

Research Article

Epidemic Change Point Detection Using Nash Equilibrium

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Abstract

The emergence and spread of infectious diseases pose significant challenges to public health systems worldwide. Understanding the dynamics of epidemic outbreaks is crucial for effective intervention strategies. The goal of change point detection is to find time steps when the mean, standard deviation, or slope of the data changes from one value to another. This paper explores the concept of epidemic change points through the lens of Nash equilibrium. We propose a mathematical model that incorporates the dynamics of epidemic the change point's model into game theory. Game Theory refers to a language used to model choices made by purposive agents, where the outcomes of each player are influenced by the actions of other agents. Game theory is the science of strategy. It attempts to determine mathematically and logically the actions that players should take to secure the best outcomes. This article is worth reading because it demonstrates the use of game theory in estimating change points. It studies interactive decision-making, where the outcome for each participant or player depends on the actions of all. And it is interesting that the Nash equilibrium points of the two actors and the researcher are for optimizing their utility functions (minimizing loss functions). We demonstrate the applicability of our approach through simulations of hypothetical epidemics, revealing how game-theoretic strategies can enhance decision-making processes in real-time.

Keywords

Change Point, Epidemic Outbreaks, Game Theory, Nash Equilibrium

1. Introduction

Detection of change points is useful in modeling and prediction of time series and is found in application areas such as medical condition monitoring, climate change detection, speech and image analysis, and financial market analysis. In this field, the epidemic change point problem involves testing the null hypothesis of no change versus the alternative that two changes occur during the data sequence which forms a segment throughout the data sequence. Parameters of baseline model differ across this segment, see [3]. Let

$$x_t = \mu_t + \varepsilon_t, t \geq 1,$$

denote the observation model at which $\mu_t = \theta_0$ for $t = 1, \dots, k_0$ and $t = k_1 + 1, \dots, n$ and it is $\mu_t = \theta_1$ for $t = k_0 + 1, \dots, k_1$ where $\theta_1 \neq \theta_0$ and $1 \leq k_0 < k_1 \leq n$, for some sample size n . Here, ε_t is a sequence of stationary time series and k_0, k_1 are unknown change points as well as θ_1 and θ_0 are unknown parameters. The μ_t forms an epidemic change point model which has many applications in practice. Let $\tau = (k_0, k_1, \theta_0, \theta_1)^T$.

Game theory is a mathematical framework used for analyzing situations in which the outcomes depend on the actions of multiple decision-makers, known as "players." It provides a way to model strategic interactions where the choices of one

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player directly affect the outcomes for others. Game theory has applications in economics, political science, biology, psychology, and computer science, among other fields. In cooperative games, players can form binding agreements and coalitions, while in non-cooperative games, they cannot. In zero-sum games, one player's gain is exactly balanced by the losses of others. In non-zero-sum games, the total payoff can vary, allowing for mutually beneficial outcomes, see [11, 13]. In simultaneous games, players make decisions at the same time, while in sequential games; players make decisions one after another, allowing later players to consider the choices of earlier players, see [7]. There are many applications of game theory including in economics for analyzing competition between firms, auction designs, and market strategies. In political science for studying voting behavior, coalition formation, and international relations. In biology for understandings evolutionary strategies and animal behavior. In this note, it is assumed that the epidemic change point has occurred and it is interested to find k_0, k_1 , using one player and two players games, see [4, 8].

1.1. One Player Game

Following [1, 5], the least square function based on observation $x_t, t = 1, \dots, n$ is given by

$$L(\tau) = \sum_{t=1}^{k_0} (x_t - \theta_0)^2 + \sum_{t=k_1+1}^n (x_t - \theta_0)^2 + \sum_{t=k_0+1}^{k_1} (x_t - \theta_1)^2.$$

Assuming k_0, k_1 are known, then estimates of θ_0, θ_1 are given by

$$\hat{\theta}_0 = \frac{\sum_{t=1}^{k_0} x_t + \sum_{t=k_0+1}^n x_t}{n - k_1 + k_0}, \hat{\theta}_1 = \frac{\sum_{t=k_0+1}^{k_1} x_t}{k_1 - k_0},$$

respectively. Then the least square function reduces to the

$$L(k_0, k_1) = \sum_{t=1}^{k_0} (x_t - \hat{\theta}_0)^2 + \sum_{t=k_1+1}^n (x_t - \hat{\theta}_0)^2 + \sum_{t=k_0+1}^{k_1} (x_t - \hat{\theta}_1)^2.$$

One can see that

$$\sum_{t=1}^n (x_t - \bar{x}_n)^2 = L(k_0, k_1) + (n - k_1 + k_0)(\hat{\theta}_0 - \bar{x}_n)^2 + (k_1 - k_0)(\hat{\theta}_1 - \bar{x}_n)^2,$$

where $\bar{x}_n$ is the sample mean of $x_t, t = 1, \dots, n$.

