i_{b,w}} &= \begin{bmatrix} \frac{\partial GV_1^2}{\partial i_{b,1}} & \frac{\partial GV_2^2}{\partial i_{b,2}} & \dots & \frac{\partial GV_W^2}{\partial i_{b,W}} \end{bmatrix} \\ &\vdots \\ \frac{\partial GV_{(1 \times W)}}{\partial i_{b,w}} &= \begin{bmatrix} \frac{\partial GV_1^n}{\partial i_{b,1}} & \frac{\partial GV_2^n}{\partial i_{b,2}} & \dots & \frac{\partial GV_W^n}{\partial i_{b,W}} \end{bmatrix} \end{aligned} \quad (15)$$

Summing up the row vectors of system (15) gives the series of Stocks's gap value elasticity in relation to the expected

Interest rate rate, the second objective of this paper. This gives the series as follows:

$$\frac{\partial GV_{(1 \times W)}}{\partial i_{b,w}} = \left[\left(\frac{\partial GV_1^1}{\partial i_{b,1}} + \frac{\partial GV_1^2}{\partial i_{b,1}} + \dots + \frac{\partial GV_1^n}{\partial i_{b,1}} \right) \dots \left(\frac{\partial GV_W^1}{\partial i_{b,W}} + \frac{\partial GV_W^2}{\partial i_{b,W}} + \dots + \frac{\partial GV_W^n}{\partial i_{b,W}} \right) \right] \quad (16)$$

Analogously, a number of vector calculations finally gives the series of Stocks's gap value elasticity in relation to the expected interest rates throughout the W weeks included in the analysis:

$$\frac{\partial GV_{(1 \times W)}}{\partial i_{e,w}} = \left[\left(\frac{\partial GV_1^1}{\partial i_{e,1}} + \frac{\partial GV_1^2}{\partial i_{e,1}} + \dots + \frac{\partial GV_1^n}{\partial i_{e,1}} \right) \dots \left(\frac{\partial GV_W^1}{\partial i_{e,W}} + \frac{\partial GV_W^2}{\partial i_{e,W}} + \dots + \frac{\partial GV_W^n}{\partial i_{e,W}} \right) \right] \quad (17)$$

Where the cells correspond to:

$$\begin{aligned} \frac{\partial GV_1^1}{\partial i_{e,1}} &= MV_1^1 + \frac{FCFF_1^1}{(1+i_{e,1})^2 \cdot (i_{e,1} - g_{e,1,1})^2} + \frac{2FCFF_3^1}{(1+i_{e,1})^3 \cdot (i_{e,1} - g_{e,1,1})} \\ \frac{\partial GV_1^2}{\partial i_{e,1}} &= MV_1^2 + \frac{FCFF_1^2}{(1+i_{e,1})^2 \cdot (i_{e,1} - g_{e,1,2})^2} + \frac{2FCFF_3^2}{(1+i_{e,1})^3 \cdot (i_{e,1} - g_{e,1,2})} \end{aligned} \quad (18)$$

and so forth until:

$$\frac{\partial GV_W^n}{\partial i_{e,W}} = MV_W^n + \frac{FCFF_1^n}{(1+i_{e,W})^2 \cdot (i_{e,W} - g_{e,W,n})^2} + \frac{2FCFF_3^n}{(1+i_{e,W})^3 \cdot (i_{e,W} - g_{e,W,n})} \quad (19)$$

Where $g_{e,w,s}$ is the expected growth for week w by the company's shareholders, $i_{b,w}$ is the benchmark interest rate (Interest rate) in week w , and $i_{e,w}$ is the expected interest rate for week w .

interest rate (Interest rate defined by Monetary Authority) is negative only in B; it is positive in all other conditions.

This means that the transmission mechanism of monetary policy will affect Stocks's GV negatively only when the companies that make up this index have, altogether, negative net cash generation. Otherwise, coeteris paribus, Monetary Authority's decisions to increase (reduce) the benchmark interest rate will invariably result in increased (reduced) Stocks's GV.

3. Corollaries of the Model

The GV results, particularly from equation (5) onwards, fall in three categories: downside, upside, and balance (IV and MV are equal). These categories are shown in Table 2. As Table 2 shows, the GV elasticity in relation to the benchmark

Table 2. Corollaries of the model.

