**Applied and Computational Mathematics**
2025; 14(6): 301-308
http://www.sciencepublishinggroup.com/j/acm
doi: 10.11648/j.acm.20251406.11

# Money and the Capital Structure of a Single Owner in Incomplete Markets with Production

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## To cite this article:

Lucio Guimaraes Moscareli, Mathias Schneid Tessmann, Alex Cerqueira Pinto. (2025). Money and the Capital Structure of a Single Owner in Incomplete Markets with Production. *Applied and Computational Mathematics, 14*(6), 301-308.
https://doi.org/10.11648/j.acm.20251406.11

**Received:** 10 September 2025 ; **Accepted:** 22 September 2025 ; **Published:** 10 November 2025

**Abstract:** This paper examines the influence of monetary policy on corporate economies, focusing on whether supply fluctuations affect the decisions of economic agents or whether money is neutral under specific conditions. To investigate this, we develop a two-period general equilibrium model with incomplete and production financial markets, capturing frictions and asymmetries typical of real financial systems. The model is then applied to a corporate finance context, analyzing a capital structure with a single owner. The findings show that monetary policy is generally not neutral when markets are incomplete. Neutrality of money emerges only as a special case when markets are fully complete and efficient, conditions rarely observed in practice. The equilibrium model demonstrates that monetary policy is not neutral or generally does not occur unless markets are complete. For the case of a single owner in a capital structure, Fisher's separation theorem is valid, reinforcing the non-neutrality of money in this economy with markets and incomplete production.In other words, being in an incomplete exchange environment affects the path of consumption of economic agents, although it does not alter the firmâs operational framework. For future research, it would be relevant to extend the analysis time horizon, relax the assumption regarding currency carryover, incorporate private or public banks with fiscal policies into the model, and also consider the application of the Modigliani-Miller theorem.

**Keywords:** Monetary Policy, Non-Neutrality of Money, Incomplete Markets, Corporate Finance

## 1. Introduction

There is no doubt that money is an important and necessary instrument for society as a whole, as it facilitates exchanges of goods and services and allows for the future enjoyment of production. It is one of the oldest and most innovative financial tools invented by humanity [1]. In essence, money has three functions: serving as a means of exchange, functioning as a unit of measurement, and providing a store of value, although the latter is questionable, given its replacement by other assets that can generate a future return.

Initially, it is a contract that facilitates the business process, but once introduced into the economy, it generates a multiplier effect by changing the behavior of economic agents. The importance of its role in economic activity is reinforced if the private market commits to closing deals and the contract system is incomplete. The idea of neutrality was initially explored by Wicksell, but referring to interest rates [2]. Wicksell referred to a situation where the market interest rate would equal a natural interest rate. His idea was that it would be possible to conceive of the problem of monetary policy as a question of interest rate policy [2].

The term ”neutrality of money” was coined by the

economist L. von Bortkiewicz in 1919. According to Snowdon and Vane, there are four main concepts on the subject [3]. The first postulates that money is a veil that permeates an exchange economy; the second is the absence of disturbances - in which monetary equilibrium is achieved and maintained; the third is represented by the quantity theory of money - in the sense of comparative statics; and the fourth is that the real economy is indifferent to the inflation rate - is considered a super neutrality.

Koopmans and Hayek assume that money is the veil of the economy, so the monetary economy behaves the same as the exchange economy. This means that the relative prices of goods and services should not differ from the quantities traded. Neutrality means that money has no effect on the real economy, but serves as a means to fill in the disturbances in the economy. Studies focus on the conditions of monetary [4, 5], equilibrium regarding excess demand and supply of money.

For Hayek, the neutrality of money means the constancy between M - quantity of money in the economy, and V - velocity of money, and he focuses on answering how much money is needed to maintain monetary equilibrium in the economy and respect Say's Law [5].

The quantity theory of money is another branch that finds as a result the effect of neutrality within a comparative static analysis. There is a dichotomy between monetary and real economics, in the sense that relative quantities and prices are determined by the real sector and the price level by the monetary sector [6]. This duality implies that demand and supply are functions only of real variables and, consequently, absolute prices should not be affected in any proportion.