Minimizing $L(k_0, k_1)$ is equivalent to maximize the below objective function

$$R^2(k_0, k_1) = (n - k_1 + k_0)(\hat{\theta}_0 - \bar{x}_n)^2 + (k_1 - k_0)(\hat{\theta}_1 - \bar{x}_n)^2.$$

Notice that

$$\hat{\theta}_0 = \left(\frac{n}{n - k_1 + k_0} \right) \bar{x}_n - \left(\frac{k_1 - k_0}{n - k_1 + k_0} \right) \hat{\theta}_1.$$

Let $l = k_1 - k_0$. Substituting $\hat{\theta}_0$ in $R^2(k_0, k_1)$, it is changed to $R(k_0, l)$, where

$$R(k_0, l) = \sqrt{\frac{n}{l(n-l)}} \sum_{t=k_0+1}^{k_0+l} (x_t - \bar{x}_n).$$

The game is defined as follows. First, it is assumed that k_1 is known, $u_1(l)$ is maximized (denote the maximizing value by $\tilde{l}$), where

$$u_1(l) = \frac{\sum_{t=k_1-l+1}^{k_1} (x_t - \bar{x}_n)}{\sqrt{l(n-l)}}.$$

By replacing $\tilde{k}_0 = k_1 - \tilde{l}$ in $u_1(l)$, then it is enough to maximize

$$\sum_{t=k_1-\tilde{l}+1}^{k_1} (x_t - \bar{x}_n),$$

with respect to k_1 to derive $\tilde{k}_1$.

Proposition 1. Maximizing values $\tilde{k}_0$ and $\tilde{k}_1$ are close to actual values k_0, k_1 .

Proof. It is known that the sequential maximization of bi-variable function is equivalent to maximization of function with respect to both variables, simultaneously, see [6, 14]. This fact and convergence (in probability) of least square estimates guarantees the convergence (in probability) of maximizing $\tilde{k}_0, \tilde{k}_1$ to true values of k_0, k_1 , as n goes to infinity, see [10].

1.2. Two Players Game

The previous section deals with the one player game. Here, consider two players who the first player (assuming k_1 is known) searches for k_0 throughout $x_t, t = 1, \dots, k_1$ and the second player assumes k_0 is known and searches for the location of k_1 starting with x_n and ending x_{k_0} , see [12]. Without loss of generality, assume that $\theta_0 < \theta_1$. Both players use the *cusum* detection process defines by

$$\hat{k}_0 = \operatorname{argmax}_{1 \leq k \leq k_1} \frac{\sum_{t=1}^k (x_t - \bar{x}_{k_1})}{\sqrt{k(k_1 - k)}},$$

$$\hat{k}_1 = \operatorname{argmax}_{1 \leq k \leq k_0} \frac{\sum_{t=1}^k (y_t - \bar{y}_{k_0})}{\sqrt{k(k_0 - k)}},$$

where $y_t = x_{n-t+1}$ and $\bar{y}_{k_0} = \frac{1}{n-k_0} \sum_{t=1}^{n-k_0} y_t$.

Nash equilibrium is a concept in game theory that describes a situation in which players in a strategic game reach a point where no player can benefit by unilaterally changing their strategy, assuming that the other players' strategies remain

unchanged. In other words, in Nash equilibrium, each player's strategy is optimal given the strategies of all other players. Nash equilibrium can exist in pure strategies (where players choose one strategy with certainty) or mixed strategies (where players randomize over strategies). Not every game has Nash equilibrium in pure strategies, but every finite game has at least one Nash equilibrium in mixed strategies, see [9]. A classic example of Nash equilibrium is the Prisoner's Dilemma, where the optimal strategy for both players, given the strategy of the other, leads them to betray each other, resulting in a suboptimal outcome for both. Understanding Nash equilibrium helps in analyzing competitive situations in economics, politics, biology, and many other fields. Here, again, the best response functions are derived and Nash equilibrium estimates of k_0, k_1 are proposed.

Proposition 2. Nash equilibriums are proposed by $\hat{k}_r = k_r; r = 0, 1$.

Proof. It is easy to see that both

$$\frac{\sum_{t=1}^k (\varepsilon_t - \bar{\varepsilon}_{k_1})}{\sqrt{k(k_1 - k)}}, \frac{\sum_{t=1}^k (\zeta_t - \bar{\zeta}_{k_0})}{\sqrt{k(k_0 - k)}}$$

are $o_p(1)$, where $\zeta_t = \varepsilon_{n-t+1}$. This fact implies that both of above expressions are close to their expectations. One can see that

$$\hat{k}_0 = \min(\hat{k}_1, k_0), \hat{k}_1 = \max(k_1, \hat{k}_0).$$

These equations are hold, if and only if $\hat{k}_r = k_r; r = 0, 1$.