Possible interactions between GV and share market conditions		
Condition	Possible results	Conclusion
1	$GV_{(i,j)} > 0 \Leftrightarrow MV_{(i,j)} > IV_{(i,j)}$ and $GV \rightarrow \infty \Rightarrow \Pi \rightarrow \infty$	Stocks' potential of losses increases: bull market, downside, or speculative bubble condition.
2	$GV_{(i,j)} < 0 \Leftrightarrow IV_{(i,j)} > MV_{(i,j)}$ and $GV \rightarrow -\infty \Rightarrow \Pi \rightarrow -\infty$	Stocks's potential of profits increases: bear market, upside, or share market downturn condition.
3	$GV_{(i,j)} \cong 0 \Leftrightarrow IV_{(i,j)} \cong MV_{(i,j)}$ and $GV \rightarrow 0 \Rightarrow \Pi \rightarrow 0$	Condition of intrinsic value balance across the shares in the Stocks theoretical portfolio. No profit nor loss potential.

Possible conditions between Stocks's cash flow, GV and GV elasticity in relation to the effects of changes to the benchmark interest rate – through transmission mechanism – on the stocks in the Stocks's theoretical portfolio.

Possible interactions between GV and share market conditions

Condition	Possible results	Conclusion
A	$MV > IV$ (downside), and $FCFF < 0$ (negative cash flow)	$\frac{\partial GV}{\partial i_b} > 0$
B	$MV < IV$ (upside), and $FCFF < 0$ (negative cash flow)	$\frac{\partial GV}{\partial i_b} < 0$
C	$MV > IV$ (downside), and $FCFF > 0$ (positive cash flow)	$\frac{\partial GV}{\partial i_b} > 0$
D	$MV < IV$ (upside), and $FCFF < 0$ (positive cash flow)	$\frac{\partial GV}{\partial i_b} > 0$

Source: Elaborated by the authors

4. Simulations

To test the agent-based computational model (ABM) while disregarding the specific drivers of recent economic events and shocks, the study utilizes data from the post-2008 financial crisis period. The dataset, applied to a theoretical stock portfolio, is derived from 816 accounting statements covering the 2008–2010 triennium (a 156-week time series). Subsequent subsections analyze the events of each year within this period, assessing their influence on the elasticity curve of the Ibovespa's IV and GV relative to changes in benchmark and expected interest rates. The simulations were conducted in Excel by calibrating the growth rate parameters (g_2) as specified in equation (11)⁶.

All the following subsections provide Figure analysis for the computer-based simulations. The subsections are divided according to the year to which they refer. The weekly series correspond to the following row vectors: 1) IV – intrinsic value vector for the Ibovespa's theoretical portfolio, which is the sum of the 52 weekly time series of the intrinsic values estimated by the computer-based model for the 68 stocks in the Ibovespa's theoretical portfolio; (2) MV – market value vector for the Ibovespa's theoretical portfolio; (3) Selic defined by COPOM short-term interest rate, represented as effective interest rates that were weekly compounded; and (4) Expected interest rate – containing expected values as published by the Central Bank of Brazil in the Focus Report for the respective weeks of the period under scrutiny.

4.1. Effect of the Interest Rates (Benchmark and Expected) on the Stocks' IV and GV in 2008

The increased expected inflation (referred to by the Central Bank of Brazil as a "benign scenario" deriving from an excess of aggregate demand over supply) led Copom to interrupt in

April 2008 the policy of progressively mild interest rates that started in September 2005. As a result, the benchmark interest rate goal, which was 11.25% since September 2007, increased 50 basis points (bps) in April 2008, and suffered other increases upon three other monetary authority meetings. It eventually reached 13.75% p.a. in December 2008. The ex-ante real interest rate, calculated by the Central Bank of Brazil for a period of one year after surveying analysts in the private sector,⁷ increased throughout the first nine months of 2008. In October, the expectations for the benchmark interest rates had a reduction tendency, which led to a rate of 7.2% p.a. by the end of the year (0.7 percentage points higher than that registered for the same month in 2007) [3, 4].

Monetary authority's contractionary monetary policy measures at the beginning of 2008 continued the decisions of previous meetings in 2007, in which the committee members assessed that the international economic conjuncture and the expanding domestic economic activity required some prudential measures from the monetary policy maker. The Committee eventually decided to keep the Interest rate at 11.25% p.a., without bias, and opted to monitor the macroeconomic conjecture and thus gain time to deliberate about potential changes to the monetary policy in subsequent meetings. As the expected inflation was increasing, the Central Bank increased the Interest rate in 50 bps in two meetings. Assuming that risks of increasing prices still existed in July, the Central Bank decided to increase the Interest rate to 13% p.a., without bias [4].