For Mankiw, the concept of neutral or non-neutral money refers to the impact that changes in the money stock have on the economy [7]. The classical dichotomy shows that real variables would not be affected by nominal variables and, therefore, an increase in the money supply would only cause an increase in prices, but not in employment, real wages, or real GDP. However, an increase in the money supply could increase spending and generate a price increase that would gradually affect the real market and, ultimately, generate more jobs [8].

Through a general equilibrium model, Patinkin showed that the price level can only be determined if the excess demand function for goods contains the real money balance - a measure of purchasing power for a given amount of money in the economy - as an argument [9]. This guarantees neutrality in comparative statics. In this way, economic mechanics works via monetary transmission driving the real sector, with a change in the money supply generating a new price level and a new equilibrium of quantities and, thus, relative prices returning to their original values

Thus, this paper seeks to investigate the influence of monetary policy on a corporate economy by determining whether money can affect real variables in an economy. To do so, the incomplete market equilibrium model of Magill and Quinzii is expanded to an economy with production and two periods. Additionally, an application is made to corporate finance with a single-owner capital structure to verify the effects of money supply on the company's activity [1].

The issues involving the effects of monetary policy on real variables are the subject of several theoretical and empirical studies. Changes in the money supply can affect the price level and the distribution of returns in money, such as in investments. Generally, changes in the money supply are not affected by government transactions in real assets (open markets). [10] found that the expansion of the money supply affects the price level only if it is accompanied by current or future transfers from the government to agents.

Friedman and Schwartz also understand that monetary policy strongly influences the price level in the economy [11]. In their classic work on the monetary history of the United States, they claim that a misguided contractionary monetary policy was a determining factor in the worsening of the American depression in 1929 and propose an inflationary control rule consisting of a constant rate of monetary issuance. At the turn of the 20th to the 21st century, Blanchard confirmed that the expansion of the monetary base determines the inflation rate, where a 1% increase in monetary expansion would lead to a 1% increase in the inflation rate [12].

The idea of rational expectations by Lucas and Sargent, and Sargent and Wallace enriches the debate on money and incorporates the agent's perception of the trajectory of monetary policy [13, 14]. Agents anticipate an expansionary policy affecting the economy (generating an increase in prices), even before such a policy occurs. Even so, in this case, money is neutral, since agents do not suffer from monetary illusion and only relative prices matter to maximize their decisions.

Feenstra placed money within the utility function of the economic agent, which guarantees the non-neutrality of money [15]. Monetary models have been developed in search of the effects of monetary policies to affect a continuous equilibrium path to the steady state. Fiore finds a permanent effect on consumption of a monetary shock, thus, there is nonneutrality of money in the short term [16]. Calvo, Brock and Woodford found that monetary policy affects the equilibrium path of the economy with different models and Divino showed that the credibility of the monetary authority for an open and small economy can guarantee that its policy is a strong economic instrument [17-20].

The use of incomplete markets in the models led to major changes in the literature. The concept of this premise is that there are not enough assets in the financial market to protect the investor from each "s" state of nature, so the economic agent cannot protect himself in the financial market from future situations. In that way, Algan, Challe and Ragot found that monetary shocks in a macroeconomic model with flexible prices and incomplete markets create persistent wealth inequality among agents [21]. While previous work by Gottardi and Dubey and Geanakoplos established non-neutrality in exchange economies, the interaction of monetary policy, corporate investment decisions, and ownership structure in a production economy remains less explored [22, 23]. This main gap will be the focus of this work.

In incomplete market settings, a recent strand of literature shows that credit constraints and limited enforcement shape

asset prices and the effects of monetary policy. Bloise and Reichlin demonstrate that borrowing limits directly affect asset valuations and generate measurable forms of âconstrained inefficiency, â meaning that competitive allocations cease to be efficient due to financial restrictions [24]. Bloise, Reichlin and Tirelli deepen this insight by introducing default risk: the possibility of default makes the the competitive equilibrium fragile and creates situations in which small institutional or enforcement changes substantial change price and credit dynamics [25]. Complementing this line, Bloise and Citanna identify technical conditions that, respectively, restore equilibrium uniqueness (under assumptions such as gross-substitution) and explain how a relative scarcity of assets can foster speculative bubbles and higher price volatility when markets lack sufficient substitute instruments [26]. Boise further characterizes recursive Markov equilibria under limited commitment, while Drèze emphasizes that nominal rigidities and rationing typically produce multiple temporary equilibria â highlighting the sensitivity of results to the presence of real and nominal frictions [27, 28].