In the next section, the simulated Nash equilibriums are derived. Section 3 concludes.

2. Simulated Equilibriums

Here, 4 simulation-based examples are studied.

Example 1. Let $n = 100, k_0 = 35$ and $k_1 = 77 \in \{60, \dots, 77\}$. Assume that ε_t 's come from normal distribution with zero mean and variance 0.5. Suppose that $\mu_0 = 0$ and $\mu_1 = 1$. The following R program gives the equilibrium values of k_0, k_1 .

```
cum <- function(y,m){
  I <- 1:(m-1)
  coef <- sqrt(I*(m-I))
  y <- y[1:(m-1)]
  z <- abs(cumsum(y-mean(y)))/coef
  amax <- which(z==max(z))
  return(amax)
}
x <- sqrt(0.5)*c(rnorm(35), rnorm(38,1,1), rnorm(27))
k0 <- 35; k1 <- c(60:77); L <- length(k1)
rpl <- rep(0,L)
for(k in 1:L){
  a <- k1[k]-1
  xk <- x[1:a]
  rpl[k] <- cum(xk,a)
```

```
}
plot(k1,rpl,type='l')
```

Figure 1 gives the best response of $\hat{k}_0$ based on various values of k_1 .

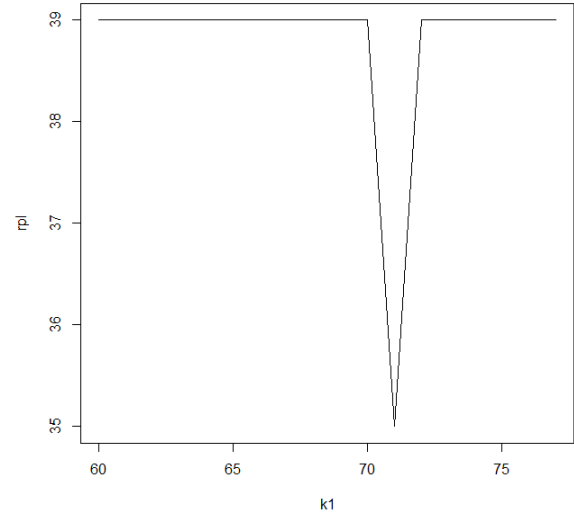


Figure 1. Best response of $\hat{k}_0$.

Example 2. Taken from scenario 3 of [2]. Consider Heavy tailed data with one signal segment where n data points were drawn from the generalized t distribution as

$$x_t = T(3) + \theta_t,$$

where $T(3)$ has t -student distribution with 3 degrees of freedom and $\theta_t = 2$ for $k_0 = 0.2n$ and $k_1 = 0.6n$ and zero, otherwise. The following figure shows the convergence of $\frac{\hat{k}_r}{n}; r = 0; 1$ to of $\frac{k_r}{n}; r = 0; 1$ based on various values of n 's. It is seen that both convergences occur in good rate of convergences.

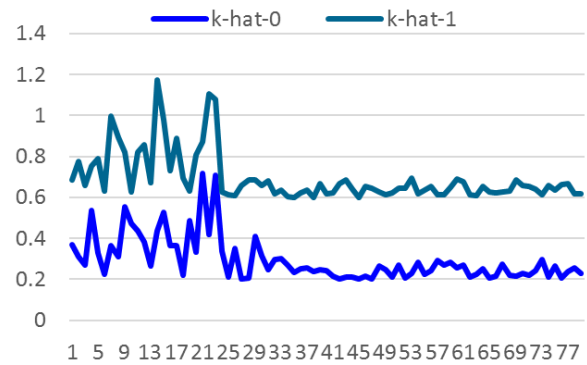


Figure 2. Rates of convergences.

Example 3. Carbon monoxide is a common air pollutant,

see [12]. It can cause harmful health effects by reducing oxygen delivery to the body's organs and tissues. The effects of its exposure can vary greatly from person to person depending on age, overall health and the concentration and length of exposure. It is a priority pollutant that is suitable as an indicator for assessing indoor air quality, see [15]. Here, daily concentrations of carbon monoxide in the Tehran are studied from November 5, 2024 through April 10, 2025, including 156 observations. Data units are $\frac{mg}{m^3}$. The data are a part of a large dataset (available at <https://aqicn.org/city/tehran/>). The average carbon monoxide is 884 for $t \leq 45$, and $106 \leq t$, and it is 933 for $46 \leq t \leq 106$. Here, ε_t 's are normally distributed with zero mean and 28 unit standard deviations. This series is plotted as follows:

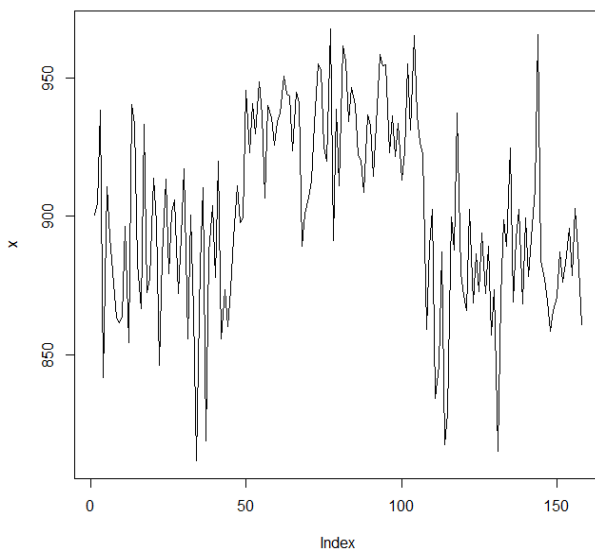


Figure 3. Carbon monoxide in average.

Using both above mentioned methods, the change points are estimated at 44, 110, respectively.

Example 4. The COVID-19 pandemic has had a profound impact on the world, causing tens of millions of deaths, overwhelming healthcare systems, and disrupting societies and economies. The rapid spread of the corona-virus led to many scientific developments and policies, from rapid testing and vaccines to social distancing and financial support, but an uneven global response, with significant disparities in healthcare access, economic responses, and outcomes. Reliable data has been crucial to effectively track and respond to the pandemic and guide public health efforts, research, and policies.

In this section, we study the monthly ratio of deaths to the population of Iran for a period before the pandemic, during the pandemic, and after the pandemic, *i.e.*, throughout 2016 to 2025, including 108 observations. For 2016, the average mortality ratio is 5.3 percent in 1000 persons while this ratio is 7.1 and 5.8 for pandemic and after pandemic periods. The

following figure presents the epidemic structure of this time series. While using the game theoretic approach, the $\hat{k}_0 = 39$ and $\hat{k}_1 = 80$ are estimated.

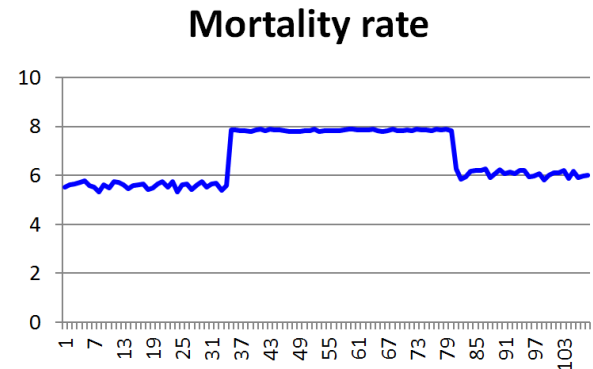


Figure 4. Monthly mortality rate.

3. Concluding Remarks

In conclusion, this paper has explored the intricate dynamics of epidemic change points through the lens of game theory, offering a novel framework for understanding the strategic interactions among various stakeholders during an epidemic. Our findings highlight the critical role of individual behavior and collective decision-making in shaping epidemic trajectories. By modeling the competition and cooperation between public health authorities, individuals, and communities, we have illuminated how game-theoretic principles can inform strategies to effectively mitigate the spread of infectious diseases. The analysis presented here underscores the potential of utilizing game theory not only to predict potential change points in epidemic progression but also to design and implement interventions that align incentives at various levels. This approach can guide policymakers in crafting communication strategies that resonate with the public, fostering compliance with health guidelines and building resilience within communities.

Moreover, our research sets the stage for further interdisciplinary work, encouraging collaboration between epidemiologists, game theorists, and behavioral scientists. As we advance our understanding of epidemic dynamics, it is essential to continue refining our models and integrating real-world data, which will enhance their predictive power and applicability. In light of recent global health crises, the urgency of developing robust frameworks for epidemic response has never been more apparent. We hope that this work inspires future studies that leverage game theory to address complex public health challenges, ultimately paving the way for more effective and adaptive interventions in the face of emerging infectious diseases. Through continued innovation and collaboration, we can better prepare for and respond to the unpredictable nature of epidemics, safeguarding public health and promoting societal well-being.

Abbreviations

<i>abs</i> (in Example 1)	Absolute Value Function in <i>R</i> Language
<i>rnorm</i> (in Example 1)	Normal Random Number Generator in <i>R</i> Language
<i>sqrt</i> (in Example 1)	Square Root Function in <i>R</i> Language
<i>cumsum</i> (in Example 1)	Cumulative Sum Function in <i>R</i> Language

Author Contributions

Reza Habibi is the sole author. The author read and approved the final manuscript.

Conflicts of Interest

The author declares no conflicts of interest.

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