The committee still believed in a tendency of increasing prices in September (the already mentioned "benign scenario"). The committee, thus, opted for a new increase on 75 bps in the Interest rate, without bias, even in a scenario of international economic recession (see Figure 1). In October and December, the monetary authority finally noticed the effects of the economic slowdown and the increased economic uncertainty resulting from the international financial crisis that was also affecting several sectors in the Brazilian economy

(especially the credit channels) as well as the consumers' and entrepreneurs' confidence. As for the dynamics of monetary aggregates, a reduction of 2.3% was registered for the average daily balance of narrowly defined money supply (M1).⁸ M1 velocity of money remained relatively stable, and the monetary base (as measured according to the average daily balances) increased 1.5% in the year.

The Central Bank of Brazil assumed other expansionary pressures on the monetary supply, which led to an increase of 933 million Brazilian Reais in the monetary base (using the criterion of available balance by the end of the period). The Monetary Authority, therefore, decided to implement changes to the rules of compulsory deposits, which translated into: reductions in rates, increases in the amounts deducted from callable reserves, changes in earnings on reserve requirements,

together with discounts on amounts to be deposited as an incentive to negotiation of assets among medium- and small-sized financial institutions.⁹

Figure 1 synthesizes the explanations provided so far and shows the negative effects of having increased real interest rates (benchmark and expected) on the intrinsic value of the Stocks theoretical portfolio. Obviously, the Stocks intrinsic value was also affected by other variables that are beyond the scope of the model such as: 1) the default of the three mentioned financial institutions, and worsening animal spirits among the entrepreneurs in the productive sector; 2) reduced volume of foreign investments in shares (Figure 2); 3) the devaluated real exchange rate's effect on the companies making up the Stocks theoretical portfolio; and 4) the remarkably reduced international trade flows.

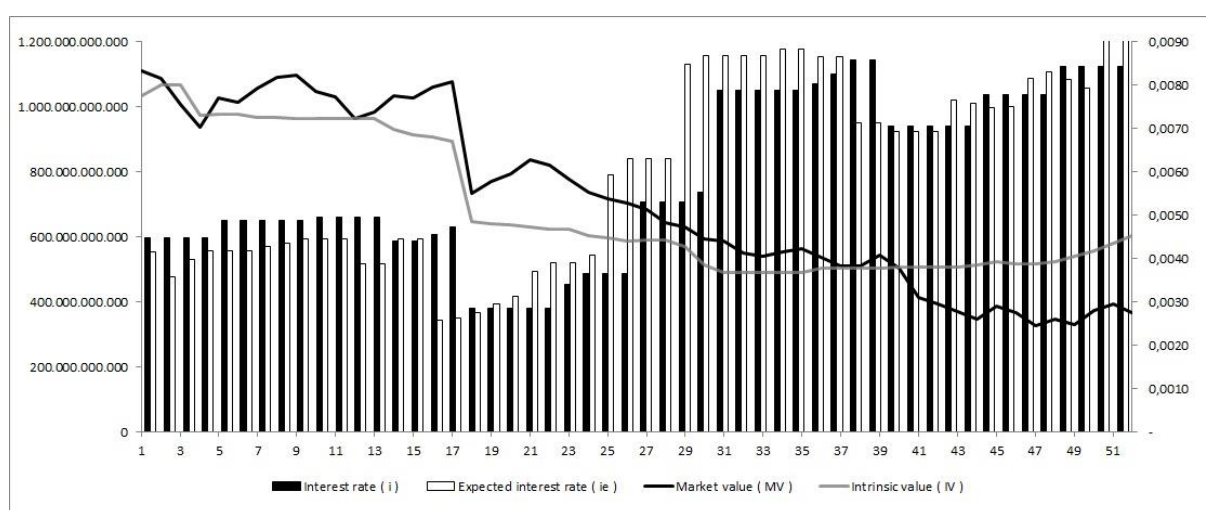


Figure 1. Computer-based weekly series of variation effects of expected and short-term interest rates on the intrinsic values in the Stocks theoretical portfolio in 2008.