What unites these contributions, is the common mechanism they isolate: money (or net liquid balances) provides liquidity services that are not perfectly substitutable by other assets in environments with incomplete markets and imperfect enforcement. Consequently, credit limits, default risk, and institutional rigidities convert monetary shocks and changes in the supply of tradable assets into real effects on prices, production, and intertemporal allocation ([24-29]). At the same time, these works show that whether money is effectively non-neutral or the equilibrium is unique depends on sharp structural assumptions, for example, substitutability conditions, the degree of commitment, and enforcement rules.

Thus, it is clear that the role of money has gained increasing traction in the academic and political spheres, but it is necessary to take a step back to understand whether money really affects an individual's consumption trajectory. The fact that the association between money and goods has gone so far can be attributed to Daniel, in which we have an associative machine that guarantees the interpretation automatically [30]. This concept refers to what he calls 'System 1', a mode of thinking that operates rapidly and automatically, with minimal cognitive effort, connecting ideas, images, and experiences in an immediate and unconscious manner. In this sense, the link between money and consumption emerges less from deliberate reasoning and more from this automatic interpretive process.

In addition to this introduction, this work has 5 more sections, where the second section presents the economic model and the third section the equilibrium. Section 4 presents the neutrality of monetary policy, section 5 presents Fisher's separation theorem and, finally, section 6 concludes.

# 2. The Economic Model

In this model, an economy with a financial market and two periods is considered, with uncertainty only in the second period, modeled by a finite set of states of nature $S$. The economy has only one good available in each period and each state of nature, thus forming the commodity space $\mathbb{R}_+^{1+S}$.. The consumption plan is the vector $(x_o, x) \in \mathbb{R} + \times \mathbb{R}_+^S$ where $x_o$ is consumption in the first period and $x_s$, the $s-$ coordinate of vector $x$,, represents consumption in the second period in state $s$.

## 2.1. Sole Owner

There are $H$ agents and $F$ firms whose owner is a single individual called an entrepreneur. Not all individuals own a firm, therefore $F \leq H$. The symblo $f(h)$ is used to denote the firm $f$ of agent $h$. Each firm $f(h)$ is represented by a set of potential investment projects $Y^{f(h)}$ defined by a differential transformation $T : R^{1+S} \to R$. That is, $Y^{f(h)} = \{y \in \mathbb{R}^{S+1}/T^f(y) \leq 0\}$. Each agent, entrepreneur or not, is represented by a utility function $u^h : R_+^{S+1} \to R$ that is assumed to be concave, strictly increasing, and differentiable.

For the sake of language, all agents are called entrepreneurs, where the consumer will be an entrepreneur, and may not have any investment project. Thus, $Y^{f(h)} = \{0\}$ if the consumer is $h$. There are $J$ investments available for exchange in the first period at prices $\pi \in R_+^J$. The promised return on each investment $j$ is measured in terms of the economy's unit of account, i.e., $N_s^j \geq 0$ in state $s$.

## 2.2. The Coin

To model money, we assume that each state of nature $s \in S$ is decomposed into three subperiods $s = (s_1, s_2, s_3)$.

Then, we describe what happens in each subperiod.

1. In $s_1$, the agent sells $w \in \mathbb{R}_+^{S(L+1)}$ in exchange for money;
2. In $s_2$, the agent makes exchanges in the financial market and invests in investment projects;
3. In $s_3$, the agent uses the balance of transactions to buy goods.

The $s-$coordinate of $w^h$, denoted by $w_s^h \in \mathbb{R}_+^L$,, is the agent's initial endowment $h$ in $s_1$.

## 2.3. The Central Bank and the Entrepreneur's Problem

The Central Bank is an institution whose function is to process and exchange the endowments of agents. Next, we will see how this process occurs. Assume that, $s_1$ of $s \in S$, each agent sells the total of his endowment $\omega_s^h$ to the Central Bank.