Source: Estimates as provided by the authors' computer-based model; Security Exchange Commission's standardized financial statements for the 68 companies making up the Stocks theoretical portfolio in fiscal year 2008; Central Bank of Brazil's Focus Report; BMF&Bovespa; Monetary Policy Committee [4].

As for the Stocks performance, it is worth noting the Petrobras¹⁰ stock split that took place in the seventh week of the year. In addition, the period from January from May witnessed a number of repeated records of increased index values, which reached 73,516 point on May 20th. From late May to the end of year, the Stocks experienced a sharp drop (of 42.1% in relation to the prevailing level by the end of 2007) and increased volatility.

Figure 2 shows that the GV is positive until the 40th week and is negatively correlated with the real interest rates

(benchmark and expected), as mentioned in the previous section. From then on, the GV is negative, reflecting Lehman Brothers', Fannie Mae's, and Freddie Mac's collapse. This moment of structural collapse in the GV series evinces that the intrinsic value of the Stocks's theoretical portfolio declined at a fast pace and started a progressive upside (tendency of profit). A characteristic phenomenon of fundamental analyses based on the notion of value investing in periods of a bear market.

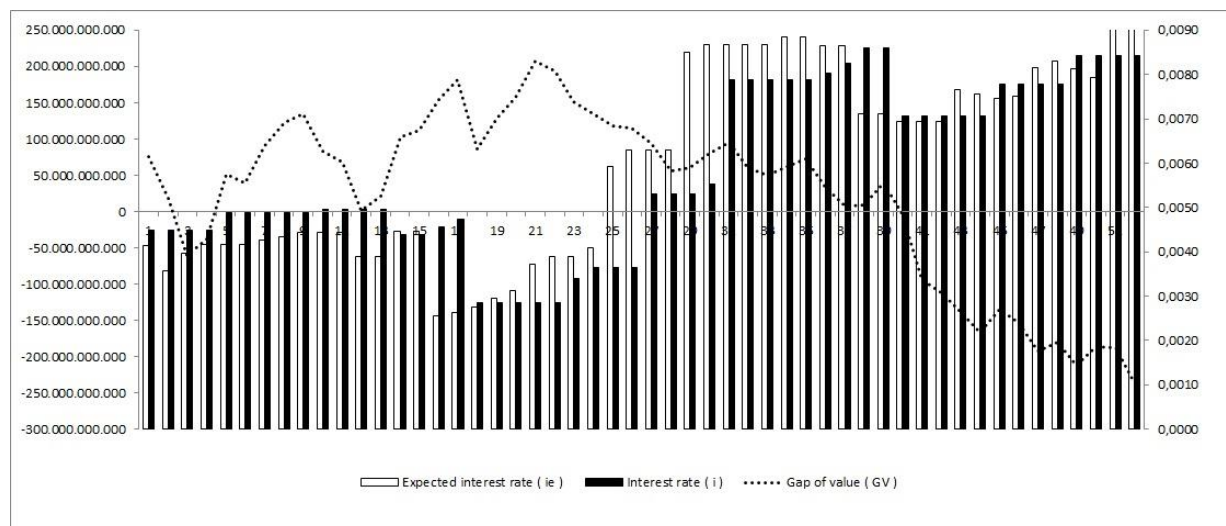


Figure 2. Computer-based weekly series of variation effects of expected and short-term interest rates on the GV ($GV = MV - IV$) in the Stocks theoretical portfolio in 2008.

Source: Estimates as provided by the authors' computer-based model; Security Exchange Commission's standardized financial statements for the 68 companies making up the Stocks theoretical portfolio in fiscal year 2008; Central Bank of Brazil's Focus Report; BMF&Bovespa; Monetary Policy Committee [4, 9].

The result described herein shows the model's power to detect and measure speculative bubbles in the stock market, as well as to identify when they emerge and disappear (Figure 2). Results of this nature are of special interest to policymakers (especially those concerned with making and executing monetary policy), financial executives, and the representative agent of the computer-based model. They help policymakers and financial executives measure and detect speculative bubbles in the share market. They also inform the model representative agent's decision: if the GV is declining and the agent trust in a positive future scenario for the Ibovespa tendency in the long term, s/he will opt for increasing investments in this index; if the value is increasing, however, s/he may decide to keep his/her levels of investments or sell shares if this increase magnitude is great enough to yield him/her the desired profit.