1. In $0_1$, let $m_0^h = p_0\omega_0^h$ be the value of $\omega_0^h$ at prices $p_0 \in R_+^L$. This value is received by $h$ after selling his endowment to the CB.
2. In $0_2$, there are $J$ nominal securities for exchange. Each asset $j$ promises $N^j \in \mathbb{R}_+^S$ on date 1 and costs $\pi_j$ reais (paid on date 0). The exchange with investments is carried out without the intervention of the CB. In addition, each individual invests in his/her projects. After purchasing portfolio $\theta$, the remaining monetary balance is applied to investment $y_0$, so $h$ has $\tilde{m}_0^h = p_0\omega_0^h - \pi\theta + p_0y_0$.

3. In $0_3$, $h$ uses $\tilde{m}_o^h$ to consume $x_0 \in \mathbb{R}_+^L$ from the CB for the price $p_0 \in \mathbb{R}_+^L$.

The constraint below summarizes the activities in each subperiod of each state of nature, including $s = 0$.

$$p_0 x_0 = \tilde{m}_0^h = p_0 \omega_0^h - \pi\theta + p_0 y_0 \tag{1}$$

$$p_s x_s = \tilde{m}_s^h = p_s \omega_s^h + N_s \theta + p_s y_s, s \in S. \tag{2}$$

In addition to the budget constraints presented, each $h$ faced the following technological constraint:

$$T^{f(h)}(y) \leq 0 \tag{3}$$

Equation (1) summarizes the economic activity of the $h's$ in the three subperiods at date 0. Similarly, Equation (2) summarizes the economic activity of the $h$ in the three subperiods in states $s \in S$. .

Since $N, M$ and $Y^{f(h)}$ are fixed, the budget set is defined for $h$,, given the asset prices $(p, \pi)$,, resulting in:

$$B^h(p, \pi) = \{x \in \mathbb{R}_+^{L(S+1)} : \text{ (1), (2) and (3) be satisfied}\} \tag{4}$$

### *2.4. Corporate Financial Policy and Money Supply*

The individual has a triple choice $(x, \theta, y)$,, satisfying the constraints determined by (4), which maximizes his utility. When this occurs, it is said that $(x, \theta, y)$ solves the entrepreneur's problem $h$,. From now on, the investment $\theta$ will be called *financial policy*, in which we have the following definition:

Definition 1. The company's financial policy, $\theta$, if (1) and (2) are satisfied, finances consumption and investment $(x, y)$.

For the above definition, we use the following notation

$$(x, \theta, y) \in B^h(p, \pi) \tag{5}$$

to describe the consumption-financial-productive plan $(x, \theta, y)$,, characterized by the financial policy $\theta$, consumption and investment $(x, y)$, separately.

The CB determines the supply of fiat money, which is injected into the economy in each state of nature $s$.

$$M_s = \sum_{h \in H} m_s^h, s \in S^* = S \cup \{0\}. \tag{6}$$

The vector $M \in \mathbb{R}_{++}^{1+S}$ is called monetary policy. While the monetary policy of economy is organized:

$$\mathcal{E} = [(\sqcap^\langle, \omega^\langle)\langle \in \mathcal{H}, \mathcal{N}, (\mathcal{Y}^\{)\{ \in \mathcal{F}, \mathcal{M}]$$

# 3. Equilibrium

The following definition is given:

Definition 2. The reduced form of monetary equilibrium for $\mathcal{E}$ is the consumption-financial-productive plan $(x^h, \theta^h, y^{f(h)})_{h \in H}$ together with the asset and commodity price system $(p, \pi)$, such that the following conditions are satisfied:

1. For each $h \in H$, $(x^h, \theta^h, y^{f(h)})$ solves the problem of entrepreneur $h$
2. Market Equilibrium. That is,

$$\sum_{h \in H} (x^h - \omega^h) = \sum_{h \in H} y^{f(h)}, \sum_{h \in H} \theta^h = 0 \tag{7}$$

3.

$$p_s \sum_{h \in H} (x_s^h - y_s^{f(h)}) = M_s, s \in S^* \tag{8}$$

The left-hand side of Equation (8) represents the demand for money, which is determined endogenously, while the right-hand side represents the supply of money and is determined exogenously. Furthermore, they serve to determine the equilibrium price level in each state of nature $s \in S^*$. The $S+1$ monetary equation (8) expresses a simpler version of the quantity theory.