4.2. Effect of the Interest Rates (Benchmark and Expected) on the Stock's Intrinsic Value and GV in 2009

The quarterly growth rates of real GDP in 2009 reveal that only the service sector remained relatively unaffected by the financial crisis, which had its epicenter in August 2008. Industrial and agricultural GDP contracted during the first three quarters of the year, showing signs of recovery only in the final quarter. The intensification of the global crisis in late 2008 triggered a domestic recession characterized by declining credit availability, deteriorating expectations, and heightened risk aversion. In response to the rapidly shifting international environment, the Central Bank of Brazil reversed its contractionary monetary stance, halting a sequence

of four rate hikes implemented between April and September 2008 and initiating a cycle of substantial interest rate reductions.

Between January and July 2009, the Monetary Policy Committee (Copom) reduced the Selic rate by a cumulative 500 basis points, bringing it to 8.75% per annum. The initial cuts of 100 and 150 basis points were based on projections that declining inflationary pressures and subdued economic activity would align the IPCA trajectory with the inflation targets set by the National Monetary Council (CMN). Subsequent reductions of 100 basis points in April and June sought to stimulate industrial production, capacity utilization, employment, and confidence among investors and consumers. In July, the rate was lowered by an additional 50 basis points, after which Copom maintained a more cautious stance, keeping the Selic unchanged through December.

The expansionary monetary policy introduced in mid-2008 led to a 5.7% increase in M1 in 2009, although its velocity remained stable. The daily average balance of the monetary base rose by 14.9%, reflecting a 14.3% increase in issued currency and a 16.8% rise in bank reserves. Broader aggregates also expanded, with M2, M3, and M4 growing by 8.6%, 15.5%, and 16.1%, respectively. In parallel, capital markets experienced a sharp contraction in late 2008, with reduced asset prices and issuance activity. However, during the second half of 2009, primary markets for equities, debentures, and promissory notes recovered, supporting improved expectations for 2010. The Ibovespa closed 2008 at 68,588 points—an 82.7% increase year-on-year—and in 2009 posted a 145.2% gain in dollar terms, driven both by stock price appreciation and the sharp depreciation of the real exchange rate.

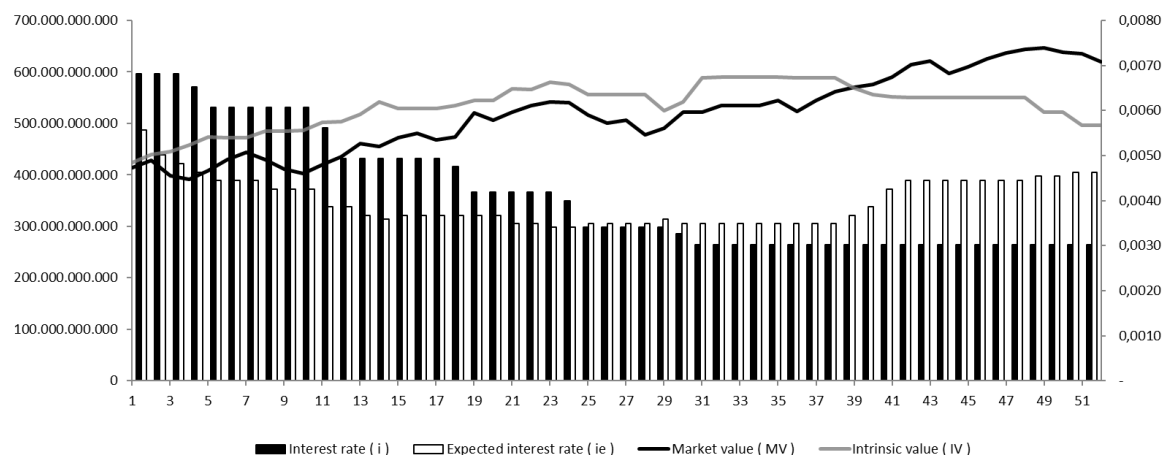


Figure 3. Computer-based weekly series of variation effects of expected and short-term interest rates on the intrinsic values in the Stocks theoretical portfolio in 2009.

Source: Estimates as provided by the authors' computer-based model; Security Exchange Commission's standardized financial statements for the 68 companies making up the Stocks theoretical portfolio in fiscal year 2008; Central Bank of Brazil's Focus Report; BMF&Bovespa; Monetary Policy Committee [4, 9].