There are several ways in which the money supply $M$ can change. This paper has refrained from discussing this aspect. The only assumption was that the monetary authority (CB) has direct control over the money supply, with $M = (M_s)_{s \in S^*}$ being the monetary rule.

Another hypothesis was that agents cannot carry money from day 0 to day 1. This assumption may be imposed by the fact that money issued on date 0 is not accepted as a means of payment on date 1. This seems artificial, but it was restricted to monetary policies that lead to equilibrium, where the nominal interest rate is positive.

# 4. On the Neutrality of Monetary Policy

To determine whether monetary policy is neutral or not in an entrepreneurial economy, we consider, for the sake of concreteness, the monetary economy with $L = 1$ goods in the economy. In this case, the budget set $B^h(p, \pi)$ will be defined by the following constraints:

$$x_0 - \omega_0^h = -\frac{\pi}{p_0}\theta + y_0, \tag{9}$$

$$x_s - \omega_s^h = \frac{N_s}{p_s}\theta + y_s, s \in S. \tag{10}$$

An equilibrium price of goods in each state (which is just the price level in each state) is determined by the monetary 8:

$$p_s \sum_{h \in H} \omega_s^h = M_s, s \in S^*. \tag{11}$$

It is defined that $\nu_s = \frac{\sum_{h \in H} \omega_s^h}{M_s}$, so (11) can be rewritten as

$$\frac{1}{p_s} = \nu_s, s \in S^*. \tag{12}$$

where $\nu_S$ represents the purchasing power of each currency in each state $s \in S^*$.

### *4.1. Existence of Monetary Equilibrium*

Defining $V_s = \nu_s N_s$ as the $s-$ vector of the payoff matrix $V \in \mathbb{R}^{S\times J}$. In matrix terms we have

$$V = [\nu]N \tag{13}$$

where $[\nu]$ is the diagonal of the matrix, which consists of $\nu_s, s = 1, \ldots, S$. Arriving at the following result.

Proposition 1. The arrangement $[(x^h, \theta^h, y^{f(h)})_{h\in H}, (p, \pi)]$ is the reduced form monetary equilibrium for $\mathcal{E} = [(u^h, \omega^h)_{h\in H}, (Y^f)_{f\in F}, N, M]$ with $L = 1$, if and only if

1. $p$ satisfies (12)
2. $[(x^h, \theta^h, y^{f(h)})_{h\in H}, \nu_0\pi)]$ is the reduced form equilibrium for the real economy with entrepreneur $\mathcal{E} = [(u^h, \omega^h)_{h\in H}, (Y^f)_{f\in F}, V]$ where $V$ is defined by (13) and $\nu$ is defined below.

Observation 1. Proposition 1 states that the existence of monetary equilibrium in reduced form follows from the entrepreneur's real equilibrium in reduced form $\mathcal{E} = [(u^h, \omega^h)_{h\in H}, (Y^f)_{f\in F}, V]$ which is guaranteed under the assumptions of utility and initial endowments, such as the differentiation of the utility functions $U^h$ and the transformation function $T^{f(h)}$ representing technology and the assumptions of concavity and convexity of these respectively.

Observation 2. The 'real economy equilibrium' refers to the equilibrium point in the markets for goods and productive factors, where the allocation of resources is consistent with agentsâ preferences and available technologies. The 'reduced form' denotes a simplification that expresses this equilibrium in a more direct manner, after eliminating intermediate variables.

The next section analyzes whether monetary policy affects monetary equilibrium or not. Regarding the "effect", we consider the case of changes in $M$ affecting consumption equilibrium. More precisely, we have the following definition.