The second quarter 2009 saw a recovery of the Brazilian balance of trade. In this period, the government also took countercyclical fiscal policy measures – i.e., reducing the tax on industrialized products (IPI) for durable consumer goods. The period also witnessed increased public expenditure and increased volume of bank credit to the private sector. Risk rating agencies Fitch Ratings and Moody's updated the country's sovereign debt rating to investment grade in the third quarter of 2009. The Brazilian economy was booming by the end of the quarter. Finally, the fourth quarter saw a remarkable increase in the Ibovespa values. Notice, however, that IV did not follow this tendency in that quarter, since the expected interest rates increased. This shows that the intrinsic value

was negatively affected by the expected interest rate variations. Figure 4 shows that these circumstances, in general, led to a MV and IV recovery. It also shows that the GV remained negative until the 40th week, i.e., it remained below the intrinsic value in the period, in an upside condition. This took place because of the financial crisis and the aforementioned events. From the 40th week on, however, expectations change in response to the economic recovery, expansionary factors as explained above, and the award of a good investment grade. The GV became thereafter positive (i.e., a downside condition).

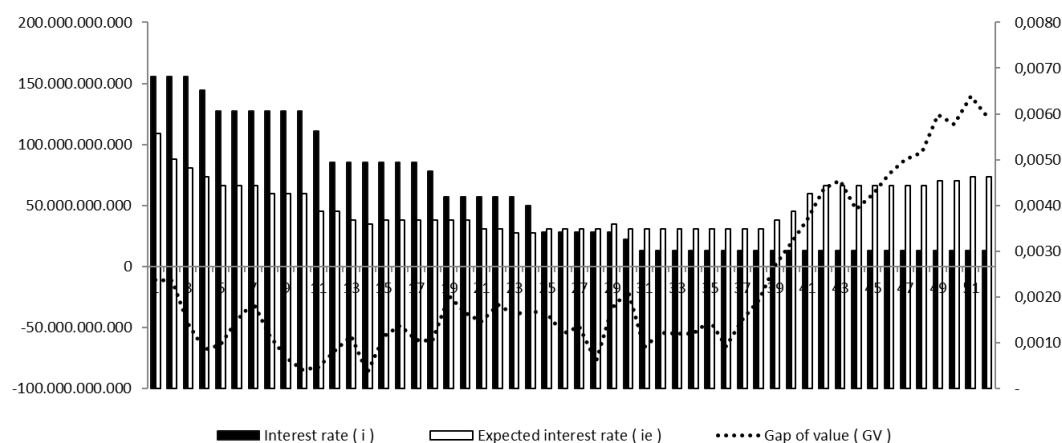


Figure 4. Computer-based weekly series of variation effects of expected and short-term interest rates on the GV ($GV = MV - IV$) in the Stocks theoretical portfolio in 2009.

Source: Estimates as provided by the authors' computer-based model; Security Exchange Commission's standardized financial statements for the 68 companies making up the Stocks theoretical portfolio in fiscal year 2008; Central Bank of Brazil's Focus Report; BMF&Bovespa; Monetary Policy Committee [4, 9].

4.3. Effect of the Interest Rates (Benchmark and Expected) on the Stock's Intrinsic Value and GV in 2010

In 2010, the Central Bank of Brazil maintained the Selic rate at 8.75% per annum throughout the first quarter, marking the lowest level since the series began in 1999. However, during the second and third quarters, the Monetary Policy Committee (Copom) implemented a cumulative increase of 200 basis points, bringing the rate to 10.75% per annum, where it remained for the last three meetings of the year. The annual IPCA inflation rate reached 5.91%, close to the upper bound of the 2.5%–6.5% target range established by the National Monetary Council (CMN). As a result, the effective accumulated interest rate stood at 9.7%, while the real accumulated rate deflated by the IPCA reached 3.7% [2].

Expected real rates rose from 5.1% at the end of 2009 to 6.2% by the end of 2010, with contracts proving highly volatile in the second semester and closing the year at 12.03% per annum. In December, the CMN and the Central Bank introduced macroprudential regulatory measures aimed at strengthening financial regulation, safeguarding institutional stability within the National Financial System (SFN), and ensuring a sustainable credit expansion, while remaining aligned with monetary policy decisions.

Figure 5 illustrates a decline in the intrinsic value (IV) and market value (MV) series during the period under analysis, as well as a negative correlation between these series and both

benchmark and expected interest rates. The capital market registered an exceptional event with Petrobras' stock offering, which reached a trading volume of BRL 120.2 billion and spurred a significant expansion in the primary market for debentures, promissory notes, and shares registered with the Brazilian Securities and Exchange Commission (CVM). Overall, the volume of primary market issues increased by 341% compared to the previous year, with the correlation between IV and MV reaching 99% in 2010.

Brazilian GDP expanded by 7.5% in 2010, the highest growth rate in over two decades. Nevertheless, international conditions introduced considerable uncertainty. The deterioration of economic expectations in the United States, coupled with credit contraction in China and fiscal crises in key Eurozone economies (Portugal, Greece, Spain, Italy, and France), heightened global market instability and weakened the market value of Brazilian companies listed on the stock exchange. These developments underscored the vulnerability of Brazilian firms whose export performance remained strongly tied to demand growth in China, Europe, and the United States, with the Chinese market gaining increasing relative importance over the preceding decade.

Amid this unfavorable external environment, Brazil also faced inflationary pressures at home, prompting Copom to sustain its cycle of interest rate increases. Higher interest rates, combined with investor uncertainty, weighed negatively on the growth value (GV) of the Ibovespa's theoretical portfolio, reflecting a more cautious market outlook.

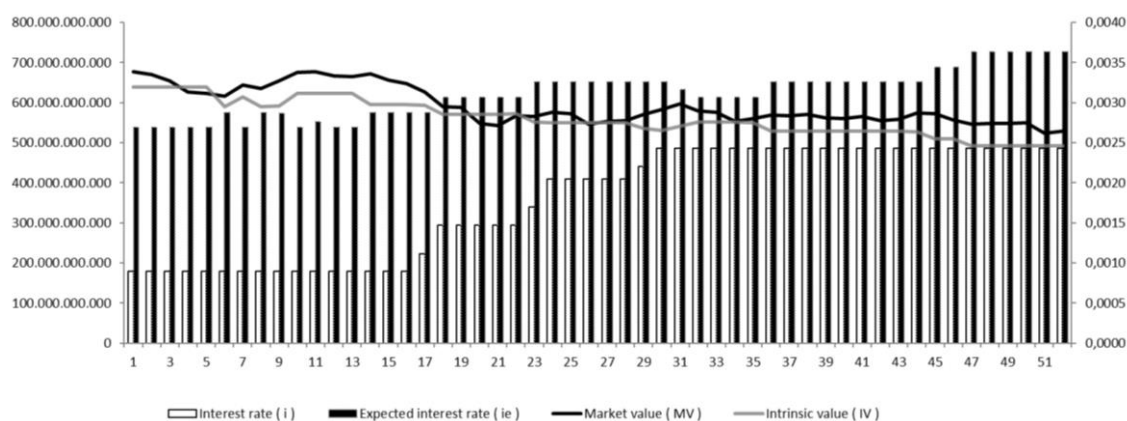
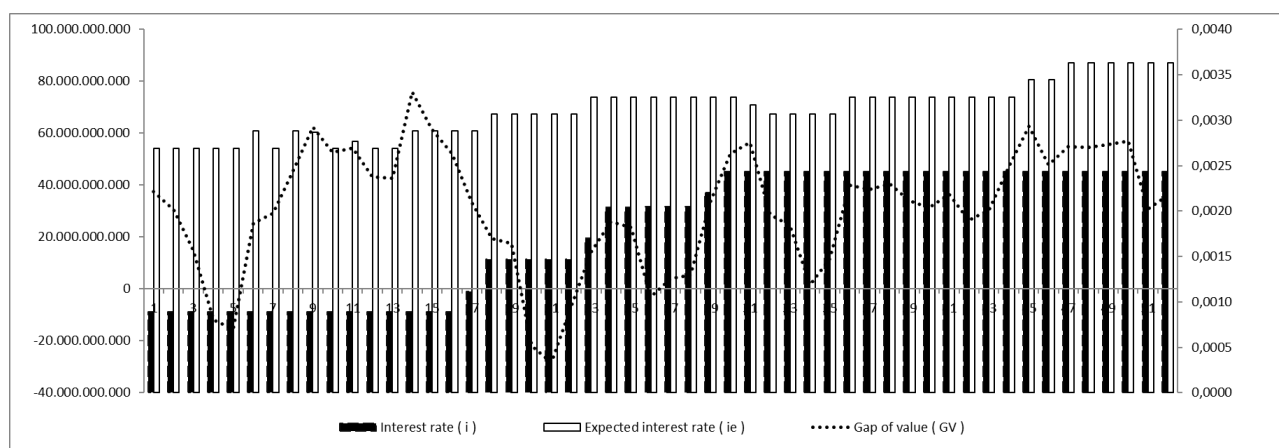