Definition 3. Let $[(x^h, \theta^h, y^{f(h)})_{h\in H}, (p, \pi)]$ be the reduced form of the economy in monetary equilibrium $\mathcal{E} = [(u^h, \omega^h)_{h\in H}, (Y^f)_{f\in F}, N, M]$. Monetary policy $M$ is said to be neutral if, for any change in monetary policy from $M$ to $M' \in \mathbb{R}^{1+S}_{++}$, there exists a financial policy $(\tilde{\theta}^h)_{h\in H}$ for agents and prices $(p', \pi')$, such that $[(x^h, \tilde{\theta}^h, y^{f(h)})_{h\in H}, (p', \pi')]$ is the monetary equilibrium of $\mathcal{E} = [(u^h, \omega^h)_{h\in H}, (Y^f)_{f\in F}, N, M']$. In another case, monetary policy is said to be non-neutral.

### *4.2. Two Types of Monetary Policy Neutrality*

Now, it is shown how the equivalence between the financial and monetary equilibrium given in Proposition 1 allows us to establish whether or not monetary policy is neutral in the sense of the definition. The result is given by the following proposition.

Proposition 2. Consider an entrepreneurial monetary economy
$\mathcal{E} = [(\sqcap^{\langle}, \omega^{\langle})\langle \in \mathcal{H}, (\mathcal{Y}^{\{})\{ \in \mathcal{F}, \mathcal{N}, \mathcal{M}]..$ Therefore, two types of changes in monetary policy will always be neutral. A change in the money supply in the first period and a uniform proportional change in the money supply in all states of the second period, such as:

1. $M \to M'(\beta_0, M), \forall\beta_0 > 0$ with $M'_o = \beta_0 M_0, M'_s = M_s, \forall s$.
2. $M \to M'(\beta, M), \forall\beta > 0$ with $M'_o = M_0, M'_s = \beta M_s, \forall s$.

Next, we define the phenomenon that occurs in the monetary economy, called inflation. Therefore, we have the following definition:

Definition 4. Inflation $\eta$ in monetary equilibrium, considered as a generalized increase in prices, $[(x^h, \theta^h)_{h\in H}, (p, \pi)]$ is a random variable defined by

$$\eta = (\eta_1, \ldots, \eta_S) = (\ldots, \frac{p_s}{p_0}, \ldots) \in R^S_+ \tag{14}$$

Proposition 2 states that inflation can be multiplied by scalars without changing the equilibrium allocations. More precisely, by $\beta$ the numerator, and by $\beta_o$, the denominator. But this is a consequence of two things: the assumption that agents do not carry money balances from one period to the next; and the fact that agents correctly anticipate the consequences of monetary policy by adjusting their portfolios to cancel the effects of anticipated inflation, $\eta$. In reality, what really matters is whether monetary policy affects the purchasing power of money $\nu$ defined in (12); and this is what Proposition 1 essentially says.

However, the neutrality or non-neutrality of money can be analyzed more conveniently in terms of the purchasing power $\nu$. The basic idea is as follows: all that matters are the equilibrium allocations of the real economy (where the financial structure is given by the matrix $V$ defined in (13)), and this has to do with the transfer space defined by the span $< V >$ which consists of all linear combinations of financial assets. Therefore, the following can be stated:

1. A change in purchasing power $\nu$ that does not affect $< V >$ is neutral since agents can adjust their portfolios to compensate for the change in the purchasing power of bond dividends.
2. Conversely, any change in purchasing power $\nu$ that âcontractsâ the market subspace $< V = [\nu]N >$ of the associated real economy generally affects the equilibrium allocations.

When $S = J$ occurs, it means that there are enough assets in the market for each state of nature. Since the position of $V = S$, since the diagonal matrix $[\nu]$ is always non-singular, monetary policy will always be neutral. In more technical terms, when financial markets are complete, in this specific dissertation model, they are always neutral. However, when markets are incomplete in the sense that the rank of $< V >$ is strictly less than $J$, monetary policy is not neutral. To prove this statement, 'differential topology' techniques are used, which are far from the scope of this masterâs dissertation.

However, below we provide detailed proof of the neutrality of money in the case of complete financial markets. Strictly speaking, the following theorem is a generalization of the results of [31] for the case of a single-owner business economy.