Figure 5. Computer-based weekly series of variation effects of expected and short-term interest rates on the intrinsic values in the Stocks theoretical portfolio in 2010.

Source: Estimates as provided by the authors' computer-based model; Security Exchange Commission's standardized financial statements for the 68 companies making up the Stocks theoretical portfolio in fiscal year 2007; Central Bank of Brazil's Focus Report; BMF&Bovespa; Monetary Policy Committee [4, 9].

Figure 6 shows the elasticity trajectory for the Stocks GV in relation to increases in the benchmark and expected interest rates. It also associates the dynamics of Stocks's GV the major economic events of the period: the lowering of the risk ratings

of Portugal's and Greece's debts in April 27th, and the expansion of the industrial and service GDP that started some weeks before April 27th.



of bear market conditions, thereby offering original insights into market behavior and the mechanisms underlying financial instability.

Another relevant conclusion of the model is that the transmission mechanism of monetary policy will affect Stocks's GV negatively only when the companies that make up this index have an aggregated negative net cash generation. As a suggestion for future research, the developed model could be used to implement the elasticities between fluctuations in global interest rates and the capital structure of companies that make up financial market indices. Furthermore, they can also use the model presented for other markets and with recent data, which was not the scope of our work, so that its focus could be concentrated on modeling in light of discussions of recent economic dynamics.

Abbreviations

CMN	National Monetary Council
Copom	Comitê de Política Monetária (Monetary Policy Committee)
CVM	Comissão de Valores Mobiliários (Brazilian Stock Market Regulatory Commission)
DCF	Discounted Cash Flow
EBIT	Earnings Before Interest and Taxes
EBIT _t	Earning Before Income and Taxes in Period <i>t</i>
FCFF	Free Cash Flow to the Firm
FCFF _t	Free Cash Flow to the Firm for Period <i>t</i>
FCFF ₁	Cash Flow in the Present Period
FCFF ₃	Free Cash Flow to the Firm in the Third Period (<i>t</i> +2)
GV	Gap Value
IPI	Tax on Industrialized Products
IT	Income Tax (Imposto de Renda)
IV	Intrinsic Value (Valor Intrínseco)
IVA	Intrinsic Value in the First Stage
IVB	Intrinsic Value In The Second Stage
MV	Market Value
SFN	National Financial System
VG	Value Gap
WACC	Weighted Average Cost of Capital

Conflicts of Interest

The authors declare no conflicts of interest.

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1 Microsoft Excel was used to calculate the cash flows, and the function PROJ.LIN was used to calculate the least squares.

2 Growth to maturity: period of transition and adjustment of a company's return and risk.

3 For a discussion of this formula, see Damodaran, McKinsey & Co. and Stickney, Weil & Schipper [10, 15, 23].

iv General formula based on Damodaran [11].

5 For the use of Mathematica 7, see Shingareva & Liz árraga-Celaya and Blachman [27, 6]. For examples applied to Economics and Finance, the most relevant references are Stinespring and Varian [17, 25]. Kendrick, Mercado & Amman also provide some examples of using the software [14].

6 The series builds on the analysis of 816 standardized financial statements included in the Brazilian Comission databank. The data are available at: <<http://www.cvm.gov.br/>>.

7 This survey data are included in the Focus Report, published weekly by the Central Bank of Brazil. These data are deflated by the IPCA and correspond to the series of expected real interest rates as described in the following graphs.

8 Considering non-seasonally adjusted and IPCA-deflated data.

9 These changes to the rules, implemented from September 24, 2008 to January 19, 2009, caused a compulsory reserve reduction of 99.8 billion Brazilian reais.

10 Brazilian distributor and marketer of petroleum derivatives and biofuel.