Theorem 1. If the financial markets of the business

monetary economy

$$\mathcal{E} = [(u^h, \omega^h)_{h\in H}, (Y^f)_{f\in F}, N, M]$$

are complete, then monetary policy is neutral. Proof. We begin by writing the fundamental theorem of "asset pricing" for the real economy generated by the monetary economy.

$$\frac{\pi_0}{p_0} = \sum_{s=1}^{S} \gamma_s \frac{N_s}{p_s} \tag{15}$$

Since markets are complete, the deflators $\gamma$ are unique.

On the other hand, arranging constraints (1) and (2) of the monetary business economy, we have:

$$p_0(x_0 - y_0 - \omega_0^h) = -\pi\theta \tag{16}$$

$$p_s(x_s - y_s - \omega_s^h) = N_s\theta \tag{17}$$

Compacting (16) e (17), using (15), we have:

$$\sum_{s=0}^{S} P_s(x_s - y_s - \omega_s^h) = 0 \tag{18}$$

where $P_o = p_o, P_s = \gamma_s p_s$.

Constraint (18) characterizes the budget set of a contingent market (or Arrow-Debreu) economy. Furthermore, the entrepreneurs' problems in reduced form are equivalent to this equilibrium in contingent markets, since they will always have a portfolio that satisfies (16) and (17) because the markets are complete.

Finally, to see that monetary policy is neutral, the following argument was made: when markets are complete, according to the analysis above, it is as if entrepreneurs signed contingent contracts in each state of nature, in particular dependent on the purchasing power of money $\nu_s = \frac{1}{p_s}$. Any change in monetary policy will lead to changes in both portfolios and asset prices, but will not affect the real economy.

In summary, let $(x^h, y^{f(h)}, \theta^h), (p, \pi)$ be an equilibrium of the monetary economy, where the injection of money is $M_0, M_1, \ldots, M_S$. Suppose there is a change in monetary policy, say that $M \to M'$. Such a monetary change will lead to a change in the purchasing power of $\nu \to \nu'$. Therefore, real returns change by $[\nu]N \to [\nu']N$. As long as financial markets are complete, entrepreneurs can always find new portfolios of $\theta \to \theta'$ that will finance the consumption plan $x_s^h - y_s^{f(h)} - \omega_s^h = \nu_s' N_s \theta'$. Asset prices will adjust by $\pi \to \pi'$ to reflect the new purchasing power of their dividends, and agents will do so in such a way that they can purchase the same consumption at date zero. That is $x_0^h - y_0^{f(h)} - \omega_0^h = -\pi'\theta'$. This completes the proof.

# 5. Fisher Separation Theorem

Fisher's separation theorem separates economic activities within the firm. More precisely, it postulates that the firm's investment decision is separate from the owner's decision. Therefore, the firm should only maximize its profit as its main motivation. This section aims to verify whether this famous and important result holds in an entrepreneurial economy with money. To achieve the objective, another equilibrium needs to be defined, in which economic activities are separated.

Definition 5. Profit maximization in the monetary economy for $\mathcal{E}$ is the financial and productive consumption plan $[(x^h, \theta^h); y^{f(h)}]_{h\in H}$ together with a price system for commodities and investments $(p, \pi)$, such that the following conditions must be satisfied:

1. Given $y^{f(h)}$, it follows that for each entrepreneur $h \in H$, the vector $(x^h, \theta^h)$ maximizes $u^h(x)$ subject to (1) and (2).
2. Given $\rho^h := \frac{\nabla u^h(x^h)}{D_{x_0} u^h(x^h)}$, for each firm $f(h)$, the vector $y^{f(h)}$ maximizes $\sum_{s=0}^{S} \rho_s^h p_s y_s$ subject to (3).
3. Market equilibrium. This is like stating,

$$\sum_{h\in H}(x^h - \omega^h) = \sum_{h\in H} y^{f(h)}, \sum_{h\in H} \theta^h = 0 \tag{19}$$

4.

$$p_s \sum_{h\in H}(x_s^h - y_s^{f(h)}) = M_s, s \in S^* \tag{20}$$

Now, we can state the result of this section. That is, understand whether a reduced-form monetary equilibrium for $\mathcal{E}$ is equivalent to a profit maximization in monetary equilibrium for $\mathcal{E}$.

Theorem 2. Consider an entrepreneurial monetary economy $\mathcal{E}$, in which the utility function $u^h$ is strictly quasi-concave and differentiable; and the transformation function $T^{f(h)}$ is quasi-convex, which is equivalent to stating that the technology $Y^{f(h)}$ is a convex set. Therefore, EMFR and MLEM are equivalent.

Proof. First, consider the Lagrangian associated with the entrepreneur's problem, when the consumption and production activities are not indistinguishable.

$$\begin{aligned} L^h = {} & u^h(x) - \lambda_0\Big(p_0(x_0 - y_0 - \omega_0^h) + \pi\theta\Big) \\ & - \sum_{s=1}^{S} \lambda_s\Big(p_s(x_s - y_s - \omega_s^h) - N_s\theta\Big) \\ & - \gamma T^{f(h)}(y) \end{aligned} \tag{21}$$

Second, let $[(x^h, \theta^h, y^{f(h)})_{h\in H}, (p, \pi)]$ be the EMFR. Thus, Item 1 of Definition 2 holds for the Kuhn-Tucker necessary conditions to be satisfied:

1. $x_s$

$$D_{x_s} u^h(x^h) = \lambda_s p_s,\ s = 0, 1, \ldots, S. \Leftrightarrow \rho_s^h = \frac{\lambda_s}{\lambda_0}\eta_s \tag{22}$$

2. $\theta$

$$-\lambda_0\pi + \sum_{s=1}^{S} \lambda_s N_s = 0 \Leftrightarrow \pi_j = \sum_{s=1}^{S} \frac{\rho_s^h N_s^j}{\eta_s}, \forall j \tag{23}$$

3. $y_s$

$$\lambda_s p_s = \mu D_{y_s} T^{f(h)}(y^{f(h)}) \Leftrightarrow \rho_s^h \eta_s = \frac{DT_s(y^{f(h)})}{DT_0(y^{f(h)})} \quad (24)$$

Lastly, let $[(x^h, \theta^h); y^{f(h)}]_{h\in H}; (p, \pi)]$. On the one hand, in Item 1 of Definition 5, it follows that the KT conditions hold and coincide with (22) e (23). On the other hand, in Item 2 of Definition 5, it follows that the KT conditions also hold and (24), consequently. Therefore, the MLEM satisfies the same conditions that EMFR does. The quasi-concavity characteristic of economic agents, entrepreneurs, and firms, are sufficient conditions to satisfy Definition 2. The converse also holds. That is, starting from EMFR, we arrive at the same condition that characterizes the MLEM. This concludes the proof of Theorem 2.

# 6. Conclusion

This study sought to investigate the impacts of monetary policy on a corporate economy, where firms have a single owner. To this end, the equilibrium model in incomplete markets of Magill and Quinzii (1994) was extended to include production in a simple monetary economy, where it is assumed that individuals cannot carry a balance in currency from one period to the next.

The results show the existence of equilibrium, the neutrality of money, and Fisher's separation theorem. The greatest contribution comes from the confirmation of the neutrality of monetary policy in the model and the validation of Fisher's separation theorem. In other words, the fact of being in an incomplete exchange environment influences the consumption trajectory of agents but does not change the operational structure of the firm.

As a suggestion for future research, changes could be made to the detailed horizon, relaxing the assumptions of currency carryover, incorporating private or state banks with fiscal policies into the model, and applying the Modigliani-Miller theorem. Such extensions could shed light on how financial intermediaries and government interventions reshape consumption dynamics, alter firm financing decisions, and potentially challenge the robustness of monetary neutrality in incomplete markets.

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# Abbreviations

| | |
|---|---|
| CB | Central Bank |
| EMFR | Reduced Form Monetary Equilibrium |
| MLEM | Monetary Equilibrium Profit Maximization |

# Acknowledgments

The authors would like to thank their colleagues from both the academic and professional environments for valuable discussions and support during the development of this work.

# Conflicts of Interest

The authors declare no conflicts of interest.

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